MATHEMATICAL MODELING OF BLOOD FLOW THROUGH AN ECCENTRIC CATHETERIZED ARTERY

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To my beloved Father and Mother

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ABSTRACT

The mathematical model of blood flow through a catheterized stenosed artery is considered. A catheter is a tube, which is used in medicine for patients who are bedridden and whose blood pressure needs to be measured and monitored continuously. An example is the use of catheter during X-ray angiography or coronary balloon angioplasty in cardiac patients. Inserting a catheter in an artery will alter some characteristics of blood flow. This project investigates the effect on blood flow characteristics such as the velocity, the wall shear stress, the resistance impedance and the streamlines when a catheter is inserted into a stenosed artery. The catheter and the artery are assumed to be in a co-axial and eccentric position while blood is assumed to be Newtonian. The governing Navier-Stokes equations are solved analytically using perturbation method. The results show that a catheter placed in an eccentric position does alter the blood flow characteristics such that the axial velocity and the wall shear stress distribution are higher while the resistance impedance values are lower compared to their values in an artery where the catheter is concentrically placed. It is also found that under the same situation, the position of trapping moves closer to the wall of the stenosis while the size of the trapped bolus increases.

ABSTRAK

Pemodelan matematik bagi aliran darah apabila kateter dimasukkan ke dalam arteri berstenosis dipertimbangkan. Kateter adalah suatu tiub yang digunakan dalam perubatan bagi pesakit yang memerlukan tekanan darah mereka diukur dan dipantau secara berterusan. Contohnya, kateter digunakan semasa prosidur *X-ray angiografi* atau *belon angioplasti* untuk pesakit jantung. Apabila kateter dimasukkan ke dalam arteri, ciri-ciri aliran darah akan berubah. Projek ini mengkaji kesan kateter terhadap ciri-ciri aliran darah seperti halaju, tekanan ricih diding, rintangan impedan dan garis arus apabila kateter dimasukkan pada kedudukan sepaksi *eksentrik* , manakala darah diandaikan sebagai bendalir Newtonan. Persamaan menakluk Navier-Stokes diselesaikan seecara analisis menggunakan kaedah usikan. Hasil kajian menunjukkan bahawa halaju sepaksi dan tekanan ricih dinidng adalah lebih tinggi sementara nilai rintangan impedan adalah lebih rendah berbanding dengan ciri-ciri aliran pada arteri di mana kateter diletakkan secara *konsentrik.* Kajian juga mendapati pada situasi serupa, kedudukan *trapping* makin hampir dengan dinding stenosis sementara saiz bolus *trapping* bertambah.

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d	-	Location of the stenosis
R_e	-	Reynolds number
$R(z,t)$	-	Radius of the tapered arterial region
R_0	-	Radius of the normal artery without stenosis
p	-	Fluid pressure
t	-	Time
ρ	-	Density of fluid
ω	-	Angular frequency
μ	-	Dynamic viscosity
ϕ	-	Angle of tapering
τ	-	Shear stress
λ	-	Resistance impedance
σ	-	Radius of catheter
ε	-	Eccentricity parameter
ψ	-	Stream function

CHAPTER 1

INTRODUCTION

1.1 Research Background

A catheter is a thin, flexible, hollow and lengthy plastic tube that is generally inserted into an artery (large blood vessel) placed in the wrist; but can also be inserted into the elbow, groin, foot or the inside of the arm. However, in theory, every artery could be used for injection of catheter but in practical situation, arteries of the mentioned organs are commonly placed for the injection. (Figure 1.1)

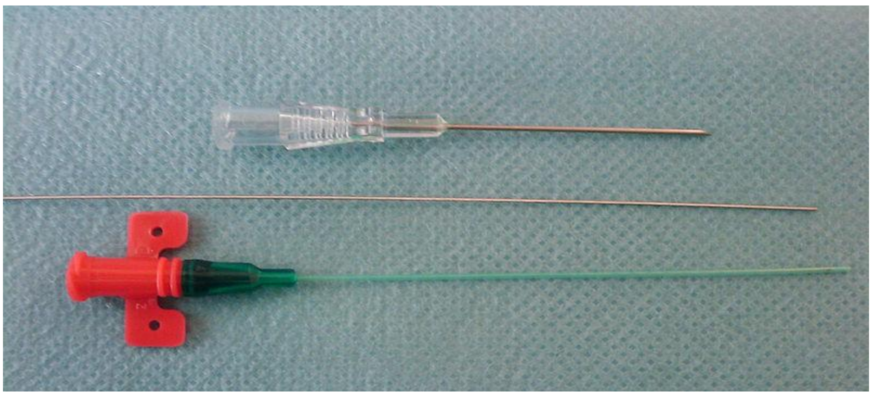

Figure 1.1 Arterial Catheter, [wikipedia.org/wiki/File:Arterial_kateter,(n.d.)]

