

FUZZY IDEALS IN ORDERED SEMIGROUPS AND THEIR
GENERALISATIONS

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A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

NOVEMBER 2013

TO MY BELOVED FAMILY ESPECIALLY MY FATHER

ACKNOWLEDGEMENT

In the name of Allah, Most Gracious, Most Merciful. Praise be to Allah, the Cherisher and Sustainer of the worlds; Most Gracious, Most Merciful; Master of the Day of Judgement.

I would like to pay deepest thanks to Allah, the Almighty who blessed me with the nerve to accomplish the task of this PhD thesis. Aside from that, fathomless gratitude goes to my supervisor, Assoc. Prof. Dr. Nor Haniza Sarmin, UTM, who has guided me throughout the process and whose wise counsels were of supreme value to this accomplishment. I am also grateful to my co-supervisor, Assist. Prof. Dr. Asghar Khan, who is always a source of inspiration and encouragement for me. He has been providing me guidance, support, advice and every kind of help in my research work and queries. Furthermore I acknowledge, with gratitude, to the School of Postgraduate Studies (SPS) UTM for the award of UTM International Doctoral Fellowship (IDF) from 2010 to 2012.

I would also like to say thanks to the faculty and staff of the Department of Mathematical Sciences and Faculty of Science, Universiti Teknologi Malaysia (UTM) whose assistance helped us get through the job with much convenience. I would like to pay cordial thanks to my colleagues and friends who lent me a helping hand at every hour of need.

Last but not the least, my warm gratefulness goes to my family members for their understanding, love and endless support throughout my studies.

Faiz Muhammad Khan

ABSTRACT

The idea of fuzzy sets has opened a new era of research in the world of contemporary mathematics. The proposed concept of fuzzy sets provided for a renewed approach to model imprecision and uncertainty present in phenomena without sharp boundaries. The fuzzification of algebraic structures, particularly ordered semigroups, play a prominent role in mathematics with diverse applications in many applied branches such as computer arithmetic, control engineering, error-correcting codes and formal languages. In this background, many researchers initiated the notion of “quasi coincident with” (q) relation between a fuzzy point and a fuzzy set in ordered semigroups. Later a new generalisation of quasi-coincident with relation symbolised as q_k where $k \in [0,1)$ has been introduced. In this thesis, new concepts including fuzzy ideals, fuzzy interior ideals, fuzzy generalised bi-ideals, fuzzy bi-ideals and fuzzy quasi-ideals of type $(\in, \in \vee q_k)$ of ordered semigroup are introduced. Further, ordinary ideals and $(\in, \in \vee q_k)$ -fuzzy ideals are linked using level subset and characteristic function. The results show that in regular, intra-regular and semisimple ordered semigroups both $(\in, \in \vee q_k)$ -fuzzy ideals and $(\in, \in \vee q_k)$ -fuzzy interior ideals coincide. The concept of upper/lower parts of $(\in, \in \vee q_k)$ -fuzzy interior ideals is also introduced and furthermore, semisimple, simple and intra-regular ordered semigroups are characterised in terms of this notion. The relation between generalised bi-ideals and $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideals is determined. Furthermore, the conditions for the lower part of $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideal to be a constant function are provided. The characterisations of ordered semigroups by the properties of semiprime $(\in, \in \vee q_k)$ -fuzzy quasi-ideals are investigated. Finally, the classification of ordered semigroups by $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy interior ideals and $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$ -fuzzy interior ideals are determined comprehensively.

