STABILITY OF TRIAXIAL WEAVE FABRIC COMPOSITES EMPLOYING FINITE ELEMENT MODEL WITH HOMOGENIZED CONSTITUTIVE RELATION

NORHIDAYAH RASIN

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Master of Engineering (Structure and Material)

Faculty of Civil Engineering
Universiti Teknologi Malaysia

JULY 2013

To my beloved parents

Rasin B. Mat @Abdul Kadir

&

Salmi Bt.Jamino

my beloved husband

Rozali Zakaria

And my beloved son

Muhammad Naim Redza

ACKNOWLEDGEMENT

First and foremost, praises to God, for giving me health and times to complete this course of research. During this research, I have given all my efforts and commitment to ensure the best for my thesis.

I would like to take this opportunity to express my appreciation to my supervisor, Dr. Ahmad Kueh Beng Hong, for his guidance, critics, and compassion. Also, I want to thank my co-supervisor, Dr. Airil Yasreen Mohd Yassin for his attention in assisting me to carry out this research. Without their supports and encouragement, it would be hard for me to complete this thesis.

I also want to thank my beloved parents, husband and family members for their prayers and moral supports during my study. This appreciation also goes to all staff of Steel Technology Centre (STC), lecturers of Civil Engineering Faculty, and colleagues for their helps either directly or indirectly.

Through this research, I hope that these precious knowledge and experiences can be used in my future career.

ABSTRACT

This study examines numerically the uniaxial stability of triaxial weave fabric (TWF) composites employing finite element (FE) model with homogenized constitutive relation. TWF, which presents high specific-strength and stiffness due to its porous and lightweight properties, was previously modelled using solid elements or plybased approach, and thus making computation considerably complex and time-consuming. To circumvent these issues, the current FE formulation is of geometrical nonlinearity employing Newton-Rhapson method where TWF unit cell is treated as a standalone non-conforming composite plate element making use of the homogenized ABD stiffness matrix, where A_{ij}, B_{ij}, and D_{ij} indicate the extensional, coupling, and bending stiffness, respectively in which degree of freedom has been greatly reduced. By means of Matlab program, the currently formulated model has demonstrated good agreement with existing numerical and experimental results from literature in terms of elastic properties. For the buckling analysis, four types of boundary conditions are explored: fully simply supported, fully-clamped, free-simply supported and free-clamped. High dependencies of post-buckling patterns of compression load against both maximum and minimum deflections on numerous aspect ratios from 0.25 to 5 are observed in TWF, from which a characteristic equation has been defined for practical convenience before the occurrence of post-buckling. Such equation is described on the basis of the critical buckling load, N_{max}, and stiffness factor, S, the best characterization of which is expressed in a logarithmic manner. The study has recognized that the buckling characteristics correlate directly to TWF's aspect ratios and level of rigidity imposed through the boundary conditions.

ABSTRAK

Kajian ini menyelidik secara berangka kestabilan satu arah komposit fabrik tenunan tiga paksi menggunakan model unsur terhingga dengan hubungan juzuk seragam. Komposit fabrik tenunan tiga paksi yang menunjukkan kekuatan tentu dan kekukuhan yang tinggi daripada ciri-cirinya yang berliang dan ringan, telah dimodelkan sebelum ini dengan menggunakan unsur pepejal atau berasaskan pendekatan berlapis dan ini menjadikan pengiraannya kompleks dan memakan masa. Oleh itu, unsur terhingga tak linear secara geometri telah dirumuskan menggunakan Kaedah Newton-Rhapson di mana sel unit komposit fabrik tenunan tiga paksi telah dianggap sebagai unsur plat komposit tak-selaras bersendiri menggunakan matriks kekukuhan ABD seragam di mana A_{ij}, B_{ij}, dan D_{ij} masing-masing merupakan kekukuhan pemanjangan, gandingan dan lenturan di mana darjah kebebasan telah dikurangkan. Dengan menggunakan program Matlab, rumusan model kajian ini telah menunjukkan persetujuan yang baik dengan keputusan berangka dan eksperimen yang sedia ada daripada literatur dalam bentuk sifat-sifat elastik . Untuk analisis lengkokan, terdapat empat jenis keadaan sempadan yang diterokai: disokong mudah penuh, diapit penuh, bebas-disokong mudah dan bebas-diapit. Kebergantungan tinggi corak pasca lengkokan daya mampatan terhadap kedua-dua pesongan maksimum dan minimum kepada nisbah aspek daripada 0.25 kepada 5 telah diperhati, yang mana satu persamaan ciri telah dirumuskan sebelum berlakunya pasca lengkokan. Persamaan tersebut dihuraikan berdasarkan beban lengkokan kritikal, N_{max}, dan faktor kekukuhan, S, di mana pencirian terbaik adalah dinyatakan dalam bentuk logaritma. Kajian ini telah mengenalpasti bahawa ciri-ciri lengkokan berhubungkait secara langsung terhadap nisbah aspek dan tahap kekukuhan komposit fabrik tenunan tiga paksi yang dikenakan melalui keadaan sempadan.