Catheters have an important role in medicine for heart problems. For instance, they are usually used to diagnose and treat heart conditions. During catheterization, small tubes (catheters) are inserted into the circulatory system under x-ray guidance in order to obtain information about blood flow and pressures within the heart and to determine if there are obstructions within the blood vessels feeding the heart muscle (coronary arteries). Usage of the catheter is well known in medicine of various arterial diseases as “coronary balloon angioplasty” (Figure 1.2a,b), “X-ray angiography”(Figure 1.3),“and “intravascular ultrasound” (Figure 1.4).

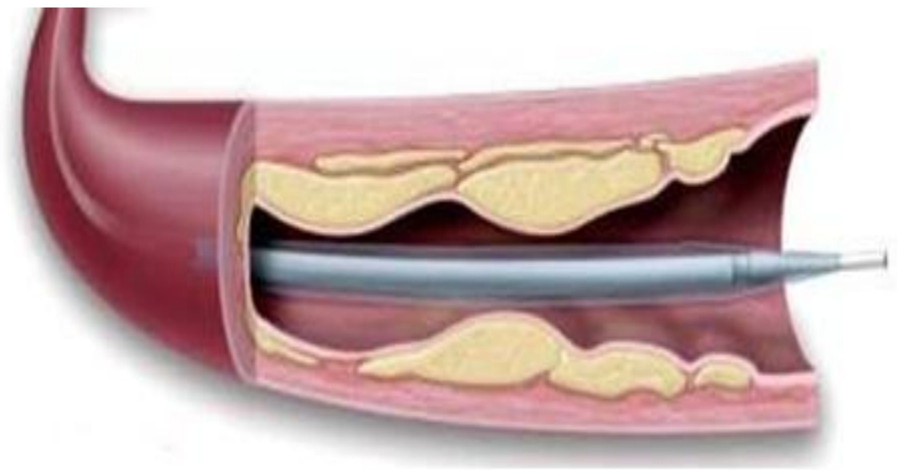

Figure 1.2a A Catheter through an artery, [arterydisease.org/(n.d.)]

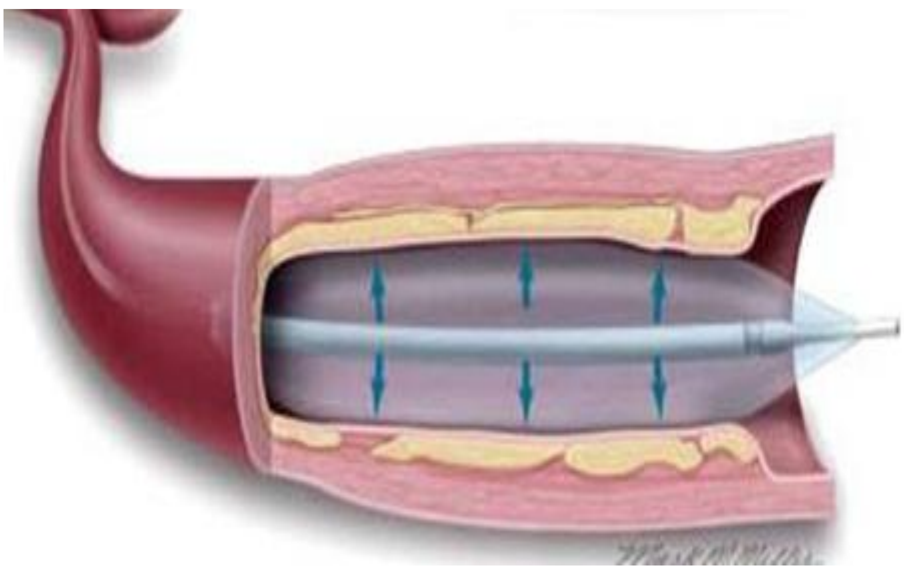

Figure 1.2b Balloon Angioplasty, [arterydisease.org/(n.d.)]

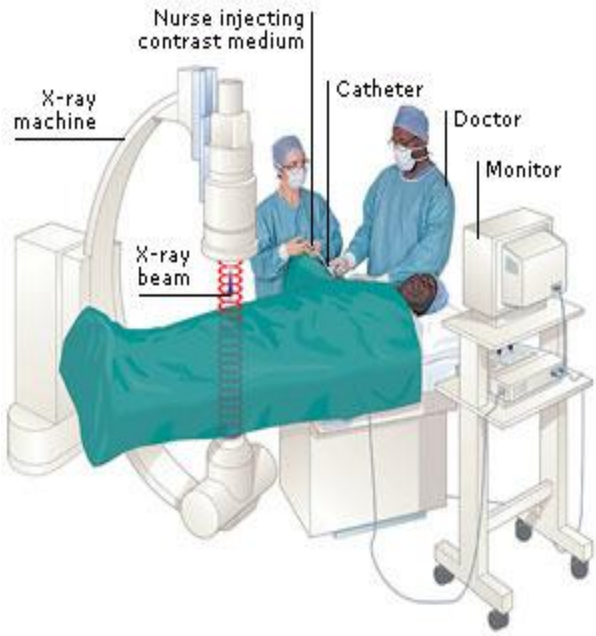

Figure 1.3 X-ray Angiography, [arterydisease.org/(n.d.)]

