

FINITE ELEMENT MODELING OF COMPOSITE PLATES WITH TOTAL
IMPERFECT INTERFACE SUBJECTED TO IMPACT LOADING

MOHAMUD ABDIRAHMAN IBRAHIM

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This project report is dedicated to my family

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ABSTRACT

This project report is devoted to the modeling of the impact behavior of laminated composite plates, with a special emphasis on incorporating interfacial imperfection. In most analytical and numerical works on composite materials, a perfect interface between adjacent laminae is assumed which implies continuous displacements and tractions across laminate. In many cases of practical interest, however, the assumption of a perfect interface is inadequate. This project report aims at the finite element formulation for the effects of different imperfect bonding intensities on the response of a laminated plate subjected to a transverse low velocity impact using MATLAB. In this study, the imperfect bonding is incorporated into the formulation by introducing an imperfection factor, R , to the interface stiffness modeled using a well-defined virtually zero-thickness interface element. The kinematically consistent mass formulation is used to formulate the mass matrix and the impact force is evaluated using a simple linear model. Newmark beta method, an implicit method of direct integration, is used in the present study for the solution of the linear transient response of the plate. From analysis, it is found that the fundamental nondimensional frequency, absorbed energy and the central deflection increase with an increase in imperfection factor. Besides, the orientation of laminae is also observed to affect the impact behavior of the laminated composite plate.

ABSTRAK

Projek ini ditumpukan kepada pemodelan tingkah laku impak plat komposit berlapis, dengan penekanan khas pada ketidaksempurnaan antaramuka. Dalam kebanyakan kerja analitikal dan berangka bahan komposit, antaramuka yang sempurna di antara lamina bersebelahan telah diandaikan yang mana sesaran dan daya tarikan berterusan merentasi lamina telah ditunjukkan. Walaubagaimanapun, dalam kebanyakan kes yang mempunyai kepentingan praktikal, andaian antaramuka yang sempurna adalah tidak mencukupi. Tesis ini bertujuan untuk menghasilkan rumusan unsur terhingga bagi kesan kekuatan berbeza ikatan tak sempurna kepada tindakbalas plat berlapis apabila dikenakan impak halaju rendah melintang dengan menggunakan MATLAB. Dalam kajian ini, ikatan tak sempurna telah dimasukkan ke dalam rumusan dengan memperkenalkan faktor ketidaksempurnaan, R , kepada kekukuhan antaramuka yang dimodelkan dengan unsur antaramuka ketebalan sifar. Rumusan jisim konsisten secara kinematik telah digunakan untuk merumuskan matriks jisim dan daya impak telah dinilai menggunakan model linear mudah. Kaedah Newmark beta, satu kaedah implisit kamiran langsung, telah digunakan di dalam kajian ini untuk penyelesaian tindakbalas transien linear plat. Daripada analisis, kajian ini mendapati bahawa frekuensi asas tanpa-dimensi, tenaga diserap dan pesongan tengah meningkat dengan peningkatan dalam faktor ketidaksempurnaan. Selain itu, orientasi lamina juga telah didapati mempengaruhi tingkah laku impak plat komposit berlapis.

TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENT	iv
	ABSTRACT	v
	ABSTRAK	vi
	TABLE OF CONTENTS	vii
	LIST OF TABLES	x
	LIST OF FIGURES	xi
	LIST OF SYMBOLS	xiv
1	INTRODUCTION	
	1.1 Background of Study	1
	1.2 Problem Statement	3
	1.3 Objectives	4
	1.4 Scope of the Study	4
	1.5 Significance of the Study	5
2	LITERATURE REVIEW	6
	2.1 Introduction	6
	2.2 Laminated Composite Plates	6
	2.3 Laminated plate theories	8
	2.3.1 Classical lamination theory	8
	2.3.2 First Order Shear Deformation Theory	10
	2.3.3 Higher Order Shear Deformation Theory	12

