RADIATION EFFECT ON LAMINAR BOUNDARY LAYER FLOW OF NANOFLUID OVER A FLAT PLATE

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To my beloved mother, father and siblings;

Mohd zin, Kamaliah, Muhammad Amsyar, Ahmad Nazirul Mubin and Mohammad Khidhir.

Thank you for everything!

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ABSTRACT

Nanofluids are a new class of advanced heat transfer fluids, which are liquids containing a dispersion of submicronic solid particles or nanoparticles. Thus, nanofluids have many applications in industry such as coolants, lubricants, heat exchanges and many more. In this study, the mathematical modelling of radiation effect on laminar boundary layer flow of nanofluid over a flat plate is investigated numerically. The basic governing nonlinear partial differential equations are transformed into a system of coupled nonlinear ordinary differential equations by using appropriate similarity transformation. These equations are solved numerically using implicit finite difference scheme known as Keller-box method. The system is discretized using finite difference method. Then, the nonlinear equations are linearized by Newton's method and solved by the block elimination method. The numerical codes in the form of software packages have been developed using Matlab to analyze the result. Finally, the numerical results are obtained for heat transfer, velocity and temperature profiles with the effect of Prandtl number, Pr and radiation parameter, N_r. These studies also discussed the effect of volume fraction on Prandtl number, density, heat capacity, thermal conductivity and viscosity for Copper and Alumina water Nanofluid respectively.

ABSTRAK

Nanobendalir merupakan satu kelas terkini permindahan haba bendalir dimana bendalir tersebut mengandungi penyerak mikro zarah pepejal atau dikenali sebagai nanopartikel. Oleh yang demikian, nanobendalir mempunyai banyak aplikasi di dalam industri seperti alat penyejuk, pelincir, pertukaran haba dan sebagainya. Di dalam kajian ini, permodelan matematik bagi kesan radiasi ke atas aliran lapisan lamina nanobendalir di atas plat yang rata dianalisis secara penyelesaian berangka. Persamaan terbitan separa tak linear yang asas diubah kepada suatu sistem persamaan terbitan biasa yang tak linear dengan menggunakan transformasi kesamaan yang bersesuaian. Persamaan ini diselesaikan secara berangka dengan menggunakan skema beza terhingga tersirat atau dikenali sebagai kaedah kotak Keller. Sistem ini diasingkan dengan kaedah beza terhingga. Kemudian, persamaan tak linear ini akan dilinearkan dengan menggunakan kaedah Newton dan akhirnya kaedah penghapusan blok digunakan bagi menyelesaikan persamaan ini. Matlab digunakan dalam menghasilkan kod berangka dalam bentuk pakej perisian bagi tujuan menganalisis keputusan. Akhirnya, keputusan berangka bagi pelbagai kesan nombor Prandtl, Pr dan parameter radiasi, N_r diperolehi untuk profil halaju dan suhu serta taburan pemindahan haba. Kajian ini turut membincangkan kesan isipadu pecahan terhadap nombor Prandtl, ketumpatan, kapasiti haba, kekonduksian terma dan kelikatan nanobendalir Kuprum dan Aluminium.

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LIST OF ABBREVIATIONS AND SYMBOLS

Roman Letters

$(C_P)_{nf}$	-	Specific heat of the nanofluid
f	-	Dimensionless stream function
k^*	-	Rosseland mean absorption coefficient
k_{nf}	-	Thermal conductivity of the nanofluid
N_r	-	Radiation parameter
$(Pr)_{nf}$	-	Prandtl number of the nanofluid
q_r	-	Radiation heat flux
T	-	Temperature
T_w	-	Temperature at surface of a plate
U_∞	-	Uniform velocity
u	-	Velocity component in x- direction
v	-	Velocity component in y- direction
x	-	Coordinate in direction of the uniform stream
y	-	Coordinate in direction normal to the plate

