DYNAMIC SIMULATION OF COLUMNS CONSIDERING GEOMETRIC NONLINEARITY

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A project report submitted in partial fulfillment of the
requirements for the award of the degree of
Master of Engineering (Civil – Structure)

Faculty of Civil Engineering
Universiti Teknologi Malaysia

JUNE 2013

To my beloved Sareh

ACKNOWLEDGEMENT

I would like to express my deepest appreciation to all those who provided me the possibility to complete this report. A special gratitude I give to my supervisor, Dr. Suhaimi Abu Bakar, whose contribution in stimulating suggestions and encouragement, helped me to coordinate my project especially in writing this report.

ABSTRACT

Dealing with slender structural columns will make the use of classic elastic dynamic analysis of structures untrustworthy. This will emphasize the importance of developing a method for dynamic analysis of structures taking geometric nonlinearities into consideration. In order to achieve this the equations of motions are formulated, based on finite strain formula and virtual work method. To solve the equation of motion, the usual central difference method is used. The developed method is applied to two cases of columns with different slenderness ratio. The effect of introducing the nonlinear approach for different slenderness ratios of columns is investigated on natural frequency of the first three modes. Also, the relationship between the maximum displacement for a known time step and loading frequency is studied. To find the suitable slenderness ratio threshold when the effect of nonlinearity becomes significant, the relationship between slenderness ratio and the maximum displacement ratio for a known loading frequency is studied. The results show that columns with higher ratio of axial loading over slenderness ratio provide a higher rate of decrease in natural frequencies. It is also found that the effect of taking geometric nonlinearity into consideration will be significant while dealing with higher slenderness ratio and smaller loading frequency. For a specific situation, these results are discussed and the slenderness ratio threshold is found equal to 110. It means that for slenderness ratios higher than 110 the effects of geometric nonlinearity becomes significant and should be taken into consideration.

ABSTRAK

Dengan mengambil kira struktur tiang adalah langsing akan membuatkan penggunaan analisis elastik dinamik klasik tidak boleh dipercayai. Kepentingan membangunkan satu kaedah untuk analisis struktur dinamik dengan mengambil kira sifat geometri tak linear perlu ditekankan. Dalam usaha mencapai matlamat ini, formulasi persamaan pergerakan berjaya dibentuk menggunakan formula terikan terhingga dan kaedah kerja maya. Bagi menyelesaikan persamaan pergerakan, kaedah perbezaan pusat yang biasa adalah digunakan. Kaedah yang dibangunkan ini akan digunakan untuk dua kes tiang dengan nisbah kelangsingan yang berbeza. Kesan penggunaan analisis tak linear untuk tiang dengan nisbah kelangsingan yang berbeza dikaji terhadap frekuensi semulajadi daripada tiga mod pertama. Selain itu, hubungan antara anjakan maksimum untuk kenaikan masa yang diketahui dan kekerapan pembebanan turut dikaji. Bagi mendapatkan nisbah kelangsingan yang sesuai untuk kesan tak linear menjadi ketara, hubungan antara nisbah kelangsingan dan nisbah anjakan maksimum untuk kekerapan pembebanan diketahui juga diselidiki. Keputusan analisis menunjukkan bahawa tiang dengan nisbah beban paksi terhadap nisbah kelangsingan yang tinggi menghasilkan kadar penurunan frekuensi semulajadi yang tinggi. Kesan mengambil kira geometri tak linear ditemui sangat penting dalam mempertimbangkan nisbah kelangsingan tiang yang tinggi dan kekerapan pembebanan yang rendah. Bagi keadaan tertentu, keputusan ini dibincang dan nisbah kelangsingan yang ditentukan ditemui sebagai 110. Ini bermakna, nisbah kelangsingan melebihi 110 menghasilkan kesan geometri tak linear struktur yang ketara dan perlu diambil pertimbangan dalam analisis.

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LIST OF SYMBOLS

u	-	Horizontal Displacements
v	-	Vertical Displacement
ε	-	Strain
N	-	Shape Functions
Π	-	The Potential Energy
E	-	Modulus of Elasticity
σ	-	Stress
U	-	Strain Energy
H	-	Hermite Shape Functions
ρ	-	Mass per Volume
$\delta u, \delta v$	-	Virtual Displacements
ω	-	Loading Frequency
ω_n	-	Natural Frequencies
$\mathbf{m}$	-	Mass Matrix
$\mathbf{K}$	-	Stiffness Matrix
p	-	Loading