2.4	Impact behavior of laminated plates	13
2.5	Impact behavior characterization	15
2.5.1	Olsson mass criterion	15
2.5.2	Impact velocity criterion	16
2.5.3	Impact characterization diagram	17
2.6	Impact damage	18
2.6.1	Matrix cracking	19
2.6.2	Delaminations	20
2.6.3	Permanent indentation	21
2.6.4	Fiber failure	22
2.7	Interface element	22
2.8	Imperfect interface	24
3	RESEARCH METHODOLOGY	26
3.1	General	26
3.2	Stiffness of Lamina	29
3.3	Stiffness of Interface	39
3.4	Element Stiffness Matrix of Laminate	45
3.5	Global Stiffness Matrix of Laminate	47
3.6	Mass matrix	49
3.7	Impact Load	49
3.8	Boundary condition	51
3.9	Newmark Beta Method	53
4	RESULTS AND DISCUSSION	53
4.1	General	53
4.2	Numerical Model	54
4.3	Validation and Convergence of Results	56
4.4	Frequency and Vibration Mode	59
4.5	Impact Behavior	62
5	CONCLUSIONS AND RECOMMENDATION	66
5.1	Conclusion	66
5.2	Recommendation	67

REFERENCES

68

APPENDICES A - C

71 - 79

LIST OF TABLES

TABLE	TITLE	PAGE
2.1	Constant correspond to FSDT and HSDT.	12
4.1	Mechanical properties of the composite plate	55
4.2	Central deflection of laminate plate	57
4.3	Comparison of nondimensional frequency	58
4.4	Non-dimensional frequencies corresponding to imperfection factor	59

LIST OF FIGURES

FIGURE	TITLE	PAGE
2.1	Coordinate System	7
2.2	Laminated plate acted upon by loads	8
2.3	Kinematics of deformation in the x - z plane	9
2.4	Kinematics of deformation in the x - z plane (FSDT)	11
2.5	Plate responses under impact loading, (a) local, (b) global	14
2.6	Comparison between boundary-controlled and wave-controlled Impacts	15
2.7	Mass criterion for impact events	16
2.8	Impact characterization diagram	17
2.9	Low velocity impact damage	18
2.10	Type of matrix cracks in a [0/90/0] laminated composite plate	19
2.11	Projected damage patterns	20
2.12	Typical distribution of peanut shaped delaminations	21
2.13	A typical interfacial layer and interface model	23
2.14	Interphases and bonding-layers	23
2.15	Influence of imperfect interfaces on transient responses	25

3.1	Physical view of the laminate structure with an interface layer	26
3.2	Discretized element and the sub element	27
3.3	Computation of ABD matrix of a lamina	27
3.4	Finite Element procedure for the analysis of a two layered cross-ply laminate with interface layer	28
3.5	Distance of lamina surface from the mid plane of laminate	32
3.6	Coordinate system and degree of freedoms (DOF) of lamina sub element	33
3.7	2 x 2 Gauss quadrature rule	39
3.8	Coordinate system and DOF of zero-thickness interface sub element	40
3.9	2 x 2 Gauss quadrature rule	44
3.10	Degree of freedom (DOF) of the nodes in each sub element	45
3.11	Coordinate system of one local laminate plate element and DOF at each node	46
3.12	Rearrangement of node numbering	47
3.13	Global node numbering for 2 x 2 elements with each node having 5 DOF	47
3.14	Global node numbering for 2 x 2 elements (lower nodes).	48
3.15	Global node numbering for 2 x 2 elements (upper nodes)	48
3.16	Fixed end at all edges of laminate plate	51
4.1	A basic model of impact problem	55
4.2	Convergence of the lowest non-dimensional frequency	56
4.3	Central deflection of laminate plate subjected to transverse loading	58

4.4	First and second mode shapes for $R=0.000001$	60
4.5	First and second mode shapes for $R=0.5$	60
4.6	First and second mode shapes for $R=0.999999$	60
4.7	Fundamental frequency against laminate angle	61
4.8	First and second mode shapes for $[0/0]$ laminate	61
4.9	First and second mode shapes for $[45/0]$ laminate	62
4.10	Effect of imperfection factor on maximum central deflection	63
4.11	Transient response of the plate for imperfection factor, $R=0.999999$ and $R=0.000001$	63
4.12	Force-displacement curve for different imperfection factors	64
4.13	Transient response for different laminate angles $[0/0]$	65
4.14	Transient response for anti-symmetric plate	65