TABLE OF CONTENTS

5 CONCLUSION AND RECOMMENDATION

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

a - Maximum value of y-axis

α^o - Directions of tow

A_{ij} - Extensional stiffness

b - Slope of curve over value of a

B - Strain-displacement matrix

B_{ij} - Coupling stiffness

D - Constitutive matrix

D_{ij} - Bending stiffness

F - Force vector

I_{Nmax} and I_s - Intercept of y-axis from N_{max} and S data

K - Stiffness matrix

l_x/l_y - Plate ratio

L_x, L_y - Length of plate in x and y directions

$m_{N,max}$ and m_s - Slope of the straight line from N_{max} and S data

M - Mass matrix

M_{xx}, M_{yy} - Moment resultants in x and y directions

M_{xy} - Twisting moment

n - Number of Gaussian Quadrature points

N	-	Axial load
N_{max}	-	Maximum buckling load
N_x	-	Compressive or axial load, N in x direction
N_{xx}, N_{yy}	-	Force resultants in x and y directions
N_{xy}	-	Shearing force resultant
S	-	Slope of curve
S_x	-	Tensile stiffness
t_{κ}	-	Tow thickness
u and v	-	In-plane displacements in the direction of x and y
V_i^f	-	Volume segmentation of tows
w	-	Transverse displacement the z-direction
x, y and z	-	Directions of Cartesian coordinates system
$\hat{C}_{ij}^{k}$	-	Stiffness terms of TWF tow sets
γ_{xy}	-	Shear strain
ε_x and ε_y	-	Strains in x and y directions
κ_x and κ_y	-	Curvatures in x and y directions
κ_{xy}	-	Twisting curvature
ν_{xy}	-	Poisson's ratio
ξ,η	-	Natural coordinates
φ_j^e	-	Interpolation functions for nonconforming rectangular elements
φ_x, φ_y	-	Rotation in the directions of y and x
ψ_j^e	-	Lagrange interpolation functions of elements in-plane displacement

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of study

Textile composite materials have been applied in large scale in numerous engineering industries such as aerospace, marine, automobile, and sporting goods. Due to the demand of lighter at the same time high performance materials in these industries, the textile materials are seen to meet such requirements from material selection process. Biaxial and triaxial fabric composites have been known as common type of materials used in the textile industry and have attracted researchers to study and investigate their unique mechanical characteristics since a few decades ago during which aerospace industry has just initiated its dominance in engineering field.

Based on previous study, triaxial weave fabric (TWF) composite has shown more advantages and potential compared to biaxial weave fabric (BWF) composite in terms of single ply comparison. Besides, TWF is often chosen as solution to problems particularly related to lightweight and shear-resistance requirements, both of which cannot be fulfilled by the conventional BWF.

Thus, the present study aims to investigate mechanical behavior of TWF by modeling a unit cell of TWF through homogenized approach such that problems such as time-consuming computation due to subtleties of its geometrical makeup, extensively discussed in literature, can be circumvent. The idea of using TWF pattern has long been applied in manmade structures as shown in Figure 1.1. For example, TWF pattern is applied on the racket and basket to gives a stiff surface and allow the structure to carry a heavy load. In advanced technology, TWF is used as spring back reflectors of MSAT-2 spacecraft where one is folded at the top and another one is deployed at the bottom as shown in Figure 1.2. Other than aerospace industry, TWF also has been used to make golf club shafts, solar panels, skis, fishing rods, and speaker cones.