CHAPTER 1

INTRODUCTION

1.1 Introduction

Most of the time, the problems in civil engineering behave in linear elastic fashion under service load. However, in some slender structures, this is not the case. In this kind of structures, before reaching the ultimate state and failure, a great deal of nonlinearity is exhibited by the structures. This nonlinearity can arise because of material nonlinearity or geometric nonlinearity. Whatever the type, there are two approaches to include them: in the first approach nonlinear behavior will not be simulated in analysis stage and it will be taken into account through some methods in the design stage. The second approach will be to make an attempt to simulate nonlinear behavior of structures. The main aim of this process is to make a more reliable prediction of the nonlinear performance of the structure and improve the initial information given to design engineer. The same line of reasoning can be applied to dynamic analysis of structures. In some applications, for example slender structures in seismic regions, a combination of these two approaches is needed to make a reliable prediction of the behavior of structure under dynamic loading. It is necessary to mention that it is highly probable that dynamic loading will result in more critical results. Therefore, combining these approaches is quite necessary for a suitable prediction of behavior of structure.

1.2 Background of Study

Geometric Nonlinearity arises when deformations are large enough to alter the distribution or orientation of applied loads, or the orientation of internal resisting forces and moments [1]. Some of the geometric effects include [2]:

- Initial imperfections such as member camber and out-of-plumb erection of a frame.
- The P-Δ effect, a destabilizing moment equal to a gravity load times the horizontal displacement it undergoes as a result of the lateral displacement of the supporting structure.
- The P-δ effect, the influence of axial force on the flexural stiffness of an individual member

Several modes of nonlinear elastic behavior are discussed in [2], i.e. (a) Bifurcation of the loading path with the system following an alternative path in post-critical state. (b) Gradually increasing nonlinearity culminating in elastic instability at a limit point. (c) Increasing stiffness either from the onset of loading or following a period of gradual softening.

As discussed before, dynamic analysis is about the solution of "Equation of Motion". In simple systems with single degree of freedom, these equations can be solved in closed form. However, when the problem involves dealing with Multiple Degree of Freedom structures or continuous structures, this is no longer a possibility. In this case, a numerical approach (i.e. finite element method) will be used. This method, which is one of the most important developments in applied mechanics[3], is based on subdivision of problem scope into some "finite elements" connected to each other through "nodes". Then, two degree of freedom will be assumed for each node. Next step will be to calculate stiffness matrix (and mass matrix for dynamic analysis) for each element and assembling them into a structure stiffness (and mass) matrix. Several methods used to derive the mass matrix include particle mass lumping, consistent mass matrix, combination matrices, HRZ lumping, and optimal lumping[1]. Then, the nodal load vector should be found based on actual loading. Knowing the nodal load vector and structure stiffness matrix, the nodal displacement vector can be obtained. Knowing the nodal displacements and using the equations developed for each kind of member, the strains and stresses for each member can be found. There are several methods of formulation used in finite element related literature, including variational methods and weighted residual methods.

Variational methods, including methods such as minimum potential energy and virtual work, are based on integral expressions called functional. The purpose of these methods is to find the values of d.o.f. which minimize these functionals. This will provide an strong basis for producing FE approximations. In structural mechanics, the most commonly used functional is that of potential energy[1]. The principle of minimum potential energy states that:

“of all the geometrically possible shapes that a body can assume, the true one, corresponding to the satisfaction of stable equilibrium of the body, is identified by a minimum value of the total potential energy.”[4]

In some problem areas, the potential function is not known or well known. Therefore, it is not possible to use the variational methods to generate FE approximations. In these cases, weighted residual methods, e.g. Galerkin method, will be used for this purpose. This method can be used for problems for which just the differential equation and boundry conditions are known. The methods available in this category are intended to select parameters in approximating trial functions so as to obtain the best approximation[1].

In Dynamic problems, the purpose of the study is to find the nodal displacements at different time increments. In other word, the problem should be discretized over time domain. The general method used for this purpose is called direct integration[4]. Direct integration refers to calculation of response history using step-by-step integration in time, without changing the form of dynamic equations, as is necessary in modal methods[1]. There are two main groups of direct integration: explicit, and implicit. An example of explicit methods is central difference method which is based on expressions discussed in finite difference. Examples of implicit methods include Newmark-Beta method and Wilson-Theta method.

There are several works in analysis regarding geometric nonlinearity and dynamic analysis, separately. However, the number of previous works aimed to combine these two methods is quite limited. Aristizabal-Ochoa uses matrix method and modified stability approach to find the first and second-order stiffness matrices of a Timoshenko beam-column with semi-rigid connections [5]. Arboleda et al. determine the dynamic stiffness matrix and load vector for a Timoshenko beam-column resting on a two parameter elastic foundation with generalized end conditions [6]. Kwak and Kim investigate the P-Δ effect in slender reinforced concrete columns. for this purpose, nonlinear dynamic analysis was used for 60 sets of horizontal and vertical earthquakes with different sets of slenderness and stability coefficient [7]. Cook et al. Describe plasticity and some conceptual procedures to deal with it. Non-linear Dynamic problems are addressed for cases in which the frequency of excitation exceeds roughly one-quarter of the structure's lowest natural frequency of vibration [1]. Simsek and Kocaturk use the geometrically dynamic approach to analyze an eccentrically prestressed damped beam when it is under a concentrated moving harmonic load. The assumptions they used are as follows,

- The Kelvin–Voigt model for the material
- Euler–Bernoulli beam theory
- The von-Karman's nonlinear strain- displacement relationships
- Using Newmark-Beta for solving the nonlinear equations of motion
- Newton–Raphson for iteration person[8].

