

FINITE ELEMENT INVESTIGATION OF TRANSMISSION CONDITIONS FOR  
THIN INTERPHASES

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FINITE ELEMENT INVESTIGATION OF TRANSMISSION CONDITIONS FOR  
THIN INTERPHASES

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**DEDICATION**

*To*

*My father and my mother*

*My beloved Wife*

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In the name of God, Most Gracious, Most Merciful. Praise is to God for His Mercy which has enabled me to complete this work.

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## ABSTRACT

Nowadays, thin interphases are used for modern technology. A composite structure obtained in such a way exhibits a wide variety of thermal and mechanical properties. In this thesis, imperfect transmission conditions (ITCs) are discussed based on the finite element method (FEM) for a soft elasto-plastic interphase in a plane stress state, as well as thin reactive heat-conducting interphases, where the transmission conditions are nonlinear. The ITCs of a thin reactive two-dimensional interphase between two bonded materials in a dissimilar strip have been investigated. The validity of the transmission conditions for the heat conducting interphases has been analysed for three formulations of a reactive layer: with no source formulation, with constant source formulation, and with a temperature-dependent source formulation. In addition, the ITCs were evaluated in the most general form for several cases, demonstrating the high efficiency of the approach. This showed that it is possible to reconstruct the full solution inside the interphase using the information available for the respective imperfect interface of zero thickness. For the case of mechanical problems, it explains a thin elasto-plastic interphase layer, which is situated between two different elastic media. The intermediate layer consists of a soft elasto-plastic material with a small Young's modulus in comparison with those of the surrounding materials. The two-dimensional nonlinear transmission conditions for the bi-material structures were investigated using an asymptotic technique. This study evaluated the ITCs for a thin interphase layer with an adhesive joint, along with the mechanical behaviour of the bonded materials. Finally, the good accuracy of the nonlinear imperfect transmission conditions of the approach presented in this thesis is shown, along with the excellent performance of the finite element analysis of the thin elasto-plastic interphases and thin heat-conducting interphases.

## ABSTRAK

Bahan nipis antara fasa banyak digunakan dalam teknologi moden oleh kerana bahan tersebut mempunyai sifat terma dan mekanikal yang luas. Struktur komposit, yang mana bentuk dan dimensinya membolehkan ia dikategori sebagai bahan antara fasa dikaji di sini. Di dalam tesis ini, keadaan transmisi tak sempurna (ITC) untuk bahan lembut elastik-plastik antara fasa dibincang berasaskan Kaedah Unsur Terhingga (FEM). Untuk kajian bebanan mekanikal, bahan ini adalah dalam keadaan tegasan satah dan untuk kajian bebanan, termal bahan antara fasa akan bertindak balas terhadap pemindahan haba yang mana keadaan transmisi adalah lurus. Bahan antara fasa adalah jalur nipis duadimensi yang dilekatkan di antara dua bahan berlainan. Kesahihan sama ada bahan mampu membuat simulasi pemindahan haba akan diketahui apabila keadaan transmisi dianalisa untuk tiga formulasi tindakbalas. Formulasi yang pertama ialah keadaan tanpa sumber haba, yang kedua ialah lapisan tindakbalas mempunyai sumber haba yang malar dan yang ketiga ialah sumber haba bergantung kepada suhu. Keadaan ITC telah dinilai secara umum untuk beberapa kes untuk menunjukkan prestasi efisien pendekatan ini. Keputusannya ialah kaedah ini membolehkan pembinaan penyelesaian menyeluruh di dalam bahan antara fasa dengan menggunakan maklumat yang terdapat pada lapisan tak sempurna yang berketebalan sifar. Untuk kes masalah mekanikal satu penyelesaian dibina untuk lapisan nipis elastik-plastik yang terletak antara dua bahan elastik. Lapisan pertengahan ini mempunyai modulus elastik yang terlalu kecil dibandingkan dengan dua bahan pengapit. Transmisi tak lurus dua dimensi untuk struktur yang diperbuat daripada dua bahan, diselidik dengan menggunakan teknik asimtotik. Kajian ini telah dapat menilai ITC dan gayalaku mekanikal untuk lapisan antara fasa nipis, dengan contoh bahan pelekat untuk menghubungkan dua bahan berlainan. Satu keputusan jitu untuk keadaan transmisi tak sempurna dan tak lurus, telah diperolehi. Keputusan ini menunjukkan prestasi FEM yang memuaskan.

