FINITE ELEMENT INVESTIGATION OF TRANSMISSION CONDITIONS FOR THIN INTERPHASES

HAMID MOZAFARI

UNIVERSITI TEKNOLOGI MALAYSIA

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HAMID MOZAFARI

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DEDICATION

To

My father and my mother

My beloved Wife

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In the name of God, Most Gracious, Most Merciful. Praise is to God for His Mercy which has enabled me to complete this work.

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ABSTRACT

Nowadays, thin interphases are used for modern technology. A composite structure obtained in such a way exhibits a wide variety of thermal and mechanical properties. In this thesis, imperfect transmission conditions (ITCs) are discussed based on the finite element method (FEM) for a soft elasto-plastic interphase in a plane stress state, as well as thin reactive heat-conducting interphases, where the transmission conditions are nonlinear. The ITCs of a thin reactive two-dimensional interphase between two bonded materials in a dissimilar strip have been investigated. The validity of the transmission conditions for the heat conducting interphases has been analysed for three formulations of a reactive layer: with no source formulation, with constant source formulation, and with a temperature-dependent source formulation. In addition, the ITCs were evaluated in the most general form for several cases, demonstrating the high efficiency of the approach. This showed that it is possible to reconstruct the full solution inside the interphase using the information available for the respective imperfect interface of zero thickness. For the case of mechanical problems, it explains a thin elasto-plastic interphase layer, which is situated between two different elastic media. The intermediate layer consists of a soft elasto-plastic material with a small Young’s modulus in comparison with those of the surrounding materials. The two-dimensional nonlinear transmission conditions for the bi-material structures were investigated using an asymptotic technique. This study evaluated the ITCs for a thin interphase layer with an adhesive joint, along with the mechanical behaviour of the bonded materials. Finally, the good accuracy of the nonlinear imperfect transmission conditions of the approach presented in this thesis is shown, along with the excellent performance of the finite element analysis of the thin elasto-plastic interphases and thin heat-conducting interphases.

ABSTRAK

Bahan nipis antara fasa banyak digunakan dalam teknologi moden oleh kerana bahan tersebut mempunyai sifat terma dan mekanikal yang luas. Struktur komposit, yang mana bentuk dan dimensinya membolehkan ia dikategori sebagai bahan antara fasa dikaji di sini. Di dalam tesis ini, keadaan transmisi tak sempurna (ITC) untuk bahan lembut elastik-plastik antara fasa dibincang berasaskan Kaedah Unsur Terhingga (FEM). Untuk kajian bebanan mekanikal, bahan ini adalah dalam keadaan tegasan satah dan untuk kajian bebanan, termal bahan antara fasa akan bertindak balas terhadap pemindahan haba yang mana keadaan transmisi adalah lelurus. Bahan antara fasa adalah jalur nipis duadimensi yang dilekatkan di antara dua bahan berlainan. Kesahihan sama ada bahan mampu membuat simulasi pemindahan haba akan diketahui apabila keadaan transmisi dianalisa untuk tiga formulasi tindakbalas. Formulasi yang pertama ialah keadaan tanpa sumber haba, yang kedua ialah lapisan tindakbalas mempunyai sumber haba yang malar dan yang ketiga ialah sumber haba bergantung kepada suhu. Keadaan ITC telah dinilai secara umum untuk beberapa kes untuk menunjukkan prestasi efisyen pendekatan ini. Keputusannya ialah kaedah ini membolehkan pembinaan penyelesaian menyeluruh di dalam bahan antara fasa dengan menggunakan maklumat yang terdapat pada lapisan tak sempurna yang berketebalan sifar. Untuk kes masalah mekanikal satu penyelesaian dibina untuk lapisan nipis elastik-plastik yang terletak antara dua bahan elastik. Lapisan pertengahan ini mempunyai modulus elastik yang terlalu kecil dibandingkan dengan dua bahan pengapit. Transmisi tak lelurus dua dimensi untuk struktur yang diperbuat daripada dua bahan, diselidik dengan menggunakan teknik asimtotik. Kajian ini telah dapat menilai ITC dan gayalaku mekanikal untuk lapisan antara fasa nipis, dengan contoh bahan pelekat untuk menghubungkan dua bahan berlainan. Satu keputusan jitu untuk keadaan transmisi tak sempurna dan tak lelurus, telah diperolehi. Keputusan ini menunjukkan prestasi FEM yang memuaskan.