LIST OF SYMBOLS

V_f, V_m	-	Volume fraction of fiber and matrix respectively
E_f, E_m	-	Young Modulus of fiber and matrix respectively
G_{12f}, G_m	-	Shear modulus of fiber and matrix respectively
ν_{12f}, ν_m	-	Poisson's ratio of fiber and matrix respectively
E_1	-	Longitudinal Young's modulus
E_2	-	Transverse Young's modulus
G_{12}	-	In-plane shear modulus
ν_{12}	-	Poisson's ratio
ζ	-	Measure of fiber reinforcement coefficient that depends on the fiber geometry, packing geometry, and loading conditions. The value of ζ is taken as 2 for E_2 calculation while 1 for G_{12} calculation.
Q_{ij}	-	Lamina stiffness matrix
\bar{Q}_{ij}	-	Transformed stiffness matrix
N	-	In-plane force
M	-	In-plane moment
ε^0	-	Mid-plane strain
κ	-	Mid-plane curvature
A_{ij}, B_{ij}, D_{ij}	-	Laminate extensional stiffness, laminate-coupling stiffness, and laminate-bending stiffness respectively

u, v, w	-	Displacement in x, y, z direction respectively
$\varnothing_y, \varnothing_x$	-	Rotation about the x, y direction respectively
N_i, N_o	-	Shape function for in-plane and out-of-plane degree of freedom respectively
$[B]$	-	Strain-displacement matrix
$[N]$	-	Shape function
$[B_i]$	-	In-plane element strain matrix
$[B_o]$	-	Out-of-plane element strain matrix
ζ_i, η_i	-	Coordinates of node
ζ, η	-	Value of Gauss point
a, b	-	Length and width of element respectively
$[K]$	-	Stiffness matrix
$\{F\}$	-	Impact load
$\{d\}$	-	Nodal displacement of the laminate
d_{lower}	-	Interpolated displacement of node at lower surface of the zero-thickness element
d_{upper}	-	Interpolated displacement of node at upper surface of the zero-thickness element
\hat{d}_{lw}	-	Nodal displacement of node at lower surface of the zero-thickness element
\hat{d}_{up}	-	Nodal displacement of node at upper surface of the zero-thickness element
$[B_{int}]$	-	Interface element strain matrix
σ	-	Stress
ε	-	Strain

D	-	Constitutive matrix
h	-	Thickness of interface element
J	-	Jacobian matrix
w_i, w_j	-	Weight of i^{th} and j^{th} Gauss point
$f(\zeta_i, \eta_j)$	-	Function of i^{th} and j^{th} Gauss point
t	-	Element thickness
$w_{,x}, w_{,y}$	-	Lateral displacement in partial differentiation with respect to x and y respectively
$\{v\}$	-	Element nodal displacement
R	-	Imperfection factor

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Fiber reinforced composites are widely used as structural components in various fields such as aerospace, civil engineering, marine, automotive, wind power industry and others. They are mainly used in applications requiring high stiffness and strength as well as low weight. They consist of matrix and reinforcement. The reinforcements carry the loads and the stresses, whereby the matrix distributes the force and stress uniformly among the reinforcement and binds the reinforcement and also prevents the fibers from external damage. Composite materials are commonly used in structures that demand a high level of mechanical performance. Their high strength to weight and stiffness to weight ratios facilitates the development of lighter structures. Basically, the two main types of fiber-reinforced composite materials are short fiber-reinforced materials and continuous fiber-reinforced materials. The latter one is usually established as layers and assemblies of layers of fibrous composite materials are known as composite laminates.

Laminated composite structures offer a variety of potential advantages such as high stiffness and strength to weight ratios, corrosive resistance, excellent fatigue resistance, long durability and many other superior properties compared to the conventional metallic materials. However, composite materials are susceptible to damage from impact loading especially those normal to the plane of the laminate. The impact damage mode in laminated composites usually consists of local permanent deformations, fiber breakage, delamination and matrix cracking. The damage due to impact can cause large drops in the strength and stability of the structure. Therefore, their behavior under impact is a concern, since impacts do occur during manufacture, operation and repairing process.

Impact on composites is a very complex phenomenon, for it is a function of many parameters. The response of laminated composites to an impact load depends on the impactor parameters, the material properties of the structure and the environmental conditions. The parameters of the impactor that significantly affect the response include the velocity, energy, shape, size, material properties and incidence angle. The structural parameters include shape, thickness, size, lamina type, material properties, density, layup sequence and boundary conditions.