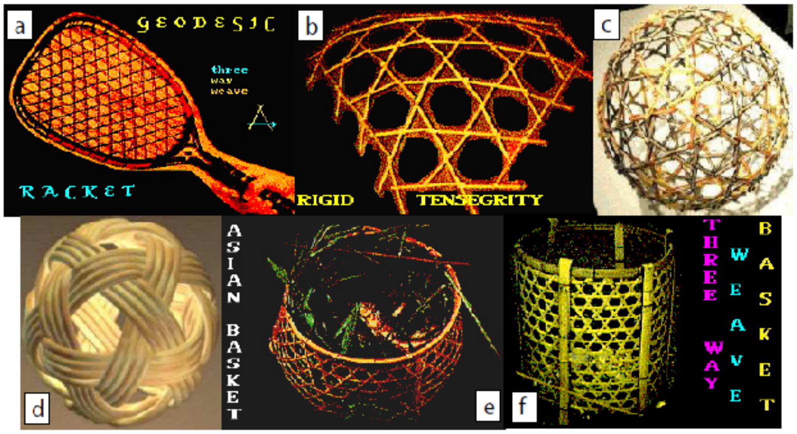

Figure 1.1: Triaxial pattern: (a) Racket (b) Rigid tensegrity (c) Sphere (d) Rattan ball (e) Asian basket and (f) Three way weave basket (Buckminster Fuller Virtual Institute, 2007; Kueh, 2007)

Figure 1.2: Spring back reflectors of MSAT-2 spacecraft (Courtesy of Canadian Space Agency)

1.2 History of TWF

TWF was discovered in the late 1950's when Francis Rogallo, a NASA project engineer constructed a paraglider for the space program purposes (Kueh, 2007). During a wind tunnel test, the structure failed by the loss of aerodynamic shape of the tail section. Investigation was carried out by Norris Dow, one of Rogallo's colleagues and discovered that the failure of tail section which was made of BWF was caused by the distortion of the material in off-axis directions. To remedy this problem, an extra direction of tows in BWF was suggested as a solution, making the material consists three sets of tows interlaced and intersected each other at 60° angles. In his opinion, this solution introduced better weaving method and gave interlacing strength due to friction created by intersecting yarns. Besides, the unique configuration of equilateral triangle made it more stable compared to rectangular. The design was patented and commercialized in US under N.F.

Doweave, Inc. Figure 1.3 shows some of the designs patented by Dow (Kueh, 2007). The number ten indicates the open holes in weave, and *x*, *y*, and *z* indicate the three axial directions.

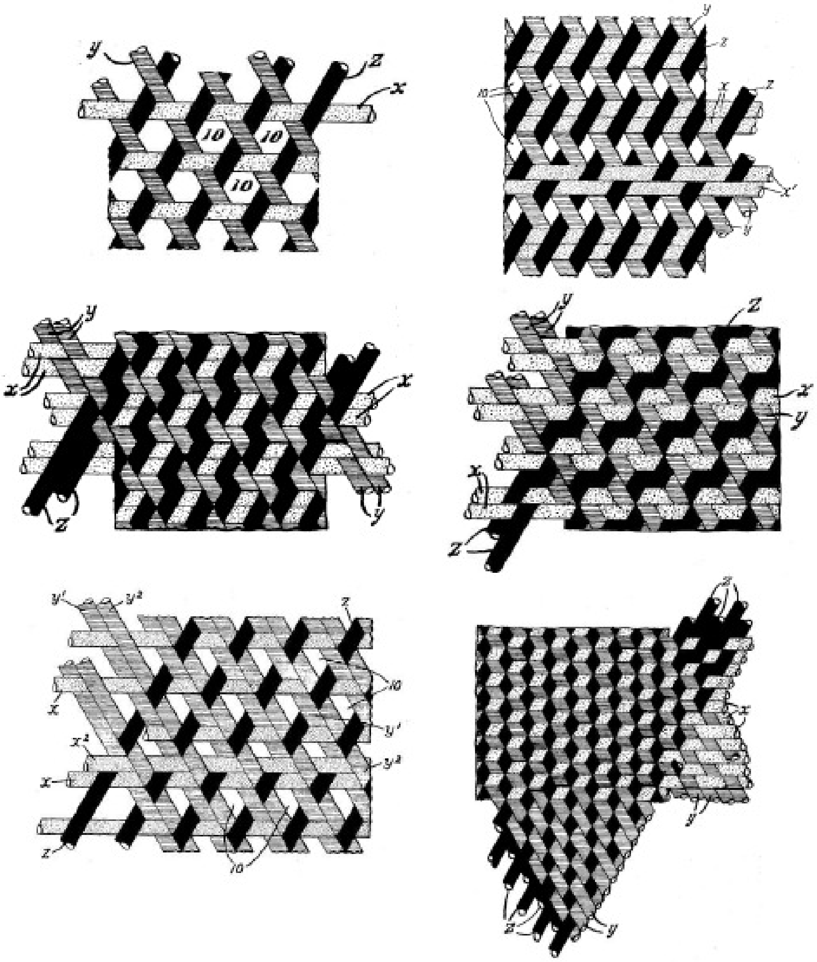

Figure 1.3: Some of TWF designs patented by Dow (1969)