ABSTRAK

Idea mengenai set kabur telah membuka suatu era baharu bagi penyelidikan dalam dunia matematik sezaman. Cadangan konsep set kabur diketengahkan untuk pembaharuan pendekatan kepada ketidaktepatan dan ketakpastian yang wujud dalam fenomena tanpa sempadan yang tepat. Pengaburan bagi struktur aljabar, khususnya bagi semikumpulan bertertib, memainkan peranan utama dalam matematik dengan pelbagai aplikasi dalam banyak cabang gunaan seperti aritmetik komputer, kejuruteraan kawalan, kod pembetulan-ralat dan bahasa formal. Dengan latar belakang ini, ramai penyelidik memulakan konsep “kuasi-kebetulan dengan” hubungan (q) antara satu titik kabur dengan satu set kabur dalam semikumpulan bertertib. Kemudian, satu pengitlakan bagi kuasi-kebetulan dengan hubungan yang ditandakan sebagai q_k di mana $k \in [0,1)$ diperkenalkan. Dalam tesis ini, konsep baharu termasuk unggulan kabur, unggulan pedalaman kabur, dwi-unggulan kabur teritlak, dwi-unggulan kabur dan kuasi-unggulan kabur dengan jenis bagi semikumpulan bertertib diperkenalkan. Selanjutnya, unggulan biasa dan unggulan kabur- $(\epsilon, \epsilon \vee q_k)$ dikaitkan menggunakan subset aras dan fungsi cirian. Keputusan menunjukkan bahawa dalam semikumpulan bertertib sekata, intra-sekata dan semi-ringkas, kedua-dua unggulan kabur- $(\epsilon, \epsilon \vee q_k)$ dan unggulan pedalaman kabur- $(\epsilon, \epsilon \vee q_k)$ adalah sama. Konsep bahagian atas/bawah bagi unggulan pedalaman kabur- $(\epsilon, \epsilon \vee q_k)$ juga diperkenalkan dan seterusnya, semikumpulan bertertib semi-ringkas, ringkas dan intra-sekata dicirikan berdasarkan konsep ini. Hubungan antara dwi-unggulan teritlak dengan dwi-unggulan kabur teritlak- $(\epsilon, \epsilon \vee q_k)$ telah ditentukan. Sebagai tambahan, syarat-syarat bagi bahagian bawah dwi-unggulan kabur teritlak- $(\epsilon, \epsilon \vee q_k)$ untuk menjadi fungsi malar diberikan. Pencirian bagi semikumpulan bertertib menggunakan sifat-sifat semiperdana kuasi-unggulan kabur- $(\epsilon, \epsilon \vee q_k)$ telah dikaji. Akhir sekali, pengelasan bagi semikumpulan bertertib oleh unggulan pedalaman kabur- $(\epsilon_\gamma, \epsilon_\gamma \vee q_\delta)$ dan unggulan pedalaman kabur- $(\overline{\epsilon_\gamma}, \overline{\epsilon_\gamma} \vee \overline{q_\delta})$ ditentukan secara menyeluruh.

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LIST OF SYMBOLS/ NOTATIONS

q	-	quasi-coincident with relation
$[x; t]$	-	fuzzy point
F	-	fuzzy subset
\in	-	belongs to relation
q_k	-	generalised quasi-coincident with relation
$\in \vee q$	-	belongs to or quasi-coincident with relation
S	-	ordered semigroup
\in_γ	-	generalised belongs to relation
$\in_\gamma \vee q_\delta$	-	generalised belongs to or quasi-coincident with relation
$U(F;t)$	-	level subset of a fuzzy set F
$\bar{\in}_\gamma$	-	negation of generalised belongs to relation
$\bar{\in}_\gamma \vee \bar{q}_\delta$	-	negation of generalised belongs to or negation of generalised quasi-coincident with relation
A	-	subsemigroup
B	-	bi-ideal
$B(a)$	-	generalised bi-ideal generated by a
I	-	interior ideal
L	-	left ideal
R	-	right ideal
σ	-	equivalence relation
$(x)_\sigma$	-	σ -class of S containing x .
χ_A	-	characteristic function
$F \circ G$	-	product of fuzzy subset F and G
$Q^k(F;t)$	-	(q_k) -level set