Greek Letters

ψ	-	Stream function
η	-	Dimensionless similarity variable
θ	-	Dimensionless temperature
δ	-	Boundary layer thickness
ϕ	-	Volume fraction

σ^*	-	Stefan-Boltzman constant
α_{nf}	-	Thermal diffusivity of the nanofluid
ν_{nf}	-	Kinematic viscosity of the nanofluid
ρ_{nf}	-	Density of the nanofluid
$(\rho C_P)_{nf}$	-	Heat capacitance of the nanofluid
μ_{nf}	-	Viscosity of the nanofluid

Superscripts

$'$	-	First differentiation with respect to η

CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter will discuss about the research background, problem statement, objectives, scopes, significance of the study and as well as introduction to chapters.

1.2 Research Background

Nanotechnology can be defined as a branch of technology dealing with manufacture of object with dimensions of less than 100 nanometres. It has been used in many different fields including engineering, chemistry, electronics and medicines. Nanofluid or so-called smart fluids is part of the nanotechnology which has been introduced by Choi on Argonne National Laboratory at 1995. The term nanofluid refer to a fluid that containing nanometer sized particles called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid, which have a better suspension stability compared to milimeter or micrometer sized one. It is typically made of metals, oxides, carbides or carbon nanotubes and usually there are two types of method that supplied the nanofluids called one and two step method.

According to Anjali Devi (2011) the goal of nanofluids is to achieve the highest possible thermal properties at the smallest possible concentrations by uniform dispersion and stable suspension of nanoparticles in host fluids. Meanwhile, nanofluids can enhance thermal conductivity of the base fluid enormously, which are also very stable and have no additional problems, such as sedimentation, erosion, additional pressure drop and non-Newtonian behaviour, due to the tiny size of nanoelements and the low volume fraction of nanoelements required for conductivity enhancement (Turkyilmazoglu, 2012). The applications of nanofluid can be found in many areas such as in transportation (engine cooling or vehicle thermal management), electronics applications (cooling microchips or microscale fluidic application), heat exchanger, nuclear reactor coolant, biomedical applications (nanodrug delivery, cancer theraupetics) and so on.

Besides that, there have been many studies in the boundary layer flow in fluid mechanics. The boundary layer refers to the layer of fluid in the immediate vicinity of a bounding surface where the effects of viscosity are significant. The concept of boundary layer was first introduced by a German engineer, Ludwig Prandtl in 1904 at the third International Congress Mathematicians in Heidelberg, Germany. He was the one who made fundamental contributions to finite wing theory and compressibility effects. The illustration of boundary layer flow is given in Figure 1.1.

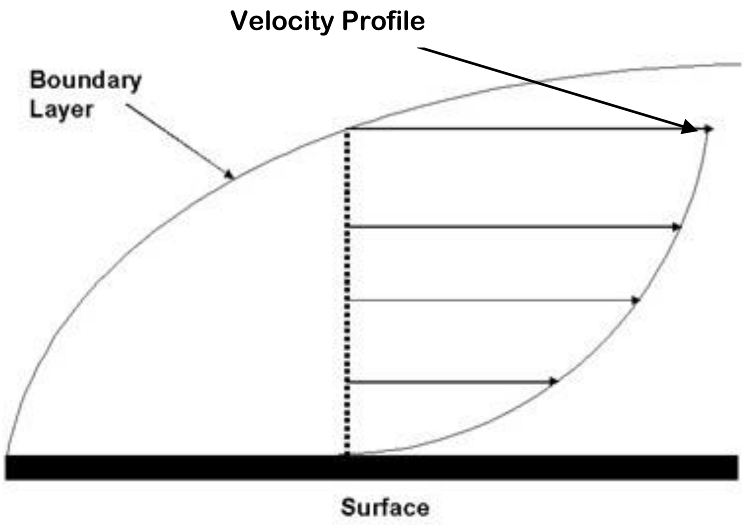

Figure 1.1: The illustration of boundary layer flow.

