

CHARACTERIZING THE SHAPLEY VALUE TO DETERMINE THE VOTING
POWER IN MALAYSIAN GENERAL ELECTION 2013

KHAIRUL AMIR BIN ANUAR

A dissertation submitted in partial fulfillment of the requirements for the award of
the degree of Master of Science (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

JANUARY 2014

Special dedication to my supportive parents,
my lovely wife,
and my brilliant kids.

ACKNOWLEDGEMENTS

The process to complete the proposal is like a captain sailor the ship in the wide ocean. Many obstacles and uncertainties cannot be solved without these tremendous people around me. First and foremost, thank to the greatest, Allah S.W.T for giving me a life to live in this amazing world. I would like to give a thousand appreciations to my lecturers, especially to my tremendous supervisor, Tuan Haji Ismail Bin Kamis for his guidance, encouragement, comments, and responsibility indeed. He helped me a lot along in many ways, especially by ensuring that I had all the necessary information as a reference to complete the process.

In the same time, I would like to express my gratitude to my lovely family for their moral support, and motivation. They play such a wonderful role to boost my effort towards the finish line. Last but not least, I would like to thank people around me for their encouragement and assistance.

ABSTRACT

This study was carried out to investigate the use of Shapley-Shubik power index in modeling decision Malaysian general elections for 2013. The selection model based on the actual results of the 13th general election using the Shapley value, as proposed in this study is to determine the power of each party in the coalition to contest either the winning or losing combination. This was done by comparing the change in percentage of votes at 5%, 10% and 15% for each constituency. By using the Shapley value for each competing party, the study found that in most cases as a set percentage, UMNO is a major player or player who are indispensable axis by a coalition to win general election. Parti Pesaka Bumiputera Bersatu (PBB) is a great partner for UMNO to form a strong coalition. In addition, the use of Shapley-Shubik power index based on the actual results of the 13th general election is effective is to identify how the value of a party whether it is needed or not in determining a winning coalition. By using the Shapley value approach, an equitable distribution of federal cabinet seats in parliament, won the seat can be verified based on the data of 2013 federal cabinet seat.

ABSTRAK

Kajian ini dilakukan bertujuan mengkaji penggunaan kuasa indeks Shapley-Shubik dalam pemodelan keputusan pilihanraya umum Malaysia bagi tahun 2013. Pemilihan model yang berdasarkan keputusan sebenar pilihanraya umum ke-13 dengan menggunakan kaedah nilai Shapley sepertimana yang dicadangkan dalam kajian ini adalah bagi menentukan kuasa setiap parti yang bertanding samada di dalam gabungan yang menang mahupun gabungan yang kalah. Kajian dilakukan dengan membandingkan dari segi peratusan perubahan undi iaitu sebanyak 5%, 10% dan 15% bagi setiap kawasan parlimen. Dengan menggunakan nilai Shapley yang diperolehi bagi setiap parti yang bertanding, kajian mendapati bahawa dalam kebanyakan kes mengikut peratusan yang ditetapkan, UMNO merupakan pemain utama ataupun pemain paksi yang sangat diperlukan oleh sesuatu gabungan parti untuk memenangi pilihanraya umum. Manakala parti komponen yang menjadi pasangan terbaik untuk UMNO bagi membentuk gabungan parti yang mantap adalah Parti Pesaka Bumiputera Bersatu (PBB). Di samping itu, penggunaan kuasa indeks Shapley-Shubik berdasarkan keputusan sebenar pilihanraya umum ke-13 ini nyata berkesan untuk mengenalpasti betapa nilainya sesuatu parti samada sangat diperlukan atau tidak dalam menentukan kemenangan sesebuah gabungan parti. Dengan menggunakan pendekatan nilai Shapley, kesamarataan pembahagian kerusi cabinet persekutuan mengikut parlimen yang dimenangi dapat ditentusahkan berdasarkan data kerusi cabinet persekutuan 2013 yang sebenar.

TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	SUPERVISOR'S DECLARATION	
	TITLE	i
	STUDENT'S DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENT	iv
	ABSTRACT	v
	ABSTRAK	vi
	TABLE OF CONTENTS	vii
	LIST OF TABLES	x
	LIST OF FIGURES	xi
	LIST OF ABBREVIATIONS	xiii
	LIST OF APPENDICES	xv
1	INTRODUCTION	1
	1.1 Introduction	1
	1.2 Background of the Problem	2
	1.3 Statement of the Problem	2
	1.4 Objectives of the Study	3
	1.5 Scope of the Study	3
	1.6 Significance of the Study	4
2	LITERATURE REVIEW	5
	2.1 Introduction	5

2.2	Coalitional Form	6
2.3	Imputation and Core	7
2.3.1	Imputation	7
2.3.2	Essential Games	8
2.3.3	The Core	9
2.4	Shapley Value	10
2.4.1	Value Functions: The Shapley Axioms	10
2.4.2	An Alternative Form of the Shapley Value	12
2.4.3	Simple Games: The Shapley-Shubik Power Index	13
2.5	The Nucleolus	14
2.5.1	Properties of the Nucleolus	15
2.6	Voting Rules	16
2.6.1	Four Examples of Voting Rules	17
2.6.2	Criteria of Quality of Voting Rules	18
2.7	Related Study	19
3	RESEARCH METHODOLOGY	21
3.1	Introduction	21
3.2	Majority Games	22
3.3	The Shapley-Shubik Power Index	23
3.4	The result of 13 th Malaysia's Parliamentary General Election	25
3.5	Game 1: Computing the Shapley Value of a Majority Game	26
3.6	Game 2: Computing the Shapley Value With A Player of Zero Weight	28
4	RESULTS AND DISCUSSIONS	32
4.1	Introduction	32
4.2	Case 1 : Coalition According to Party	33
4.3	Case 2 : Minority into Majority	37

4.3.1	Effect of 5% Swing Voters into Winning Coalition	38
4.3.2	Effect of 10% Swing Voters into Winning Coalition	40
4.3.3	Effect of 15% Swing Voters into Winning Coalition	43
4.4	Case 3 : Majority the Minority	47
4.4.1	Effect of 5% Swing Voters into Losing Coalition	48
4.4.2	Effect of 10% Swing Voters into Losing Coalition	50
4.4.3	Effect of 15% Swing Voters into Losing Coalition	53
4.5	Fair Distribution of Malaysia Cabinet Seat 2013	56
5	SUMMARY AND CONCLUSION	58
5.1	Introduction	58
5.2	Summary	58
5.3	Conclusion	60
5.4	Recommendations	61
	REFERENCES	62
	APPENDICES A-H	64-91

LIST OF TABLES

TABLE NO.	TITLE	PAGE
3.1	The number of Seats According to Party and State, the 13th Parliament	25
3.2	Example of Game 1	26
3.3	Coalitions and Weight For Example of Game 1	27
3.4	Example of Game 2	29
3.5	Coalitions and Weight For Example of Game 2	29
4.1	List of Player and Seats for Case 1	33
4.2	Weight, Seat and Shapley Value According to the Party for Case 1	34
4.3	Weight, Seat and Shapley Value According to 5% of Swing Voters (Case 2)	38
4.4	Weight, Seat and Shapley Value According to 10% of Swing Voters (Case 2)	41
4.5	Weight, Seat and Shapley Value According to 15% of Swing Voters (Case 2)	44
4.6	Weight, Seat and Shapley Value According to 5% of Swing Voters (Case 3)	48
4.7	Weight, Seat and Shapley Value According to 10% of Swing Voters (Case 3)	51
4.8	Weight, Seat and Shapley Value According to 15% of Swing Voters (Case 3)	53

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
4.1a	Seat Percentage for Case 1	35
4.1b	Shapley Value for Case 1	35
4.2a	Seat Percentage According to 5% of Swing Voters (Case 2)	39
4.2b	Shapley value According to 5% of Swing Voters (Case 2)	39
4.3a	Seat Percentage According to 10% of Swing Voters (Case 2)	42
4.3b	Shapley value According to 10% of Swing Voters (Case 2)	42
4.4a	Seat Percentage According to 15% of Swing Voters (Case 2)	45
4.4b	Shapley value According to 15% of Swing Voters (Case 2)	45
4.5a	Seat Percentage According to 5% of Swing Voters (Case 3)	49
4.5b	Shapley value According to 5% of Swing Voters (Case 3)	49
4.6a	Seat Percentage According to 10% of Swing Voters (Case 3)	51
4.6b	Shapley value According to 10% of Swing Voters (Case 3)	52
4.7a	Seat Percentage According to 15% of Swing Voters (Case 3)	54

4.7b	Shapley value According to 15% of Swing Voters (Case 3)	54
4.8a	the Percentage of Cabinet Seat According the Party	56
4.8b	the Shapley Value of Cabinet Seat According the Party	57