Simsek uses a nonlinear dynamic approach to analyze fuctionally graded beam. They used Timoshenko beam theory[9].

1.3 Problem Statement

When dealing with slender columns, the results of linear dynamic analysis is not trustworthy anymore because, for matrix analysis, the stiffness matrix of structure will be changed due to geometric nonlinearity; however, this effect is neglected in linear analysis.

The procedure of determining the stiffness matrix of a structure with geometric non-linearity is a repetitious process based on the internal force of the member and methods such as Secant method and Newton-Raphson method. Using a dynamic approach will increase the complexity of analysis by introducing the response spectra, especially in under-damped cases. So, the main problem will be how to alter the methods available so that they can describe the behavior of the structure over time. Dynamic analysis of columns is discussed thoroughly in literature review. However, taking the effects of Geometric Nonlinearity in dynamic analysis of columns is quite limited in studies. In this study, the effect of geometric nonlinearity will be examined in the results of dynamic analysis.

1.4 Objectives of Study

- Developing a method for Geometric Nonlinear Dynamic Analysis of Columns using Virtual Work Method and Finite Element Approach
- Applying the Proposed Method to some case studies and discussing the effect of various variables, including slenderness ratio, loading frequency and vertical loading

1.5 Scope of Study

-This study deals with Elastic Material and Geometric Nonlinearity.

- In this study, central difference method will be used for discretization over time.

1.6 Significance of study

There are several factors which result in nonlinear behavior of frame structures, i.e. shear deflections, the shear component of the applied axial force as each member deflects, lateral deflections (P-Δ effect) along each element, and the effects of the flexural moments on the axial stiffness [5]. Non-linear behavior will result in additional bending moments, rotations, and displacements. These factors change the stiffness matrix, as well as buckling capacity of each element and the frame.

Dynamic analysis of structures is based on the solution of the following differential equation,

$$m\ddot{u} + Ku = p(t)$$

For solving these equations, the stiffness matrix should be determined. As discussed, nonlinear behavior of structure will result in change of stiffness matrix. In case of members with significant nonlinear effects, e.g. slender columns, the effect of this change should be taken into consideration when performing dynamic analysis; otherwise, the accuracy of the results will be lost.

The results of this study will provide a strong theoretical foundation for developing an accurate method which can be integrated into design process. It will improve our analysis and design results whenever P-delta effect is an issue and through that, it will improve the safety factor of our design.

REFERENCES

1. Cook, R.D., *Concepts and applications of finite element analysis*. 4th ed. 2001, New York, NY: Wiley. xvi, 719 p.
2. McGuire, W., R.H. Gallagher, and R.D. Ziemian, *Matrix structural analysis*. 2nd ed. 2000, New York: John Wiley. xx, 460 p.
3. Chopra, A.K., *Dynamics of structures : theory and applications to earthquake engineering*. 3rd ed. 2007, Upper Saddle River, N.J.: Pearson/Prentice Hall. xxxiv, 876 p.
4. Logan, D.L., *A first course in the finite element method*. 4th ed. 2007, United States: Thomson. xvii, 808 p.
5. Aristizabal-Ochoa, J.D., *Matrix method for stability and second-order analysis of Timoshenko beam-column structures with semi-rigid connections.* Engineering Structures, 2012. **34**(0): p. 289-302.
6. Arboleda-Monsalve, L.G., D.G. Zapata-Medina, and J.D. Aristizabal-Ochoa, *Timoshenko beam-column with generalized end conditions on elastic foundation: Dynamic-stiffness matrix and load vector.* Journal of Sound and Vibration, 2008. **310**(4–5): p. 1057-1079.
7. Kwak, H.-G. and J.-K. Kim, *effect of slender RC columns under seismic load.* Engineering Structures, 2007. **29**(11): p. 3121-3133.
8. Şimşek, M. and T. Kocatürk, *Nonlinear dynamic analysis of an eccentrically prestressed damped beam under a concentrated moving harmonic load.* Journal of Sound and Vibration, 2009. **320**(1–2): p. 235-253.
9. Şimşek, M., *Non-linear vibration analysis of a functionally graded Timoshenko beam under action of a moving harmonic load.* Composite Structures, 2010. **92**(10): p. 2532-2546.
10. Timoshenko, S., *Theory of elastic stability*. 2d ed. Engineering societies monographs. 1961, New York,: McGraw-Hill. 541 p.
11. Gere, J.M. and B.J. Goodno, *Mechanics of materials*. 7th ed. 2009, Toronto, ON ; Clifton Park, NY: Cengage Learning. xviii, 1022 p.