According to Prandtl theory, when a real flows past a stationary solid boundary, the flow are divided into two regions which are one inside the boundary layer dominated by viscosity and creating the majority of drag experienced by the boundary body. While the second one is an outer region, where the viscous force is very small and can be neglected. The possible areas for the applications of boundary layer theory includes aerodynamics (airplanes, rockets, projectiles), hydrodynamics (ships, submarines, torpedoes), transportations (automobiles, trucks, cycles), wind engineering (buildings, bridges, water towers), and ocean engineering (buoys, breakwaters, cables).

In addition, the study of thermal radiation also included in this research. According to Anbuchezhian *et al.* (2012), radiation comes from solar energy, and the resultant solar energized resources, such as wave power, wind, biomass, and hydroelectricity, all give an explanation for most of the accessible renewable energy that is present on the Earth. Meanwhile, thermal radiation refers to electromagnetic radiation generated by the thermal motion of charged particles in matter. It consists of ultraviolet rays, infra-red and light rays follows a nuclear explosion. The examples of major radiation exposure in real live events are Hiroshima and Nagasaki, Three

Mile Island, Chernobyl and so on. Radiation effects can be divided into two categories which are high dose and low dose. Normally, high doses tend to kill cells and producing acute or short term effects. Meanwhile, low doses tend to damage or change the cells and will cause chronic and long term effects as well.

Basically, rem is the unit to measure radiation dosage and it is stands for roentgen equivalent in man. It represents the amount of radiation needed to produce a particular amount of damage to living tissue. Examples of high dose effects are skin burn, hair loss, sterility, cataracts and acute radiation syndrome including hematopoietic, gastrointestinal and central nervous system. Besides that, the low doses effects will causes genetic effect, somatic effect- such as lung cancer, bone cancer, breast cancer, leukemia, skin cancer and in-utero effects- such as intrauterine death, growth retardation, developmental abnormalities and childhood cancers.

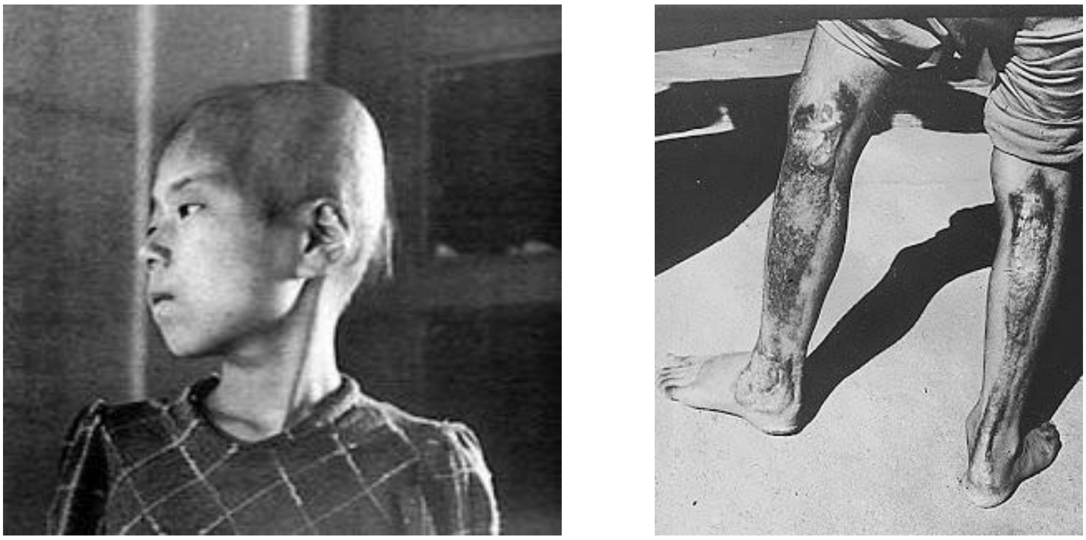

Figure 1.2: Illustrations of radiation effects on human.