$[F]_t^k$	-	$(\in \vee q_k)$ -level set
$(F \circ^k G)^-$	-	generalised product of fuzzy subsets
\bar{F}^k	-	lower part of fuzzy subset
$F^+{}^k$	-	upper part of fuzzy subset
χ_A^{-k}	-	lower part of characteristic function
χ_A^{+k}	-	upper part of characteristic function
F_0	-	subset of ordered semigroup which contain fuzzy image greater than 0
$B(a^2)$	-	generalised bi-ideal generated by a^2
G	-	$(\in, \in \vee q_k)$ -fuzzy left ideal of ordered semigroup
H	-	$(\in, \in \vee q_k)$ -fuzzy ideal of ordered semigroup
J	-	$(\in, \in \vee q_k)$ -fuzzy right ideal of ordered semigroup
Y	-	semilattice
N	-	semilattice congruence
I	-	maximum fuzzy subset
O	-	minimum fuzzy subset
F_γ	-	subset of S which have fuzzy image greater than γ
$\bar{\in}$	-	negation of belongs to relation
\bar{q}	-	negation of quasi-coincident with relation

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The importance of fuzzy algebraic structures is increased due to the concept of quasi-coincident with relation (q) [1] of a fuzzy point with a fuzzy set. A "quasi-coincident with" relation between a fuzzy point $[x; t]$ and a fuzzy set F denoted by $[x; t]qF$, is defined as $F(x) + t > 1$ where $t \in (0, 1]$. The notion of quasi-coincident with relation is crucial to generate new types of fuzzy subgroups.

The new idea proposed by Bhakat and Das [1] revised the world of contemporary mathematics which have been explored at length to bring out novel vistas for future researchers. In contribution to this idea, Jun [2] investigated a more generalised form of quasi-coincident with relation in BCK/BCI -algebra and introduced a new notion (q_k) where $k \in [0, 1)$ between a fuzzy point and a fuzzy set.

The present research introduces the concept of $(\in, \in \vee q_k)$ -fuzzy subsystem and some new fuzzy interior ideals of type $(\in_\gamma, \in_\gamma \vee q_\delta)$ and $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$ in ordered semigroup. Several notions like fuzzy ideals, fuzzy interior ideals, fuzzy generalised bi-ideals, fuzzy bi-ideals, fuzzy quasi-ideals of type $(\in, \in \vee q_k)$, $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy interior, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$ -fuzzy interior ideals are defined and supported by suitable examples. The relationships between ordinary ideals and fuzzy ideals of type $(\in, \in \vee q_k)$ are provided.

New characterisations of regular (resp. completely regular, intra-regular, semi-simple and simple) ordered semigroups in terms of fuzzy left (resp. right, interior, bi-, generalised bi-, quasi-) ideals of type $(\in, \in \vee q_k)$ are determined in detail.

1.2 Research Background

The major advancements in the fascinating world of fuzzy set started with the work of renowned scientist Zadeh [3], in 1965 with new directions and ideas. A fuzzy set can be defined as a set without a crisp and clearly sharp boundaries which contains the elements with only a partial degree of membership. Fuzzy sets are the extensions of classical sets, and the latter is denoted as a container that wholly includes or excludes any given element. In 1971, Rosenfeld's [4] method of fuzzification of algebraic structures represented a quantum jump in the history of fuzzy sets and related mathematics, and most of the later contributions in this field are the validations of this work. Rosenfeld introduced the concept of fuzzy groups and successfully extended many results from groups into fuzzy groups. The idea of a quasi-coincidence of a fuzzy point with a fuzzy set is initiated by Bhakat and Das [5, 6] and Bhakat [7] which played a significant role to generate different types of fuzzy subgroups. Later, they [5, 6] reported the concept of (α, β) -fuzzy subgroups by using "belongs to" relation (\in) and "quasi-coincident with" relation (q) between a fuzzy point and a fuzzy set. In particular, $(\in, \in \vee q)$ -fuzzy subgroup is an important and useful generalisation of the Rosenfeld's fuzzy subgroup [4].

From the time that fuzzy subgroups gained general acceptance over the decades, it has provided a central trunk to investigate similar type of generalisations of the existing fuzzy subsystems of other algebraic structures. A contributing factor for the growth of fuzzy groups is increased by the reports from Davvaz [8], who introduced the concept of $(\in, \in \vee q)$ -fuzzy sub-near-rings (R -subgroups, ideals) of a near-ring and provided some of their interesting properties. Later, Jun and Song [9] studied general forms of fuzzy interior ideals in semigroups whereas Kazanci and Yamak initiated the concept of a generalised fuzzy bi-ideal in semigroups [10] and highlighted properties of ordered semigroups in terms of $(\in, \in \vee q)$ -fuzzy bi-ideals.