LIST OF ABBREVIATIONS

ABBREVIATIONS	TITLE
TU	Transferable Utility
NTU	Non Transferable Utility
BN	Barisan Nasional
UMNO	United Malays National Organization
MCA	Malaysian Chinese Association
MIC	Malaysian Indian Congress
GERAKAN	The Malaysian People's Movement Party
PBB	Parti Pesaka Bumiputera Bersatu
SUPP	Sarawak United Peoples' Party
SPDP	Sarawak Progressive Democratic Party
PRS	Parti Rakyat Sarawak
UPKO	United Pasokmomogun Kadazandusun Murut Organization
PBS	Parti Bersatu Sabah
PBRs	Parti Bersatu Rakyat Sabah
LDP	The Liberal Democratic Party
PPP	Parti Progresif Penduduk
PR	Pakatan Rakyat
DAP	Democratic Action Party
PAS	Parti Islam Se-Malaysia
PKR	Parti Keadilan Rakyat
SAPP	Sabah Progressive Party
STAR	State Reform Party
PSM	Parti Sosialis Malaysia

KITA	Parti Kesejahteraan Insan Tanah Air
SWP	Sarawak Workers Party
BERJASA	Barisan Jemaah Islam Se Malaysia
BERSAMA	Parti Bersatu Sasa Malaysia

LIST OF APENDICES

APPENDIX	TITLE	PAGE
A	Code for the Shapley value by considering zero weight player	64
B	Code for the Shapley value by not considering zero weight player	65
C	List of Federal Cabinet 2013	66
D	Results of 13th State Parliamentary General Election	70
E	Logo of the Political Parties in Malaysian General Election 2013	87
F	Results of Malaysian General Election 2013 Based on Component Party	91

CHAPTER 1

INTRODUCTION

1.1 Introduction

Coalition formation is the process of joining together of two or more agents so as to achieve goals that individuals on their own cannot, or to achieve them more efficiently. Often, in such situations, there is more than one possible coalition and a player's payoff depends on the coalition it joins [1]. The Shapley Value, one of the game theory component will be used to analyze the study. The components for this chapter are the statement of the problem, the objectives, the scope, and the significance of the study. In game theory, a manner of fairly distributing both gains and costs to several players working in coalition. The Shapley value applies primarily in situations when the contributions of each player are unequal. The Shapley value ensures each player gains as much or more as they would have from acting independently. This is important, because otherwise there is no incentive for players to collaborate.

1.2 Background of the Problem

Game theory is one of the branches of the mathematical research that models and analyzes various situations of issues. More or equal to two individuals, or players, with either similar or different interests are taking actions or making decisions.

The theory is generally divided into two parts, the non-cooperative theory and the cooperative theory. In non-cooperative theory, communication between the players is allowed but the players are forbidden to make binding agreements. Different with cooperative theory, which allows communication of the players and allows binding agreement to be made. To figure out the solutions, the Shapley value provides a unique solution to coalition games and is used to evaluate a player's prospects of playing a game.

The cooperative theory is further divided into two classes of problems which are transferable utility (TU) and non-transferable utility (NTU). The feature of the transferable utility (TU) case is that the players may make side payments of utility as part of the agreement while the non-transferable utility (NTU), side payments are forbidden. The players may argue, threaten, and make agreements with each other but cannot agree to make side payments as there is no monetary unit available to them.

1.3 Statement of the Problem

This study is to evaluate the Shapley value power index held by the political parties in Malaysia as a player, by considering the coalitional form theory. The power index of political party is computed using the Shapley value of the party based

on the results of Malaysia's 2013 General Election. The General Election payoff is defined as the distribution of seats in the Cabinet.

1.4 Objectives of the Study

The objectives of the study are:

- 1) To use Shapley value to solve weighted majority game modeled based on the results of Malaysia's 2013 General Election.
- 2) To determine the effects of voters, either swing into the current government or opposition and identify the key player (if any).
- 3) To determine the effect of voters needed by both coalitions to get simple majority and two third majority.

1.5 Scope of the Study

The study focused on the cooperative theory with transferable-utility (TU). The Shapley value as the method used to solve the problem based on the results of Malaysian General Election 2013.

1.6 Significance of the Study

The study is made to analyze the effectiveness of game theory used in seat distribution in the Malaysian Cabinet. From the result, we can see either the distribution is fair or not. From there, we can determine either the payoff received by each party is appropriate.

The organization of this dissertation as follows, Chapter 1 describe the background of the problem, statement of the problem, the objectives, the scope and the significance of the study. Chapter 2 will analyzes the literature review of coalitional form, imputation, the Shapley value, the nucleolus, voting rules and past researches. Chapter 3 is about research methodology. Chapter 4 will describe the results and some discussion while chapter 5 will summarize and gives the conclusion of this study.