Considerable research has been performed concerning the response of laminated plates to impact load. There is a tendency to assume the bond between the laminates of a laminated plate is perfect, in practice, however, the bonding between layers is imperfect due to the existence of some micro-voids, micro-cracks, and other kinds of defects at the interface. Theoretically, this imperfect bonding in composite laminate causes discontinuous displacement across the interface which is also known as interlayer slip. The existence of this interlayer slip will thus contribute to the stiffness degradation of composite laminate since the components in bonded layers are separated. Therefore, an accurate prediction of the behavior of a laminated composite plate requires the consideration of the effect of imperfect bonding rather than making an assumption that the bonding between layers is completely perfect.

1.2 Problem Statement

Laminated composites have been extensively used in different fields including civil engineering due to their inherent advantages, like high strength to stiffness ratio. In many cases, thin and large rectangular plates are subjected to impacts during the fabrication, operation, and repairing processes. The impact load may cause significant damage embedded within the materials of the laminated composite which can reduce the strength and stiffness of the materials depending on the extent of the damage. Therefore, the knowledge of impact response in laminated composites is critically important for the application of the materials in structural design.

There are conventional theories and refined theories of laminated composite materials. In most of the theories, a perfect interface between adjacent laminae is assumed which implies continuous displacements and tractions across the laminates. Therefore, the interface properties and structures are eliminated, despite the fact that the behavior of composite materials is significantly affected by the properties of interfaces. However, the assumption of a perfect interface is not always sufficient. There are many ways in which the imperfection can exist in laminated plates. Such as the presence of interfacial damage caused by fatigue and environmental or interphase material which may be due to chemical interaction between the constituents.

It is especially important to know how the interface imperfection affects the impact response of the laminated plate, whether it occurs as part of the intended use of structure or unintentional as a result of dropping or striking it with another object. Therefore, perfect bonding is an ideal condition of laminated composite plates but it is never a realistic condition since micro crack and cavities may be introduced into the bond in the process of manufacture or service. This might affect the stability of the structure during the service lifetime. Therefore, to avoid the local failure of bond or the whole collapse of structure, the effect of imperfect interfaces should be taken into consideration in design or analysis.

1.3 Objectives

This study is concerned with the behavior of laminated composite plates when subjected to impact loading. The main objectives of this study can be summarized as follows:

- 1) To formulate a laminated composite plate with an interface element by applying finite element method and develop the corresponding MATLAB code.
- 2) To study the impact response of a laminated plate with different degrees of interface imperfection and fiber orientation.

1.4 Scope of the Study

The study focuses on the modeling of laminated composite plates with interface imperfection subjected to transverse impact loading using a commercial programming language, MATLAB. The interface imperfection considered is uniformly distributed over the interface. The materials of the rectangular plate considered are generally fiber and matrix with linear elastic material behavior. The unidirectional fiber in the lamina is E-Glass and the matrix material is Epoxy (3501-6). The plate was considered as a combination of two laminae with the same boundary condition and thickness. Kirchoff's Classical Laminate Theory was adopted to model the flat, thin plate. Each of the nodes of laminate plate element consists of five degrees of freedoms, which are 3 displacement degrees of freedom and 2 rotational degrees of freedom. In this study, the stiffness matrix of the interface element is calculated by applying 2x2 Gauss quadrature rule. The impact event considered in this study is that of low velocity impact applied at the center of a plate with fully fixed boundaries.

1.5 Significance of the Study

Fiber reinforced composites are finding increasing applications as primary structural components in many different fields. In some of the applications, composites are subjected to impact loads. In order, to design laminated composites for resistance to foreign object impact damage, an understanding of the dynamic response of composite structures under impact loading becomes a requirement. In most analytical and numerical work on composite materials, a perfect interface between adjacent lamina is assumed which implies continuous displacements and tractions across it. Therefore the interface properties and structures are eliminated, despite the fact that the behavior of composite materials is significantly influenced by the properties of interfaces. In many cases of interest, however, the assumption of a perfect interface is inadequate.

An analysis with higher accuracy, relatively to the reality condition, is to be established to have better accuracy in structural performance prediction. Therefore, the formulation for a laminate plate element should take into account the effect of imperfect bonding in the prediction of material behaviors of laminated composite plate. The presence of an imperfect interlaminar interface gives a more realistic description of all phenomena occurring at interlayer and the behavior of the laminated composite plate.

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