

PARALLEL COMPUTATION OF FINITE DIFFERENCE METHOD FOR HEAT
CONDUCTION IN DUCT PROBLEM

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The thesis is submitted in partial fulfillment of the
requirements for the award of
Master of Science (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

JANUARY 2012

To my beloved family, thanks for your love and support

For Azhari and all friends, thank you for fully guidance to complete this thesis.

ACKNOWLEDGEMENT

First of all, I thank Allah (SWT), the Lord Almighty, for giving me good health and strength ability to complete this thesis.

I would like to express my deepest gratitude to my supervisors, Dr Yeak Su Hoe, for his guidance and suggestion from the start until the end of this research. I also like to thank upon his thoughtful and constructive criticisms that lead me to achieve the objectives of my research.

I would like to convey my thanks to all my friends that help me a lot in finding information, checking my grammar and their support toward me throughout this research. Their help is valuable motivation for me to keep on doing this research. Thank you very much.

Also not forget to my parents and family members for their support and love for me. Their love and never ending support toward me is the strength that keeps me complete this research.

ABSTRACT

This dissertation is about to find a heat conduction in a curvature structure of duct. This problem will be solved using finite difference method and since the heat transfer in curvature is different from straight duct, heat equation must to be transformed into modified polar coordinate. OpenMP parallel was applied in this problem to speed up the calculation and the method used in parallel is Gaussian Elimination. Speedup for each processor will be calculated using formula $s(N) = \frac{T(1)}{T(N)}$ in order to know how fast this problem will be solved after applying the parallel method. The result shows that the temperature was decreasing gradually and became smaller at the end of duct and the graph show that area at inlet is bigger than area at outlet of duct. Obviously, results show that the geometry of shape affect the flow of heat transfer. Time reduction in parallel computation is due to parallel Gauss elimination method as well as parallel file handling technique.

ABSTRAK

Disertasi ini adalah bertujuan untuk mencari pengaliran haba di dalam saluran yang berbentuk melengkung. Kaedah beza terhingga digunakan untuk menyelesaikan masalah ini, namun persamaan haba haruslah diubah kepada ubah suai koordinat kutub memandangkan pengaliran haba di dalam saluran yang berbentuk melengkung adalah tidak sama dengan saluran yang berbentuk lurus. OpenMP selari telah diaplikasikan bagi menyelesaikan masalah ini dimana ianya bertujuan untuk mempercepatkan pengiraan suhu di dalam saluran dan kaedah dipilih untuk proses selari ini adalah penghapusan Gaussian. Kelajuan bagi setiap pemproses yang terlibat akan dikira dengan menggunakan formula $s(N) = \frac{T(1)}{T(N)}$ dan ianya adalah bertujuan untuk melihat kepantasan penyelesaian masalah ini setelah kaedah selari diaplikasikan. Keputusan menunjukkan bahawa suhu telah berkurang secara berperingkat-peringkat dan ianya menjadi semakin rendah apabila berada di hujung kawasan saluran. Selain itu graf juga telah menunjukkan saiz saluran pada bahagian awal adalah lebih besar berbanding di bahagian akhir. Kesimpulannya menunjukkan bahawa bentuk geometri mempengaruhi pengaliran haba di dalam saluran. Pengurangan masa dalam pengiraan selari adalah disebabkan oleh kaedah penghapusan Gaussian selari serta teknik pengendalian fail selari.

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NOMENCLATURE

θ	-Angle
$^{\circ}\text{C}$	-Celsius
π	-pi
m	-Gradient
R_i	-Radius form origin to any point at the region
Σ	-Summation
Δ	-The difference operator
T	-Temperature
a	-Radius from origin
b	-Width of duct outlet
c	-Width of duct inlet
r_{12}	-Width along the duct
R	-Length 0 to any point region
r	-Radius
$S(N)$	-Speedup
T_s	-The original single processor serial time

- T_{is} - The average of serial time
- T_p - The original single processor parallelizable time
- T_{ip} - The average of parallelizable time
- ∇^2 -The operators

CHAPTER 1

INTRODUCTION

1.0 Background of the Study

Heating is an important process especially in industry of raw material. Heat transfer (Heat conduction) is one of the processes that transfer heat from the low into high temperature. According to the First Law of Thermodynamics heat transfer changes the internal energy of both systems. The process of heat transfer is higher in solid because the network of the relatively fixed special relationship between atoms helps to transfer energy between them by vibration. However, this project we will more concentrate on heat transfer in duct geometry.