However, the enterprise was closed in the late 1970's due to high industrial competition. The business then was taken over by Sakase Adtech Co. Ltd. of Fukui, Japan. This company is thus far known as the only manufacturer of TWF and managed by its founders, Sakai Brothers, Ryoji and Yoshiharu.

1.3 Statement of Problem

Unlike the solid composite, especially unidirectional material, the TWF cannot be assumed as a flat composite because of the waviness of tow. The complexity of its geometry makes the behaviors of TWF difficult to predict. For a laminated composite, laminae when modeled in the standard finite element software are presented in a layered form. Hence, to model a composite material that is made up of several plies, a number of elements are needed through thickness.

In previous work on woven fabrics, a repeating unit cell (RUC) was adopted in modeling, assuming a uniform repeating building block throughout the whole volume of composites. But the main concern was given to plied and thicker materials. The model was treated as a solid which contains only fibers and matrix, and also, the open voids exist in material were usually neglected. As a result, the built-in elements provided in the finite element software are more applicable for modeling flat laminated materials. These assumptions cannot be used for TWF because of the waviness of tow. Also, hexagonal voids spread across the volume have to be modeled properly. Such features can be noticed in Figure 1.4 which shows the unit cell of TWF with its dimension in mm, where the rectangular area within the dash lines shows the unit cell with the hexagonal hole of 1.8 mm high, covering about half of the area of unit cell. In addition, there exist efforts in modeling TWF using solid elements, the degree of freedom of which can be of high intensity. Due to this complexity, the formulation of the unit cell of TWF as a standalone planar element is highly important to ease the difficulty in modeling such a material in the commercial finite element software.

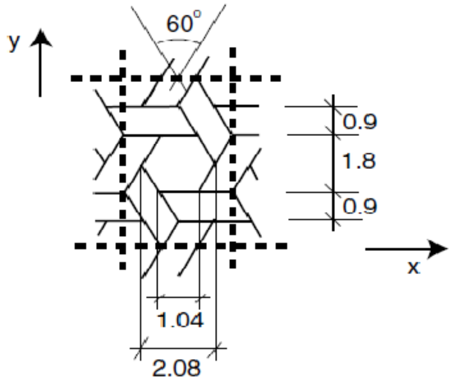

Figure 1.4: A unit cell of TWF (Kueh and Pellegrino, 2007)

1.4 Objectives of Study

The objectives of this research are:

1. To formulate the TWF unit cell macromechanically adopting available constitutive relation by homogenization method based on micromechanics of curved composite and hence, treat the unit cell as a standalone finite element.
2. To program the script of the formulation of the unit cell of TWF for linear and nonlinear analyses.
3. To generate the general equation of buckling from the proposed model.

1.5 Scope of the Study

This research is focused on the single-ply TWF composite. The fundamental of finite element formulation is used to generate a standalone element for the unit cell of TWF composite plate. 2D plate elements consisting five degrees of freedom per node are employed. In this research, the assumption of Kirchhoff hypothesis for thin plates is used in the formulation. Besides, a perfect bonding between tows is assumed in the formulation. Only the elastic mechanical behaviors are of concern in this study. Fracture of material is not considered. The Matlab software is used to program the finite element formulation. The linear and nonlinear analyses of buckling are conducted for numerous aspects ratios (0.25-5) and boundary conditions. As mentioned in the objectives, one of the purposes of the analyses is to produce some important parameters which can be used to represent the general characteristics of buckling of TWF.

1.6 Significance of Research

By establishing the formulation of standalone composite element for unit cell of TWF, it helps reduce the time-consuming and tedious process in the material modeling, which is to date computationally expensive due to solid element meshing. Considerably costly experimental method could be the main obstacle for composite analysts to make characterization of material. Hence, it is very useful to apply currently developed composite element adopting homogenized model and program script, which give choices and freedoms to composite analysts, in any structure that uses TWF as its reinforcing constituent. Furthermore, a list of characteristic terms that expresses the buckling of TWF for numerous cases can be used instantaneously to assess stability of this material without having to go through the lengthy nonlinear computation process.