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**LIST OF ABBREVIATIONS**

ABS	-	Acrylonitrile butadiene styrene
Al	-	Aluminum
BC	-	Boundary condition
BEM	-	Boundary element method
CFRP	-	Carbon fiber-reinforced composite
CTD	-	Constant temperature distribution
EP	-	Epoxy resin
FDM	-	Finite difference method
FEM	-	Finite element method
FRP	-	Fiber reinforced plastics
GFRP	-	Glass fiber-reinforced composite
ITC	-	Imperfect transmission condition
LHS	-	Left hand side
LTD	-	Linear temperature distribution
PMMA	-	Polymethylmethacrylate
PTD	-	Parabolic temperature distribution
PVC	-	Polyvinyl chloride
RHS	-	Right hand side
SMC	-	Sheet moulding compound
St	-	Steel
TC	-	Transmission condition

## LIST OF SYMBOLS

$A$	-	Area
$B_1, B_2, C$	-	Constant values
$c$	-	Specific heat
$E$	-	Young's modulus
$F$	-	Force
$F_x, F_y$	-	Transmission condition function
$G$	-	Shear modulus
$2h$	-	Thickness of interphase
$h_*, \tilde{h}$	-	Position parameters
$H$	-	Thickness
$k$	-	Thermal conductivity
$k^*, \tilde{k}, k_\lambda$	-	Arbitrary functions
$k_{t,0}$	-	Huber-Mises stress
$\bar{k}$	-	Spring stiffness
$n$	-	Unit normal to interface
$q$	-	Heat flux
$q_x$	-	$x$ -Component of heat flux
$q_y$	-	$y$ -Component of heat flux
$q^*$	-	Arbitrary parameter



$Q$	-	Thermal source
$R$	-	Constant parameter
$t$	-	Time
$T^*, \tilde{T}$	-	Arbitrary parameters
$u, u_x, u_y$	-	Components of displacement
$L$	-	Length
$T$	-	Temperature
$V$	-	Volume
$x, y, z$	-	Coordinate systems
$\Delta T$	-	Temperature difference
$\alpha, \beta, \chi$	-	Small dimension parameters
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	-	Shear strain
$\varepsilon$	-	Strain
$\varepsilon_{eff}^p$	-	Effective plastic strain
$\varepsilon_{eq}$	-	Equivalent strain
$\zeta, \xi$	-	Variable components
$I(D_\varepsilon)$	-	Invariant of strain tensor
$I(D_\sigma)$	-	Invariant of stress tensor
$\lambda^*, \mu^*$	-	Lame's coefficient
$\lambda_\pm, \Gamma_\pm$	-	Upper and lower interface
$\nu$	-	Poisson's ratio
$\rho$	-	Density
$\sigma$	-	Stress
$\sigma_{eff}$	-	Effective stress

$\sigma_{eq}$	-	Equivalent stress
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	-	Shear stress
$\kappa$	-	Engineering constant parameter
$\phi, \varphi, \psi$	-	Respective functions
$\Theta, \Phi$	-	Auxiliary functions
$\Omega, \Pi$	-	Domains

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## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Introduction**

This chapter consists of four sections that provide an overview of the research. First, it discusses the background of the problem. Then, it states the problem and explains the scopes and objectives of the research. And lastly, it delineates the chapters of the thesis.

#### **1.2 Background of the Problem**

Thin interphases such as adhesive layers are commonly used in the modern technology industry. A composite structure obtained in such a way exhibits a wider variety of thermal and mechanical properties. On the other hand, finite element modeling (FEM) of composites with thin interphases is still a difficult numerical task as it requires high inhomogeneity of the constructed mesh, which can lead to a loss of accuracy and even numerical instability. This explains the high interest in modeling the interphase as a zero-thickness object described by specific so-called “transmission conditions” along the infinitesimal interface.

The obtained transmission conditions may be used to derive new finite element formulations in order to overcome the above-mentioned problems in the scope of a finite element approach.

### 1.3 Statement of the Problem

For many practical thin interphase layer problems it is not possible to obtain a solution by means of analytical techniques. Instead, solving them requires the use of numerical methods, which in many cases allow such problems to be solved quickly. Often, an engineer can easily see the effect of changes in parameters when modeling a problem numerically. This way is much faster, and tends to be more inexpensive than assembling and working with the actual experimental apparatus. In this project, transmission condition modeling of a thin intermediate layer between two bonded materials in a dissimilar strip will be derived and analyzed for heat conduction problems and mechanical problems. The validity of these transmission conditions for heat conduction problems and mechanical problems will be investigated with the finite element method (FEM) for several formulations of a thin interphase layer.

Consider a bi-material domain with a thin interphase layer between two materials (Figure 1.1) described by our research.