REFERENCES

1. Shaheen S., Fatima, Wooldridge, Michael & R. Jennings, Nicholas. *An Analysis of the Shapley value and Its Uncertainty for the Voting Game*. Springer-Verlag Berlin Heidelberg. 2006.
2. Bachrach, Y., Elkind, E., Meir, R., Pasechnik, D., Zuckerman, M., Rothe, J. & S. Rosenschein, J. *The Cost of Stability in Coalitional Games*. Springer-Verlag Berlin Heidelberg. 2009.
3. Kolokoltsov, V.N. & Malafayev, O.A. Understanding Game Theory: Introduction to the Analysis of Many Agent Systems with Competition and Cooperation. *Coalition and distribution*. World Scientific Publishing Co. Pte. Ltd. 2010. 81-93.
4. Van den Brink, Rene & Van der Laan, Gerard. *Axiomatizations of the normalized Banzhaf value and the Shapley value*. Social Choice and Welfare © Springer-Verlag Berlin Heidelberg. 1998.
5. Hinojosa, M.A. & Marmol, A.M. Game Theory: Strategies, Equilibria and Theorems. *Transferable Utility Games with Individual Goals*. Nova Science Publisher, Inc. 2009. 231-240.
6. Kalai, E. & Samet, D. *On Weighted Shapley Values*. International Journal of Game Theory, Vol. 16, Issue 3, page 205 -222. 1985
7. Radzik, Tadeusz. *A new look at the role of players' weights in the weighted Shapley value*. European Journal of Operational Research, page 407-416. 2012.
8. Ferguson, T.S. Game Theory. *Games In Coalitional Form*. Los Angeles: University of California. 2003.
9. Kolokoltsov, V.N. & Malafayev, O.A. Understanding Game Theory: Introduction to the Analysis of Many Agent Systems with Competition and Cooperation. *Presidents and dictators*. World Scientific Publishing Co. Pte. Ltd. 2010. 95-102.
10. Van Deemen, A. & Rusinowska, A. *Paradoxes of voting power in Dutch politics*. Kluwer Academic Publishers. 2003.

11. Holler J., M. & Nurmi, H. Power, Voting, and Voting Power : 30 Years After. *Reflections on Power, Voting, and Voting Power*. Springer Heidelberg New York Dordrecht London. Page 1-26. 2013.
12. Bolle, F. & Breitmoser, Y. Power, Freedom and Voting E-Book. *Coalition Formation, Agenda Selection and Power*. Springer-Verlag Berlin Heidelberg. Chapter 10, page 185-208. 2008.
13. Carreras, F. & Magana, A. *The Shapley-Shubik index for simple games with multiple alternatives*. Springer Science+Business Media, LLC. 2007.
14. *Official Results of Malaysian General Election 2013*. Retrieved at June 5th, 2013 from <http://www.spr.gov.my/>
15. *Official Website of the Parliament of Malaysia*. Retrieved at June 5th, 2013 from <http://www.parlimen.gov.my/statistik.html?uweb=dr&>
16. *Website of the list of Federal Cabinet 2013-2018*. Retrieved at June 5th, 2013 http://www.utusan.com.my/utusan/images/banner/senarai_kabinet_2013.PDF

APPENDIX A

Code for the Shapley value by considering zero weight player

```

clear all;
reply_n=input('Enter values of n:\n');
n=reply_n;
for i=1:n
    i
    Q(i,:)=input('Enter value of Q for row #');
end
for i=1:2^n-1
    i;
    Bin(i,:)=bitget(i,1:n);
end
Bin_times_Q=Bin*Q;
reply_T=input('Enter values of T\n');
T=reply_T;
for i=1:length(Bin)
    if Bin_times_Q(i)>T
        Bin_times_Q(i)=1;
    else
        if Bin_times_Q(i)==T
            switch Bin(i,n)
            case 1
                Bin_times_Q(i)=1;
            otherwise
                Bin_times_Q(i)=0;
            end
        else
            Bin_times_Q(i)=0;
        end
    end
end
Shapley_result=shapleyE(Bin_times_Q')

```

APPENDIX B

Code for the Shapley value by not considering zero weight player

```

clear all;
reply_n=input('Enter values of n:\n');
n=reply_n;
for i=1:n
    i
    Q(i,:)=input('Enter value of Q for row #');
end
for i=1:2^n-1
    i;
    Bin(i,:)=bitget(i,1:n);
end
Bin_times_Q=Bin*Q;
reply_T=input('Enter values of T\n');
T=reply_T;
for i=1:length(Bin)
    if Bin_times_Q(i)>T
        Bin_times_Q(i)=1;
    else
        Bin_times_Q(i)=0;
    end
end
Shapley_result=shapleyE(Bin_times_Q')