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LIST OF ABBREVIATIONS

ABS - Acrylonitrile butadiene styrene
Al - Aluminum
BC - Boundary condition
BEM - Boundary element method
CFRP - Carbon fiber-reinforced composite
CTD - Constant temperature distribution
EP - Epoxy resin
FDM - Finite difference method
FEM - Finite element method
FRP - Fiber reinforced plastics
GFRP - Glass fiber-reinforced composite
ITC - Imperfect transmission condition
LHS - Left hand side
LTD - Linear temperature distribution
PMMA - Polymethylmethacrylate
PTD - Parabolic temperature distribution
PVC - Polyvinyl chloride
RHS - Right hand side
SMC - Sheet moulding compound
St - Steel
TC - Transmission condition

LIST OF SYMBOLS

A - Area

B_1, B_2, C - Constant values

c - Specific heat

E - Young’s modulus

F - Force

F_x, F_y - Transmission condition function

G - Shear modulus

$2h$ - Thickness of interphase

$h_*, \tilde{h}$ - Position parameters

H - Thickness

k - Thermal conductivity

$k^*, \tilde{k}, k_\Lambda$ - Arbitrary functions

$k_{t,0}$ - Huber-Mises stress

$\bar{k}$ - Spring stiffness

n - Unit normal to interface

q - Heat flux

q_x - x-Component of heat flux

q_y - y-Component of heat flux

q^* - Arbitrary parameter

Q - Thermal source
R - Constant parameter
t - Time
$T^{*}, \tilde{T}$ - Arbitrary parameters
u, u_x, u_y - Components of displacement
L - Length
T - Temperature
V - Volume
x, y, z - Coordinate systems
ΔT - Temperature difference
α, β, χ - Small dimension parameters
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$ - Shear strain
ε - Strain
ε_{eff}^{p} - Effective plastic strain
ε_{eq} - Equivalent strain
ζ, ξ - Variable components
$I(D_{\varepsilon})$ - Invariant of strain tensor
$I(D_{\sigma})$ - Invariant of stress tensor
λ^{*}, μ^{*} - Lame’s coefficient
$\lambda_{\pm}, \Gamma_{\pm}$ - Upper and lower interface
υ - Poisson’s ratio
ρ - Density
σ - Stress
σ_{eff} - Effective stress

σ_{eq}	-	Equivalent stress
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	-	Shear stress
κ	-	Engineering constant parameter
ϕ, φ, ψ	-	Respective functions
Θ, Φ	-	Auxiliary functions
Ω, Π	-	Domains

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter consists of four sections that provide an overview of the research. First, it discusses the background of the problem. Then, it states the problem and explains the scopes and objectives of the research. And lastly, it delineates the chapters of the thesis.

1.2 Background of the Problem

Thin interphases such as adhesive layers are commonly used in the modern technology industry. A composite structure obtained in such a way exhibits a wider variety of thermal and mechanical properties. On the other hand, finite element modeling (FEM) of composites with thin interphases is still a difficult numerical task as it requires high inhomogeneity of the constructed mesh, which can lead to a loss of accuracy and even numerical instability. This explains the high interest in modeling the interphase as a zero-thickness object described by specific so-called “transmission conditions” along the infinitesimal interface.

The obtained transmission conditions may be used to derive new finite element formulations in order to overcome the above-mentioned problems in the scope of a finite element approach.

1.3 Statement of the Problem

For many practical thin interphase layer problems it is not possible to obtain a solution by means of analytical techniques. Instead, solving them requires the use of numerical methods, which in many cases allow such problems to be solved quickly. Often, an engineer can easily see the effect of changes in parameters when modeling a problem numerically. This way is much faster, and tends to be more inexpensive than assembling and working with the actual experimental apparatus. In this project, transmission condition modeling of a thin intermediate layer between two bonded materials in a dissimilar strip will be derived and analyzed for heat conduction problems and mechanical problems. The validity of these transmission conditions for heat conduction problems and mechanical problems will be investigated with the finite element method (FEM) for several formulations of a thin interphase layer.

Consider a bi-material domain with a thin interphase layer between two materials (Figure 1.1) described by our research.