12. Chandrupatla, T.R. and A.D. Belegundu, *Introduction to finite elements in engineering*. 3rd ed. 2002, Upper Saddle River, N.J.: Prentice Hall. xviii, 453 p.

13. Kassimali, A., *Matrix analysis of structures*. 2nd ed. 2012, Australia ; Stamford, CT: Cengage Learning. xii, 641 p.

14. Tongue, B.H., *Principles of vibration*. 2nd ed. 2002, New York: Oxford University Press. x, 518 p.

15. Aristizabal-Ochoa, J., *First- and Second-Order Stiffness Matrices and Load Vector of Beam-Columns with Semirigid Connections.* Journal of Structural Engineering, 1997. **123**(5): p. 669-678.

16. Aristizabal-Ochoa, J., *Timoshenko Beam-Column with Generalized End Conditions and Nonclassical Modes of Vibration of Shear Beams.* Journal of Engineering Mechanics, 2004. **130**(10): p. 1151-1159.

17. Cheng, F. and C. Pantelides, *Dynamic Timoshenko Beam-Columns on Elastic Media.* Journal of Structural Engineering, 1988. **114**(7): p. 1524-1550.

18. Cheng, F. and C. Pantelides, *Static Timoshenko Beam-Columns on Elastic Media.* Journal of Structural Engineering, 1988. **114**(5): p. 1152-1172.

19. Morfidis, K. and I.E. Avramidis, *Formulation of a generalized beam element on a two-parameter elastic foundation with semi-rigid connections and rigid offsets.* Computers & Structures, 2002. **80**(25): p. 1919-1934.

20. Mullapudi, R. and A. Ayoub, *Nonlinear finite element modeling of beams on two-parameter foundations.* Computers and Geotechnics, 2010. **37**(3): p. 334-342.

21. Bernal, D., *Amplification factors for inelastic dynamic p–Δ effects in earthquake analysis.* Earthquake Engineering & Structural Dynamics, 1987. **15**(5): p. 635-651.

22. Tremblay, R., C. Duval, and P. Léger, *Chapter 10 - Effects of Viscous Damping Models, Hysteretic Models and Ground Motion Characteristics on Seismic P-Delta Strength Amplification Factors*, in *Stability and Ductility of Steel Structures (SDSS'97)*, U. Tsutomu and I. Yoshito, Editors. 1998, Pergamon: Oxford. p. 103-118.

23. Williamson, E., *Evaluation of Damage and P-Δ Effects for Systems Under Earthquake Excitation.* Journal of Structural Engineering, 2003. **129**(8): p. 1036-1046.

24. Button, M., C. Cronin, and R. Mayes, *Effect of Vertical Motions on Seismic Response of Highway Bridges.* Journal of Structural Engineering, 2002. **128**(12): p. 1551-1564.

25. Ju, S., C. Liu, and K. Wu, *3D Analyses of Buildings under Vertical Component of Earthquakes.* Journal of Structural Engineering, 2000. **126**(10): p. 1196-1202.

26. Şimşek, M., T. Kocatürk, and Ş.D. Akbaş, *Dynamic behavior of an axially functionally graded beam under action of a moving harmonic load.* Composite Structures, 2012. **94**(8): p. 2358-2364.

27. Almeida, C.A., et al., *Geometric nonlinear analyses of functionally graded beams using a tailored Lagrangian formulation.* Mechanics Research Communications, 2011. **38**(8): p. 553-559.

28. Mata, P., S. Oller, and A.H. Barbat, *Dynamic analysis of beam structures considering geometric and constitutive nonlinearity.* Computer Methods in Applied Mechanics and Engineering, 2008. **197**(6–8): p. 857-878.

29. Virgin, L.N. and R.H. Plaut, *Postbuckling and vibration of linearly elastic and softening columns under self-weight.* International Journal of Solids and Structures, 2004. **41**(18–19): p. 4989-5001.

30. Esmailzadeh, E. and N. Jalili, *Parametric response of cantilever Timoshenko beams with tip mass under harmonic support motion.* International Journal of Non-Linear Mechanics, 1998. **33**(5): p. 765-781.

31. Briseghella, L., C.E. Majorana, and C. Pellegrino, *Dynamic stability of elastic structures: a finite element approach.* Computers & Structures, 1998. **69**(1): p. 11-25.

32. Kiusalaas, J., *Numerical methods in engineering with MATLAB*. 2005, Cambridge England ; New York: Cambridge University Press. viii, 426 p.