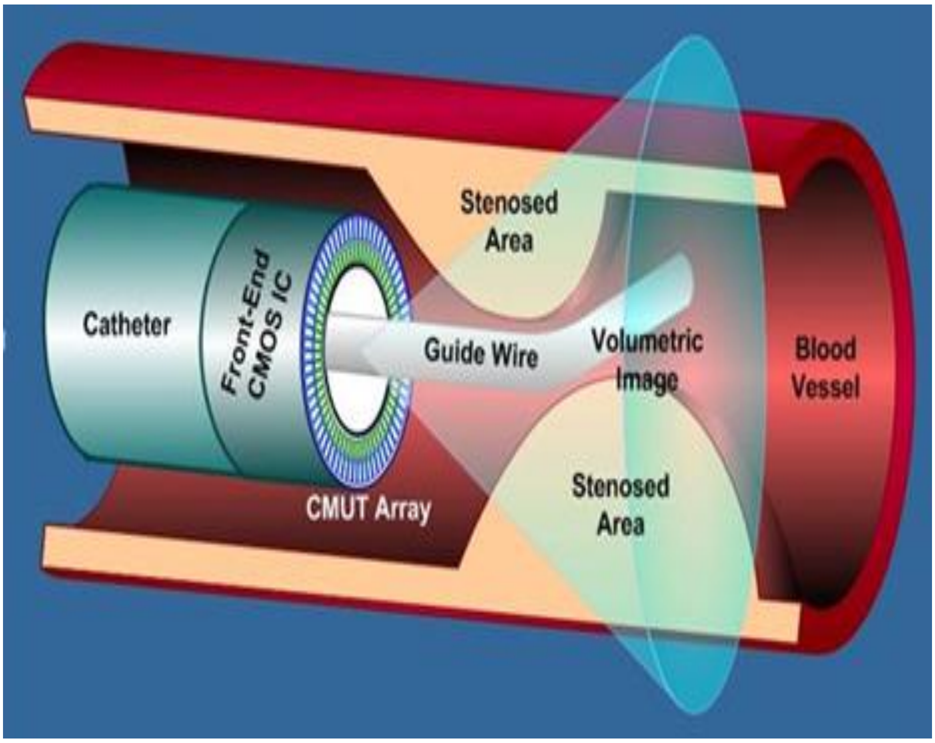

Figure 1.4 Intravascular Ultrasound and Using Catheter, [Tekes *et al*.,(2009)]

In addition, a catheter is used for the measurement of various physiological flow characteristics such as arterial blood pressure or pressure gradient and flow velocity or flow rate. Examples include patients in the intensive care unit (ICU) requiring isotropic support or patients with severe cardiovascular disease undergoing surgery.

Furthermore, catheter can be used for periodic arterial blood gas analysis in patients with respiratory failure, or severe acid/base disturbance. When a patient has a lung problem that is so severe it requires checking the levels of oxygen or carbon dioxide of the blood more than 3 to 4 times a day regularly, the arterial catheter is used to draw blood without having to repeatedly stick a needle into the patient body.

1.2 Problem Statement

When a catheter is inserted into a blood vessel, blood clots can be formed on the tips of arterial catheters, the clots can block blood flow. Furthermore, bleeding can occur at the time of inserting the catheter. Patient becomes uncomfortable resulting from the injection. However, more important, the injection of a catheter alters the flow field and disturbs the hemodynamic conditions in the artery. In practical situation, the injection of a catheter cannot be exactly concentric with the artery. It is in fact placed in an eccentric position. Hence, this study investigates what happen to the blood flow characteristics when catheter is inserted eccentrically through an artery.

1.3 Objectives of the Study

The main objective of this research is to determine the effects of eccentric catheterization on blood flow characteristics in a tapered and stenosed artery. Specific objectives are to calculate the axial velocity, stream function and trapping, resistance impedance and wall shear stress.

1.4 Scope of the Study

This study is based on the work of Mekheimer and Kot (2012). The blood is considered Newtonian fluid, incompressible, steady and laminar; In addition, catheter and artery supposed to be two tubes, which are co-axial together, and inner tube is catheter that assumed to be in eccentric position with the outer tube (tapered artery).

1.5 Significance of the Study

A motivation of the present analysis is the hope that such a problem will be applicable in many clinical applications. It is hoped that with this investigation, the injection can be carried out more proficiently. In addition, the results are used to obtain the estimates of increased or decreased of blood flow characteristics across an artery stenosis during catheterization.

1.6 Outline of Dissertation

In Chapter 1, the nature and use of catheter the problem statement and objectives are presented. In Chapter 2, overview of previous studies on catheterized artery and blood flow changes are discussed. In Chapter 3, problem is formulated mathematically. In Chapter 4, method to solve the problem is expressed and the equations of some characteristics of blood flow are obtained. In Chapter 5, the results are displayed graphically. In Chapter 6, conclusions and comments about the results and recommendation for future studies are interpreted.

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