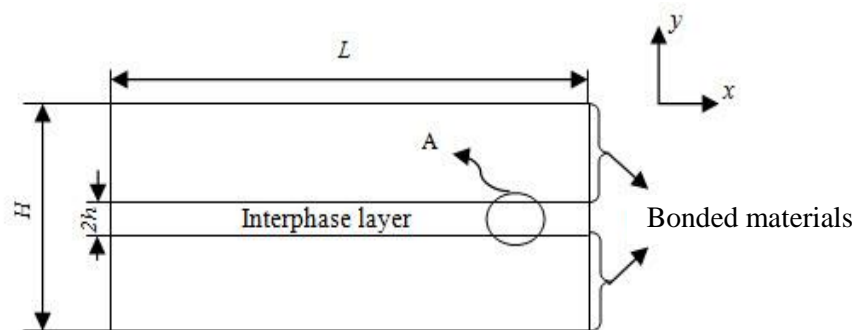


Figure 1.1: Illustration of the specimen problem

The interphase is assumed to be very thin ( $h \ll H$ ) so that  $h = \beta h_0$  while  $\beta$  is a small dimension parameter  $\beta \ll 1$ .

Two materials are applied at the top and the bottom of the interphase layer in the same elements size as shown in Figure 1.1.

The value  $2h$  in this case is the small parameter  $\beta$ . The two-dimensional FE-mesh is built from 16 and 18 elements along the  $y$ -axis into interphase layer (Figure 1.2) which can be evaluated by the transmission conditions for heat-conducting and mechanical problems.

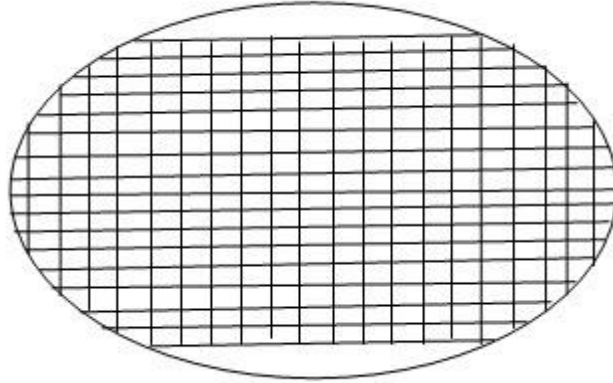


Figure 1.2: Schematic representation of the problem from Figure 1.2 (part of A)

In this research, we will investigate all the possible interfaces which can be evaluated by the transmission conditions for all of these cases. We would like refer here to other methods to deal with thin interphase, as well as to construct effective homogenized properties of composite materials.

#### 1.4 The Research Objectives

The objectives of this study can be summarized as follows:

- 1) To find transmission conditions for heat-conducting problems in thin interphase layers.
- 2) To investigate transmission conditions for mechanical problems in thin interphase layers.
- 3) Validation of obtained result by comparing with earlier research findings.
- 4) To Investigative different interfaces and evaluate the transmission conditions and validation of obtained result with analytical method

## **1.5 The Scopes of Research**

The scope of this study can be summarized as follows:

- 1) Finite element modeling of a bi-material structure.
- 2) Transmission conditions modeling a thin reactive 2D intermediate layer between two bonded materials.
- 3) Finite element modeling of a thin elasto-plastic interface between different materials

## **1.5 The Arrangement of Chapters**

In this thesis, we apply FEM to investigate the transmission conditions for thin interphase layers. The challenges fall into two categories – heat-conducting problems and mechanical problems. The thesis is organized as follows:

Chapter 2 discusses a literature review of interphases and their applications in technology and then, it defines the transmission conditions and gives a history of literature review for thin interphases.

Chapter 3 describes methodology for the overall research project. Comprehensive flowcharts of the research methodology show the objectives of the proposed research project.

Chapter 4 investigate transmission conditions for thin reactive heat-conducting interphases and universal transmission conditions for several interphase cases such as; without source, constant source and temperature-dependent source formulations.

Chapter 5 explains a thin elasto-plastic interphase layer which is situated between two different elastic media. Thin adhesive layer consists a soft elasto-plastic material behavior whose Young's modulus is small enough in comparison with those of surrounding materials.

In chapter 6, imperfect transmission conditions simulate a thin two-dimensional interphase layer between two bonded materials based on the finite element method (FEM). The numerical results validate the equations of chapter 4 for the different cases, namely thin reactive adhesive layers for various sources and temperature distributions that are explained in chapter 4.

In chapter 7, imperfect transmission conditions simulate for a thin elasto-plastic interphase on plane stress case based on FEM. The numerical results validate the equations of chapter 5 for the different cases and boundary conditions.

Finally, Chapter 8 presents some conclusions and recommendations for the future work that could be done in this field.