Differential equations can be studied by three different approaches which are analytical, qualitative techniques and numerical analysis. In this study, numerical analysis is a tool that we interested with to analyze our models. Numerical analysis is one of area of mathematics and computer science that creates, analyzes, and implements algorithms for obtaining numerical solutions to problems involving continuous variable. Examples of numerical methods are Euler method, Runge-Kutta method, Finite Difference method, Newton method, Keller box method and many more.

There are many applications involved numerical analysis such as in modern business it makes use of optimization method in deciding how to allocate resources most efficiently, atmospheric modelling (computational fluid mechanics and numerical solutions of differential equations), computer aided design (CAD) and computer aided manufacturing (CAM) used of differential-algebraic systems. Moreover, it can also be found in computer software likes spreadsheet software, LAPACK project, Fortran, Fortran 95, C, C++, Java, MATLAB, Maple and so on.

Matrix laboratory or well known as MATLAB is one of the mathematical algebra software which are developed by Cleve Moler in late 1970's. MATLAB is a numerical computing environment and fourth-generation programming language developed by MathWorks. It allows matrix manipulations, plotting of functions and data, implementation of algorithm and many more. MATLAB can be used in many areas including signal processing and communications, image and video processing, control systems, test and measurement, computational finance and computational biology. Last but not least, MATLAB will help us in this research to plot several graphs in order to analyze our models.

1.3 Problem Statement

This research is actually extended from Anjali Devi (2011), which has studied about the laminar boundary layer flow of nanofluid over a flat plate. We extend the problem by added the effect of radiation. The problem statements that will be discussed in this research are as follows. How to transform the partial differential equation into ordinary differential equation? How to solve the ordinary differential equation numerically using Keller box method? What are the velocity and temperature profiles when the parameter of radiation and Prandtl number are considered?

1.4 Objectives of the Study

The objectives of the study are:

i. To transform partial differential equation into ordinary differential equation.
ii. To find the solutions for the equations by using Keller box method.
iii. To analyze the effect of radiation on laminar boundary layer flow of nanofluid over a flat plat using MATLAB software.

1.5 Scope of the Study

This research focus on the models of copper-water nanofluid and alumina-water nanofluid with the radiation effect on it. These models are considered to be in steady, incompressible, viscous and two dimensional. Then, the Keller box method is used in order to solve the nonlinear boundary layer equations numerically. Lastly, the velocity and temperature profile, as well as the heat transfer are plotted by using mathematical software, MATLAB.

1.5 Significance of the Study

This research gives us better understanding about the behaviour of these models in the presence of radiation effect on it. Furthermore, we can sharpen our knowledge about the mathematical software MATLAB and learn a new numerical method which is Keller Box method in order to solve this kind of problems. Lastly, it also can give us more information about the nanotechnology especially in the fields of mathematics and engineering.

1.7 Introduction to chapters

This report consists of five chapters counting this introductory chapter. Basically, Chapter 1 tells about the research background, problems statements, objectives of the study, scopes as well as the significance of the study. While, in Chapter 2 we look at the some works and achievements which have been done by some past researchers throughout their journals and papers. This reference will support our study with strong evidence from the result published.

Further, the next chapter is about the mathematical model. The aim of this chapter is to reduce the partial differential equations into second order ordinary differential equations by using the similarity transformation. The Keller-box method is explained in details in Chapter 4. Generally, there are a few steps in the Keller-box method which are finite difference method, Newton's method and block elimination method.

Meanwhile, Chapter 5 is about the result and discussion. The final set of matrices from Chapter 4 will be used to the mathematical algebra software, MATLAB. In order to get the results and visualization, an algorithm will be created and used to the programming. Finally, the analysis about the temperature and velocity profiles and other results will be the last part of this chapter. Last but not least, the summary of the research and recommendation of this study will be in the Chapter 6.

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