REFERENCES

Aoki, T. and Yoshida, K. (2006). Mechanical and Thermal Behaviors of Triaxially-Woven Carbon/Epoxy Fabric Composite. *47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Material Conference*. Newport, Rhode Island.

Buckminster Fuller Virtual Institute website. http://www.buckminster.info

Chou, T.W. and Ko, F.K. (1989). *Textile Structural Composites, Volume 3* Composite Materials Series. New York, U.S.A.: Elsevier Science Publishers, Amsterdam.

Cox, B. N. and Flanagan, G. (1997). *Handbook of analytical methods for textile composites*. NASA Contractor Report 4750.

Dow, N. F. (1969). *Triaxial fabric*. United States Patent Office 3,446,251.

Fujita, A. , Hamada, H. and Maekawa, Z. (1993). Tensile Properties of Carbon Fiber Trixial Woven Fabric Composites. *Journal of Composite Materials,* 27. 1428 – 1441.

Hoa, S.V., Sheng, S.Z. and Ouellette, P. (2003). Determination of Elastic Properties of Triax Composite Materials. *Composites Science and Technology*. 63, 437-443.

Karkkainen, R.L. and Sankar, B.V. (2006). A Direct Micromechanics Method for Analysis of Failure Initiation of Plain Weave Textile Composites. *Composite Science and Technology*, 66 (2006) 137 – 150.

Kueh, A.B.H (2007). *Thermo-Mechanical Properties of Triaxial Weave Fabric Composites*. Ph.D. Thesis. University of Cambridge, United Kingdom.

Kueh, A.B.H. (2012). Fitting-Free Hyperelastic Strain Energy Formulation for Triaxial Weave Fabric Composites. *Mechanics of Materials*. 47, 11-23.

Kueh, A.B.H. (2013).Compressive Response of Sandwich Columns Reinforced by Triaxial Weave Fabric Composite Skinsheets. *International Journal of Mechanical Science,* 66(2013) 45-54.

Kueh, A.B.H., and Pellegrino, S. (2007). ABD Matrix of Single-Ply Triaxial Weave Fabric Composites. *48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Material Conference*. Honolulu, Hawaii: AIAA-2007-2161.

Mahat, M.N.H (2013). *Thermo-Mechanical Buckling of Triaxial Weave Fabric Composites*. Master Thesis. Universiti Teknologi Malaysia, Skudai, Malaysia.

Peng, X.Q. and Cao, J. (2000). Numerical Determination of Mechanical Elastic Constants of Textile Composites. *15th Annual Technical Conference of the American Society for Composite*. College Station, TX.

Reddy, J.N. (2004). *Mechanics of Laminated Composite Plates and Shells Theory and Analysis*. (2nd ed.). Boca Raton : CRC Press LLC.

Schwartz, P. (1982). Bending Properties of Triaxially Woven Fabrics. *Textile Research Journal*, 52, 604 – 606.

Skelton, J. (1971). Triaxially Woven Fabrics: Their structure and Properties. *Textile Research Journal*, 41, 637 – 647.

Xu, D. , Ganesan, R. and Hoa, S.V. (2005). Buckling Analysis of Tri-axial Woven Fabric Composite Structures. Part I: Non-linear Finite Element Formulation. *Compos Struct*. 67, 37-55.

Xu, D. , Hoa, S.V. and Ganesan, R. (2006). Buckling Analysis of Tri-axial Woven Fabric Composite Structures. Part II: Parametric Study – Uni-directional Loading. *Composite Structure*. 72, 236-253.

Zhao, Q. and Hoa, S.V. (2003). Triaxial Woven Fabric (TWF) Composites with Open Holes (Part I): Finite Element Models For Analysis. *Journal of Composite Materials*. 37(9), 763-789.

Zhao, Q., Hoa, S.V. and Ouellette, P. (2003). Triaxial Woven Fabric (TWF) Composites with Open Holes (Part II): Verification of the Finite Element Models. *Journal of Composite Materials*. 37, 10.

Zienkiewicz, O.C. and Cheung, Y.K. (1964). The Finite Element Method for Analysis of Elastic Isotropic and Orthotropic Slabs. *Proceeding of the Institute of Civil Engineers*, London, UK, 28, 471-488.