## REFERENCES

- [1] Housenick J., Pascarella V. Recent Advances in Structural Adhesive Technology for Bonding SMC and other Forms of Interphase. *SPI Composite Institute 49th Annual Conference*. 7<sup>th</sup>-10<sup>th</sup> Feb. Cincinnati, 1994. 627.
- [2] Dunn D. J. *Adhesives and Sealants-Technology, Application and Markets*. Shawbury, Rapra Technology Ltd. 164-176; 2003
- [3] Hashim S. A., Loke K. L. The Behavior of Adhesively Bonded-Beams Versus Their Welded Equivalent. *Proceedings of a Conference Held Glasgow*. 10-13 September. London. 2002. 39-42.
- [4] Banks W. M., Boinard P., Pethrick R. A. Use of Dielectric Spectroscopy to Assess Adhesively Bonded Composite Structures. III. Investigation of Mechanical Strength. *Journal of Adhesion*. 2002. 78(12): 1027-38.
- [5] Mehrkam P. A., Cochran R. Evaluation of Materials for Composite Repair Applications. *Journal of Thermoplastic Composite Materials*. 1997. 10(1): 51-60.
- [6] Tsukruk V. V. Molecular Lubricants and Glues for Micro and Nano Devices. *Journal of Advanced Materials*. 2001. 13(2): 95-108.
- [7] Connell J. W., Smith J. G., Hergenrother P. M. Chemistry and Adhesive Properties of Phenylethynyl-Terminated Phenylquinoxaline Oligomers. *Journal of Adhesion*. 1997. 60(1-4): 15-26.
- [8] Gouri C., Nair C. P. R., Ramaswamy R. Adhesive Characteristics of Alderene Adduct of Diallyl Bisphenol a Novolac and Bisphenol a Bismaleimide. *High Performance Polymers*. 2002. 12(4): 497-514.

- [9] Gouri C., Reghunadhan Nair C. P., Ramaswamy R. Adhesive and Thermal Characteristics of Maleimidefunctional Novolac Resins. *Journal of Applied Polymer Science*. 1999. 73(5): 695-705
- [10] Chuang C. H. K. *Aromatic Diamines and Polyimides Based On 4, 4'*. Patent No. US6069278. 2000.
- [11] Tan B., Vasudevan V., Lee Y. J., Gardner S., Davis R. M., Bullions T., Loos A. C., Parvatareddy H., Dillard D. A., McGrath J. E., Cella J. Design and Characterization of Thermosetting Polyimide Structural Adhesive and Composite Matrix Systems. *Journal of Polymer Science*. 1997. 35(14): 2943-54
- [12] Wolfe K. L., Dillard J. G., Harp S. R., Grant J. W. Plasma-Sprayed Aluminium and Titanium Adherends. ii. Durability Studies for Wedge Specimens Bonded with Polyimide Adhesive. *Journal of Adhesion*. 1997, 60(1-4): 141-52
- [13] Ellison M. M., Taylor L. T., Sulfur Versus Non-Sulfur Containing Polyimide Adhesives for Bonding Steel. *Journal of Adhesion*. 1997, 60(1-4): 51-69
- [14] Jensen B J; Hou T H; Wilkinson S P. Adhesive and Composite Properties of Larc-8515 Polyimide. *High Performance Polymers*. 1995. 7(1): 11-21
- [15] Hergenrother P. M., Smith J. G., Connell J. W. Synthesis and Properties of Poly (Arylene Ether Benzimidazole). *Journal of Polymer*. 1993. 34(4): 856-65
- [16] D'Antonio C. Adhesives for Aerospace Applications. *Rivestimenti & Materiali Compositi*. 1994. 2(3): 25-30
- [17] Snowdon I. Adhesive Technology Takes to The Road. *Reinforced Plastics*. 1995. 39(3): 18-22.
- [18] Schwartz J. Sticking with a Winner. *Adhesives Age*. 2001. 44(9):10
- [19] Schwartz J. Driving for Profits. *Adhesives Age*. 2001. 44(9): 9
- [20] Velero G. Automotive. *Adhesives Age*. 2000. 43(7):26