```

APPENDIX C

LIST of FEDERAL CABINET 2013

Prime Minister : Datuk Seri Najib Razak

Deputy Prime Minister : Tan Sri Muhyiddin Yassin

Minister of Prime Minister's Department:

Datuk Seri Jamil Khir Baharom

Datuk Seri Abdul Wahid Omar

Datuk Seri Idris Jala

Tan Sri Joseph Kurup Datuk Seri Shahidan Kassim

Nancy Shukry

Datuk Joseph Entulu Belaun

Datuk Paul Low Seng Kuan

Deputy Minister of Prime Minister's Department:

Datuk Razali Ibrahim

P.Waythamoorthy

Ministry of Finance

Minister I : Datuk Seri Najib Tun Razak

Minister II : Datuk Seri Ahmad Husni Hanadzlah

Deputy Minister : Datuk Ahmad Maslan

Ministry of Education

Minister I : Tan Sri Muhyiddin Yassin

Minister II : Datuk Seri Idris Jusoh

Deputy Minister I : Datuk Mary Yap

Deputy Minister II : P.Kamalanathan

Ministry of Home Affairs

Minister : Datuk Seri Ahmad Zahid Hamidi

Deputy Minister : Datuk Wan Junaidi Tuanku Jaafar

Ministry of Communications and Multimedia

Minister : Datuk Seri Ahmad Shabery Cheek
Deputy Minister : Datuk Jailani Johari

Ministry of International Trade and Industry

Minister : Datuk Seri Mustapa Mohamed
Deputy Minister : Hamim Samuri

Ministry of Natural Resources and Environment

Minister : Datuk Seri G. Palanivel
Deputy Minister : Datuk James Dawos Mamit

Ministry of Transport

Minister : Datuk Seri Hishammuddin Hussein (acting)
Deputy Minister : Datuk Aziz Kaprawi

Ministry of Tourism and Culture

Minister : Datuk Seri Mohamed Nazri Abdul Aziz
Deputy Minister : Datuk Joseph Salang

Ministry of Agriculture and Agro-Based Industry

Minister : Datuk Seri Ismail Sabri Yaakob
Deputy Minister : Datuk Tajuddin Abdul Rahman

Ministry of Defence

Minister : Datuk Seri Hishammuddin Hussein
Deputy Minister : Datuk Abdul Rahim Bakri

Ministry of Works

Minister : Datuk Fadillah Yusof
Deputy Minister : Datuk Rosnah Abdul Rashid Shirlin

Ministry of Health

Minister : Datuk Seri Dr S.Subramaniam
Deputy Minister : Datuk Seri Dr Hilmi Yahaya

Ministry of Youth and Sports

Minister : Khairy Jamaluddin Abu Bakar
Deputy Minister : Datuk M Saravanan

Ministry of Human Resources

Minister : Datuk Richard Riot
Deputy Minister : Datuk Ismail Abdul Muttalib

Ministry of Domestic Trade, Co-Operatives and Consumerism

Minister : Datuk Hasan Malek
Deputy Minister : Datuk Ahmad Bashah Md Hanipah

Ministry of Women, Family and Community Development

Minister : Datuk Rohani Abdul Karim
Deputy Minister : Datuk Azizah Mohd Dun

Ministry of Foreign Affairs

Minister : Datuk Seri Anifah Aman
Deputy Minister : Datuk Hamzah Zainuddin

Ministry of Urban Well-Being, Housing and Local Government

Minister : Datuk Abdul Rahman Dahalan
Deputy Minister : Datuk Halimah Sadique

Ministry of Plantation Industry and Commodities

Minister : Datuk Seri Douglas Uggah Embas
Deputy Minister : Datuk Noriah Kasnon

Ministry of Rural and Regional Development

Minister : Datuk Seri Mohd Shafie Apdal
Deputy Minister : Datuk Alexander Nanta Linggi

Ministry of Federal Territories

Minister : Datuk Seri Tengku Adnan Tengku Mansor
Deputy Minister : Datuk Dr. J. Loga Bala Mohan

Ministry of Energy, Green Technology and Water

Minister : Datuk Seri Maximus Ongkili
Deputy Minister : Datuk Dr Abu Bakar Mohamad Diah

Ministry of Science, Technology and Innovation

Minister : Datuk Dr Ewon Ebin
Deputy Minister : Datuk Dr Abu Bakar Mohamad Diah