Ducts are the pipe systems that are use as a transporting fluid. The conveying fluid that flow through a duct system is air. Duct is also instrumental in capturing the material into the flow system. Heat transfer in a duct is dependent on the shape of a duct and that was proven by Shah and London (1978). They made progress on computational technique for this problem and they review that the heat transfers under developing and fully develop laminar flow in duct with many cross-sectional shapes.

The problem of heat transfer can be solved by many ways from the finite difference method which is by estimating the differential term with algebraic expression. For example the problem can be transformed into Cartesian coordinates, cylindrical coordinates or spherical coordinates.

A Laplace equation is the simplest example of explicit the partial differential equation. Potential theory is a general theory for the Laplace equation. A Laplace equation is a steady state heat equation for the heat conduction problem and the Laplace equation as follows

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$$

It is very hard to find the value of temperature by using a heat equation for a duct having curve structure. In this case the heat equation may be transformed into modified polar system, so that it will become much simpler to determine the temperature distribution in the structure. Besides that, grid point in duct is to be created by using C+ programming. The main purpose of grid in duct geometry is to approximate the heat equation using finite difference. The parallel computation which is OpenMP has been used to increase the speed up of calculation. From this, the result of the temperature in duct can be obtained faster and quickly.

1.2 Statement of the Problem

In this project we want to discuss the heat transfer in a curvature structure of duct. This problem will be solved by using finite difference method. The heat transfer in curvature structure is different from the straight duct. From this we need transform the heat equation into modified of polar coordinate system in order to solve the problem. The initial stage of approximating the temperature is the generation of a grid points using modified polar coordinate system. The language of C++ has been chosen to solve the problem. OpenMP has been used to speed up the calculations with parallel approaches.

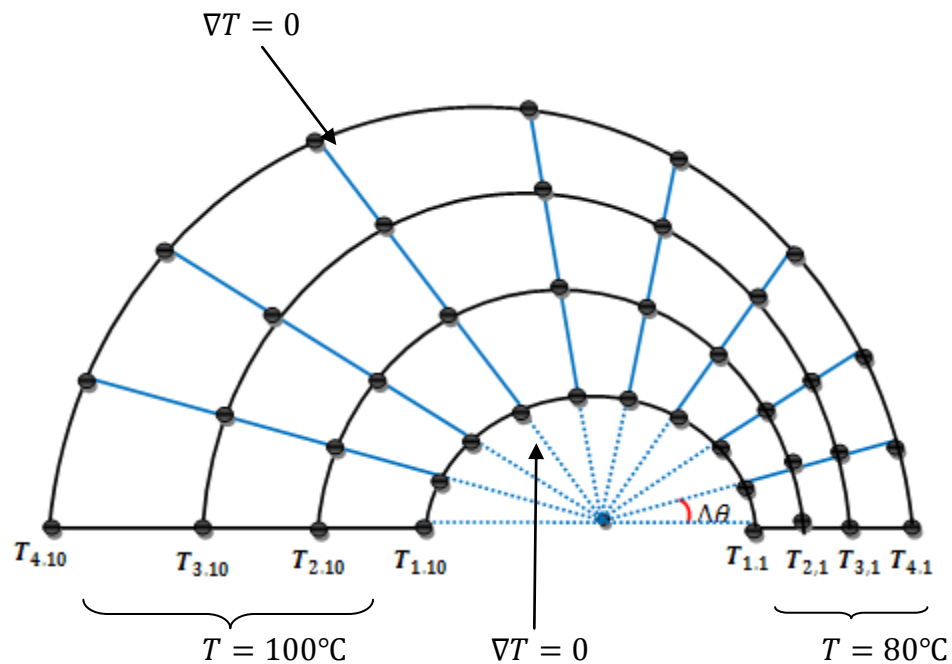


Figure 1.1 Grids in Duct for Heat Conduction

1.3 Objectives of the Study

The objectives of the study are:

- 1) To study the temperature in this geometry duct by using finite difference method
- 2) To represent the geometry of duct using modified polar coordinate system
- 3) To solve the problem of temperature in duct by using programs C++
- 4) To speed up calculation by using parallel computation based on OpenMP.

1.4 Significant of Study

Heat transfer problem is one of the most important problems in engineering field. It is hoped that the present numerical method will help all researchers and engineers to calculate the temperature distribute especially in duct geometry besides producing an accurate result. The solutions and result of this research can also be used as references by other future researchers.

1.5 Scope of the Study

In this research, the transformation of heat equation into modified polar system is been use to find the temperature in curve structure of duct. Thermal properties involved such as temperature and space are fixed, while the effect of radiation and small changes in the structure of the duct are negligible. Moreover a computer program based on C++ is developed to generate all the grids in order to solve heat equation.

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