- [21] Gascoigne B. Automotive Adhesives - From Design to Repair. *Machine Design*. 2000. 72(6): 60-1
- [22] Loven W. E. Structural Bonding of Composites in The Transportation Market. *Reinforced Plastics*. 1999. 43(6): 40-3
- [23] Gascoigne B. Driver's ED. *Adhesives Age*. 1999. 42(10):16-8
- [24] Russell E. Stickers for Efficiency. *Journal of Automotive Engineer*. 1999 24(10), p.73-4
- [25] Habenicht G., Koch S. Elastic Bonding in Vehicle Construction (Part I). *Adhasion Kleben & Dichten*. 1997. 41(10): 35-8
- [26] Lathrop R; Miric A. Flexible Method for Applying Adhesives to Pcb's. *Machine Design*. 1998. 70(4): 214-6
- [27] Keller H. High-Performance Bonding in Vehicle Manufacture. *European Adhesives & Sealants*. 1996. 13(3): 14-6
- [28] Prince K. Boats Benefit from Adhesive Bonding. *Reinforced Plastics*. 2002. 46(3): 46-8
- [29] Winkle I. E. Role of Adhesives. *Composite Materials in Maritime Structures. Vol.II*. 1993, 627(6): 43-62.
- [30] Higgins A. Adhesive Bonding of Aircraft Structures. *International Journal of Adhesion & Adhesives*. 2000. 20(5): 367-76
- [31] Ford P. Icarus and Allsorts - The Thomas Hawksley Memorial Lecture. *European Adhesives & Sealants*. 1997. 14(1): 23
- [32] Krieger R B. Bonding Structural Composites for Aircraft. *Composites Design Conference Proceedings*. 12<sup>th</sup>-16<sup>th</sup> July. Madrid: 1993. 15-19. 627
- [33] Courgey A; Rousseau J; Gong X J; Landrot A G. Experimental and Numerical Study of an Adhesively Bonded Joint for Railway Applications. *Structural Adhesives in Engineering VI. Conference Proceedings*. 4<sup>th</sup>-6<sup>th</sup> July. Bristol: 2001. 275-2788

- [34] Iwainsky H. Railway Vehicle Construction-Acoustic and Economic Advantages through Adhesives. *Adhasion Kleben & Dichten*. 1999. 43(10): 25-31
- [35] Dengler M., Lemm K. Adhesion in Rail Vehicle Construction. *Adhasion Kleben & Dichten*. 1997. 41(6): 30-34
- [36] Sue H. Composite Wall Systems Assembled with Structural Adhesive *Adhesives & Sealants Industry*. 2002. 9(1): 36
- [37] Revyakin O; Zicans J; Kalnins M; Bledzki A K. Adhesion Properties of Materials Based on Post-Consumer Polyurethane Waste to Metals. *Polymer Recycling*. 1999. 4(2): 93-100
- [38] Beasley J. Advances In Uv Technology for Medical Device Fabrication *RadTech 98 Conference proceedings*. 19<sup>th</sup>-22<sup>nd</sup> April. Chicago: 1998.133-9
- [39] Verhoff J; Ramani K; Blank N; Rosenberg S. Moisture Durability of Four Moisture Cure Urethane Adhesives. *Journal of Adhesion Science and Technology*. 2002. 16(4): 373-93
- [40] Hodkinson R. Adhesives Hold Electric Motors together. *Machine Design*. 2001. 73(20): 90
- [41] Shelford R. Through Adhesives Technology to Highest Performance in Sport. *Adhasion Kleben & Dichten*. 1997. 41(7-8): 13-16
- [42] Sastre A. Elastic Adhesives in The Sports Sector. *Revista de Plasticos Modernos*. 1999. 78(517): 55-58
- [43] Darwish S. M., Soliman M. S., Al-Faheed A. M. Characteristics and Variables of Spot Welding and Weldbonding Bimaterials. *Materials & Manufacturing Processes*. 1997. 12(2): 175-86
- [44] Pearson I. Adding Welded/Mechanical Fastening to Adhesive-Bonded Joints. *Automotive Engineer*. 1995 20(4): 24-5
- [45] Dunn J. *Engineering and Structural Analysis*. Second ed. UK: 2004
- [46] Patrick E.P. and Sharpe M.L. *Automotive Engineering*. 1992. 100(5): 31.