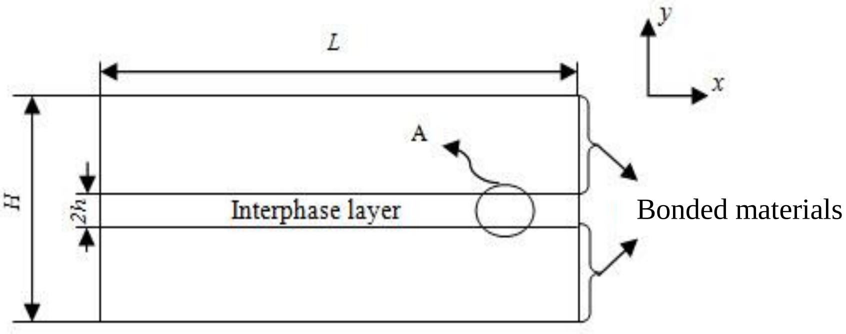

Figure 1.1: Illustration of the specimen problem

The interphase is assumed to be very thin ($h << H$) so that $h = \beta h_0$ while β is a small dimension parameter $\beta << 1$.

Two materials are applied at the top and the bottom of the interphase layer in the same elements size as shown in Figure 1.1.

The value $2h$ in this case is the small parameter β. The two-dimensional FE-mesh is built from 16 and 18 elements along the y-axis into interphase layer (Figure 1.2) which can be evaluated by the transmission conditions for heat-conducting and mechanical problems.

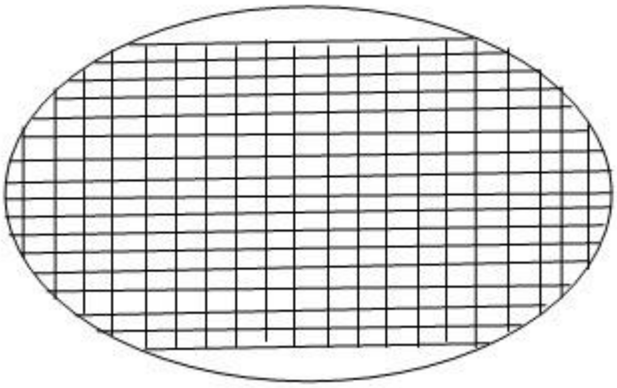

Figure 1.2: Schematic representation of the problem from Figure 1.2 (part of A)

In this research, we will investigate all the possible interfaces which can be evaluated by the transmission conditions for all of these cases. We would like refer here to other methods to deal with thin interphase, as well as to construct effective homogenized properties of composite materials.

1.4 The Research Objectives

The objectives of this study can be summarized as follows:

1) To find transmission conditions for heat-conducting problems in thin interphase layers.

2) To investigate transmission conditions for mechanical problems in thin interphase layers.

3) Validation of obtained result by comparing with earlier research findings.

4) To Investigative different interfaces and evaluate the transmission conditions and validation of obtained result with analytical method

1.5 The Scopes of Research

The scope of this study can be summarized as follows:

1) Finite element modeling of a bi-material structure.

2) Transmission conditions modeling a thin reactive 2D intermediate layer between two bonded materials.

3) Finite element modeling of a thin elasto-plastic interface between different materials

1.5 The Arrangement of Chapters

In this thesis, we apply FEM to investigate the transmission conditions for thin interphase layers. The challenges fall into two categories – heat-conducting problems and mechanical problems. The thesis is organized as follows:

Chapter 2 discusses a literature review of interphases and their applications in technology and then, it defines the transmission conditions and gives a history of literature review for thin interphases.

Chapter 3 describes methodology for the overall research project. Comprehensive flowcharts of the research methodology show the objectives of the proposed research project.

Chapter 4 investigate transmission conditions for thin reactive heat-conducting interphases and universal transmission conditions for several interphase cases such as; without source, constant source and temperature-dependent source formulations.

Chapter 5 explains a thin elasto-plastic interphase layer which is situated between two different elastic media. Thin adhesive layer consists a soft elasto-plastic material behavior whose Young’s modulus is small enough in comparison with those of surrounding materials.

In chapter 6, imperfect transmission conditions simulate a thin two-dimensional interphase layer between two bonded materials based on the finite element method (FEM). The numerical results validate the equations of chapter 4 for the different cases, namely thin reactive adhesive layers for various sources and temperature distributions that are explained in chapter 4.

In chapter 7, imperfect transmission conditions simulate for a thin elasto-plastic interphase on plane stress case based on FEM. The numerical results validate the equations of chapter 5 for the different cases and boundary conditions.

Finally, Chapter 8 presents some conclusions and recommendations for the future work that could be done in this field.

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