- [47] Warren A.S. *Aluminum*. 1991. 67(11): 1078.
- [48] Lachmann E. Durability of Metal Bonded Joints in Motor Vehicle Construction. *Euradh '94 Conference Proceedings*. 12<sup>th</sup>-15<sup>th</sup> Sept. Mulhouse: 1994. 277-83.
- [49] Nakano Y., Temma K., Sawa T. Stress Analysis Of Adhesive Butt Joints Of Dissimilar Materials Subjected To Cleavage Loads. *Journal of Adhesion*. 1991. 34(1-4): 137-51
- [50] Lawrenceville. N.J., Advanced Adhesive Materials for Electronic, Microelectronic and Semiconductor Assembly. *AI Technology*. 1994. 8(10): 6-11
- [51] Dyane T. Raleigh Builds a Winner With 3M's Help. *3M Briefing*. 1995 4(1): 5
- [52] Novak I., Simonikova J., Chodak I. Adhesive Properties of Epoxy Resin with Chromic Hardener. *Macromolecular Materials And Engineering*. 2001. 286(3): 191-5
- [53] Villoutreix G., Villoutreix J., Bretton-Pieters C. Influence of The Surface Composition of Metallic Substrates on The Reactivity And Bonding Properties of Structural Adhesives. *Journal of Adhesion*. 1996. 59(1-4): 241-50
- [54] Moussiaux E., Luegering A. Adhesive Bonding Brings Lasting Performance. *Reinforced Plastics*. 2000. 44(6): 46-50
- [55] Righettini R., Durso S. Using Adhesives in Composites Assembly. *Reinforced Plastics*. 1999. 43(6): 34-38
- [56] Moss N. S., Wilson R. G. Development of Novel Adhesives for Bonding Engineering Thermoplastics. *Euradh '96. Adhesion '96 Conference proceedings*. 3<sup>rd</sup>-6<sup>th</sup> Sept. Cambridge: 1996, 361-364
- [57] Mehrkam P. A., Cochran R. Evaluation of Materials for Composite Repair Applications. *Journal of Thermoplastic Composite Material*. 1997. 10(1): 51-60
- [58] Wang Shijie., Zhang Ruixian. Study of Fibre Composite Plates for Strengthening Reinforced Bridges. *Composites Properties and Applications. Conference Proceedings*. 12<sup>th</sup>-16<sup>th</sup> July. Madrid: 2003, 224-231

- [59] Hovan G., Carbutt P., Wang X., Rosselli F. Large Part Assembly Using Long-Open Time Methacrylate Structural Adhesives. *Composites in the global market Proceedings of a conference*. 5<sup>th</sup>-6<sup>th</sup> Sept. Kuala Lumpur: 2002, 175-193
- [60] Valersteinas P. Introduction of Composites into Automotive Structures. *Composites Plastiques Renforces Fibre*. 1995. 8(1): 85-91
- [61] Balkova R., Holcnerova S., Cech V. Testing of Adhesives for Bonding of Polymer Composites. *International Journal Of Adhesion & Adhesives*. 2002. 22(4): 291-5
- [62] Mikeska H. Bead of Adhesive on The Track. *Adhasion Kleben & Dichten*. 2002. 46(1-2): 32-4
- [63] Pomposo J. A., Rodriguez J., Grande H. Polypyrrole-Based Conducting Hot Melt Adhesives for Emi Shielding Applications. *Synthetic Metals*. 1999. 104(2): 107-111
- [64] Boyd J. Quick-Dry Experts. *Journal of Rubber And Plastics News*. 2003. 32(12): 10
- [65] Mishuris G., Oechsner A. Edge Effect Connected with Thin Interphase in Composite Materials. *Composite Structures*. 2005. 68:409-417
- [66] Mishuris G., Oechsner A., Kuhn G. FEM Analysis of Nonclassical Transmission Conditions Elastic Structures, Part 1: Soft Imperfect Inteface. *Tech Science Press*. 2005. 2(4):227-238
- [67] Mishuris G., Oechsner A., Kuhn G. Imperfect Interface in Dissimilar Elastic Body: FEM-Analysis. *Advanced Computational Engineering Mechanics*. October 9-11. Maribor:2003, 101-110
- [68] Zienkiewicz O. C. Cheung Y. K. Finite Element in the Solution of Field Problems. *The Engineer*. 1965, 507-510.

- [69] Wilson E. L., Nickell R. E. Application of the Finite Element Method of Heat Conduction Analysis. Nucl. Eng. Des. 1966
- [70] Gallagher R. H., Mallett R. H. Efficient Solution Processes for Finite Element Analysis of Transient Heat Conduction. *Journal of Heat Transfer*. 1971. 93:257-263.
- [71] Lee H. P. Application of Finite Element Method in the Computation of Temperature with Emphasis on Radiative Exchange. In: Tein C. L. ed. *Thermal Control and Radiation*. Cambridge: MIT Press. 491-520;1973
- [72] Beckett R. E., Che S. C. Finite Element Method Applied to Heat Conduction With Nonlinear Boundary Conditions. *Journal of Heat Transfer*. 1973. 95:126-129
- [73] Lee H. P., Jackson C. E. Finite Element Solution for Radiative Conductive Analysis with Mixed Diffuse-Specular Surface. In: Smith A. M. Radiative Transfer and Thermal Control. New York: AIAA. 25-46; 1976.
- [74] Fiedler T. *Numerical and Experimental Investigation of Hollow Sphere Structures in Sandwich Panels*. PhD Dissertation. University of Aveiro, Portugal, 2007.
- [75] Ziegler, H. Some extremum principles in irreversible thermodynamics with application to continuum mechanics, in: Sneddon I. N. ed. *Progress in Solid Mechanics*. Amsterdam: North-Holland. 4-24;1963.
- [76] Rice J. R. Inelastic Constitutive Relations for Solids: an Internal-Variable Theory and Its Application to Metal Plasticity, *Journal of the Mechanics and Physics of Solids*. 1971. 19:433-455.
- [77] Hill, R. and Rice, J. R. Elastic Potentials and the Structure of Inelastic Constitutive Laws, *SIAM Journal of Applied Mathematics*, 1973. 25:448-461.
- [78] Halphen, B. and Son, N. Q. Sur les matériaux standards généralisés, *Journal de Mécanique*. 1975. 14:39-63.

- [79] Germain, P., Nguyen, Q. S. and Suquet, P. Continuum Thermodynamics. *Journal of Applied Mechanics*. 1983. 50:1010-1020.
- [80] Lekhnitskii, S. G. *Theory of Elasticity of an Anisotropic Body*. San Francisco: Holden-Day. 1963
- [81] Cowin, S. C. and Mehrabadi, M. M. Anisotropic Symmetries of Linear Elasticity. *Applied Mechanics Reviews*. 1995. 48:247–285.
- [82] Rosselli F., Carbutt P. Structural bonding applications for the transportation industry. *Sampe Journal*. 2001. 37(6):7-13.
- [83] Mal, A.K., Bose, S.K., Dynamic elastic moduli of a suspension of imperfectly bonded spheres. *Proc. Cambridge Philos. Soc.* 1975. 76:587–600.
- [84] Benveniste, Y. The Effective Mechanical Behavior of Composite Materials with Imperfect Contact between the Constituents. *Mechanics of Materials* 1985. 4:197–208.
- [85] Achenbach, J. D., Zhu, H. Effect of Interfacial Zone on Mechanical Behavior and Failure of Fiber-Reinforced Composites. *Journal of the Mechanics and Physics of Solids*. 1989. 37: 381–393.
- [86] Achenbach, J.D., Zhu, H., Effect of Interphases on Micro and Micromechanical Behavior of Hexagonal-Array Fiber Composites. *Journal of Applied Mechanics*. 1990. 57:956–963.
- [87] Hashin, Z., Thermoplastic Properties of Fiber Composites with Imperfect Interface. *Mechanics of Materials* 1990. 8:333–348.
- [88] Hashin, Z., Thermoplastic Properties Of Particulate Composites with Imperfect Interface. *Journal of the Mechanics and Physics of Solids*. 1991. 39:745–762.
- [89] Hashin, Z., The Spherical Inclusion with Imperfect Interface. *J. Appl. Mech.* 1991. 58:444–449.



- [90] Hashin, Z., Extremum Principles for Elastic Heterogeneous Media with Imperfect Interface and Their Application to Bounding of Effective Elastic Moduli. *J. Mech. Phys. Solids*. 1992. 40:767–781.
- [91] Hashin, Z., Thin Interphase/Imperfect Interface in Conduction. *J. Appl. Phys.* 2001. 89:2261-2267.
- [92] Lipton R., Vernescu B. Variational Methods, Size Effects and Extremal Microgeometries for Elastic Composites with Imperfect Interface. *Math. Models Meth. Appl. Sci.* 1995. 5:1139-1173.
- [93] Benveniste, Y., Miloh, T. Imperfect Soft and Stiff Interfaces in Two-Dimensional elasticity. *Mechanics of Materials*. 2001. 33:309-324.
- [94] Hashin Z. Thin Interphase/Imperfect Interface in Elasticity with Application to Coated Fiber Composites. *Journal of the Mechanics and Physics of Solids*. 2002. 50: 2509-2537
- [95] Wang, J., Duan, H. L., Zhang, Z. & Huang, Z. P. An anti-interpenetration model and connections between interphase and interface models in particle-reinforced composites. *International Journal of Mechanical Science*. 2005. 47:701-718.
- [96] Duan H. L., Wang J., Huang Z. P. and Karihaloo B. L. Size-Dependent Effective Elastic Constants of Solids Containing Nano-Inhomogeneities with Interface Stress. *Journal of the Mechanics and Physics of Solids*. 2005. 53:1574-1596.
- [97] Rubin, M. B. & Benveniste, Y. A Cosserat Shell Model for Interphases in Elastic Media. *Journal of the Mechanics and Physics of Solids*. 2004. 52:1023-1052.
- [98] Benveniste, Y. A general interface model for a three-dimensional curved thin anisotropic interphase between two anisotropic media. *Journal of the Mechanics and Physics of Solids*. 2006. 54:708-734.

- [99] Vu-Quoc, L. & Tang, X. G. Optimal Solid Shells for Non-Linear Analysis of Multiplayer Composites. *I. Statics. Comput. Methods Appl. Mech. Eng.* 2003. 192:975-1016.
- [100] Mishuris G., Miszuris W., Oechsner A. Evaluation of Transmission Conditions for a Thin Heat-Resistant in homogeneous Interphase in Dissimilar Material. *Materials Science Forum.* 2007. 553:87-92.
- [101] Mishuris G., Miszuris W., Oechsner A. Finite Element Verification of Transmission Conditions for 2D Heat Conduction Problems. *Materials Science Forum.* 2007. 553:93-99.
- [102] Oechsner A., Mishuris W. Finite Element Verification of Transmission Conditions for Thin Reactive Heat-Conducting Interphases. *Defect and Diffusion Forum.* 2008. 273(276): 400-405.
- [103] Mishuris G., Miszuris W., Oechsner A. Transmission Conditions for Thin Reactive Heat-Conducting Interphases: General case. *Defect and Diffusion Forum.* 2009. 283(286): 521-526.
- [104] Oechsner A., Mishuris G. A New Finite Element for Thin Non-Homogeneous Heat-Conducting Adhesive Layers. *Journal of Adhesion Science and Technology.* 2008. 22:1365-1378.
- [105] Lee S., Wang S., George M. P., Xu H. Evaluation of Interphase Properties in A Cellulose Fiber-Reinforced Polypropylene Composite by Nanoindentation and Finite Element Analysis. *Composites: Part A.* 2007. 38:1517-1524.
- [106] Klöppel A., Movchan A. B. Asymptotic Modeling of Adhesive Joints. *Mechanics of Materials.* 1998. 28:137-145.
- [107] Mishuris G. Imperfect Transmission Conditions for a Thin Weakly Compressible Interface. 2D Problems. *Arch. Mech.* 2004. 56(2):103-115.
- [108] Antipov Y. A., Avila-Pozos O., Kolaczowski S. T., Movchan A. B. Mathematical Model of Delamination Cracks on Imperfect Interface. *International journal of solids and structures.* 2001. 38:6665-6697

- [109] Mishuris G., Kuhn G. Asymptotic Behavior of The Elastic Solution Near the Tip of a Crack Situated at A Nonideal Interface. *ZAMM*. 2001. 81(12):811-826.
- [110] Ikeda T., Yamashita A., Lee D., Miyazaki N. Failure of a Ductile Adhesive Layer Constrained by Hard Adherents. *Trans ASME Journal of Engineering Materials and Technology*. 2000. 122: 80-85.
- [111] Jeandrau J. P. Analysis and Design Data for Adhesively Bonded Joints. *International Journal of Adhesions and Adhesives*. 1991. 11(2):71-79.
- [112] Mishuris G., Oechsner A. Transmission Conditions for a Soft Elasto-Plastic Interphase between Two Elastic Materials. Plane Strain State. *Arch Mech*. 2005. 57(2-3):157-169
- [113] Kachonov L. M. *Fundamental of the Theory of Plasticity*. Second ed. Moscow: MIR. 1974.
- [114] Mishuris G., Oechsner A. 2D Modelling of a Thin Elasto-Plastic Interphase between Two Different Materials: Plane Strain Case. *Composite Structures*. 2007. 80: 361-372
- [115] Tsai C. L., Weinstock H., Overton W.C. Low Temperature Thermal Conductivity of Stycast 2850 FT. *Cryogenics*. 1978. 18:562-563.
- [116] Movchan A.B., Movhan N.V. *Mathematical Modelling of Solids of with Nonregular Boundaries*. CRC Press, Boca Raton, 1995.
- [117] Kachanov L. M. *Fundamentals of the theory of plasticity*. Second ed. Moscow: MIR Publishers; 1974.
- [118] Mishuris G., Mishuris W., Oechsner A. Transmission Condition for Thin Elasto-Plastic Interphase. *Fourth International Conference in Mathematical Modeling and Computer Simulation of Material Technologies*. September 11 - 15, Ariel:2006. 37-46.
- [119] Blassiau S, Thionnet A, Bunsell AR. Micromechanisms of load transfer in a unidirectional carbon fibre-reinforced epoxy composite due to fibre failures.

Part 1: Micromechanisms and 3D analysis of load transfer: the elastic case. *Composite Structure*. 2006;74(3):303–18.

[120] Blassiau S, Thionnet A, Bunsell AR. Micromechanisms of load transfer in a unidirectional carbon fibre-reinforced epoxy composite due to fibre failures. Part 2: Influence of viscoelastic and plastic matrices on the mechanisms of load transfer. *Composite Structure*. 2006;73(3):319–31.

[121] Palacz M, Krawczuk M, Ostachowicz W. The spectral finite element model for analysis of flexural-shear coupled wave propagation. Part 1: Laminated multilayer composite beam. *Composite Structure*. 2005. 68:37–44.