However, Jun *et al.* [11] has thrown interesting light on the concept of a generalised fuzzy bi-ideal in ordered semigroups and characterisation of regular ordered semigroups in terms of $(\in, \in \vee q)$ -fuzzy bi-ideals.

For a fuzzy subset F of an ordered semigroup S , the set of the form

$$F(y) := \begin{cases} t & \text{if } y = x, \\ 0 & \text{if } y \neq x, \end{cases}$$

is called a fuzzy point with support x and value $t \in (0, 1]$ and is denoted by $[x; t]$. A fuzzy point $[x; t]$ is said to belong to F , written as $[x; t] \in F$ if $F(x) \geq t$ and $[x; t]$ is quasi-coincident with F denoted by $[x; t] qF$ if $F(x) + t > 1$. The notation $[x; t] \in \vee qF$ means that $[x; t] \in F$ or $[x; t] qF$. The symbol $\overline{\in \vee q}$ stands for $\in \vee q$ does not hold.

In another pioneered contribution, Jun [2] generalised the concept of $(\in, \in \vee q)$ -fuzzy subalgebra of a BCK/BCI -algebra and introduced a new concept of $(\in, \in \vee q_k)$ -fuzzy subalgebras followed by the basic properties of BCK -algebras. In continuation of this idea, Shabir *et al.* [12] and Shabir and Mahmood [13] reported the concept of generalised forms of (α, β) -fuzzy ideals and defined $(\in, \in \vee q_k)$ -fuzzy ideals of semigroups and hemirings comprehensively. Recent developments in fuzzy ideals related to semigroups and hemirings have prompted the formulation of a precise description of numerous classes of semigroups and hemirings and their characterisation.

In this project, novel fuzzy ordered semigroup theory, called $(\in, \in \vee q_k)$ -fuzzy ordered semigroups are developed. Further, it has been extended to $(\in, \in \vee q_k)$ -fuzzy subsystems. This also leads to the characterisation of ordered semigroups in terms of $(\in, \in \vee q_k)$ -fuzzy left (right) ideals, $(\in, \in \vee q_k)$ -fuzzy interior ideals, $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideals, $(\in, \in \vee q_k)$ -fuzzy bi-ideals and $(\in, \in \vee q_k)$ -fuzzy quasi-ideals. In addition, several new types of fuzzy interior ideals called $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy interior ideals and $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$ -fuzzy interior ideals are introduced.

1.3 Statements of the Problem

The focus of this research is to answer the following questions satisfactorily.

How to introduce new types of fuzzy left (right, interior, generalised bi-, bi-, quasi-) ideals based on reported idea [2]? How to initiate the concept of $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy interior ideals and $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$ -fuzzy interior ideals of ordered semigroup? How to establish a clear relationship between ordinary fuzzy left (right, interior, generalised bi-, bi-, quasi-) ideals and fuzzy left (right, interior, generalised bi-, bi-, quasi-) ideals of type $(\in, \in \vee q_k)$ of ordered semigroups? How to determine a new characterisation method for ordered semigroups and their different classes (regular, completely regular, intra-regular and simple) in terms of $(\in, \in \vee q_k)$ -fuzzy left (right, interior, generalised bi-, bi-, quasi-) ideals? How to characterise ordered semigroups by the properties of fuzzy interior ideals of type $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ and $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$? and, what will be the type of fuzzy left (right, interior, generalised bi-, bi-, quasi-) ideal F , if level subset $U(F; t)$ is an empty set or a left (right, interior, generalised bi-, bi-, quasi-) ideal for $t \in (0, \frac{1-k}{2}]$ with $k \in [0, 1)$?

1.4 Research Objectives

The objectives of this research are as follows:

1. To introduce a new kind of fuzzy ideals called $(\in, \in \vee q_k)$ -fuzzy left (right) ideals in ordered semigroups and to present different characterisations of numerous classes of ordered semigroups.
2. To introduce a concept of $(\in, \in \vee q_k)$ -fuzzy interior ideals and establish the relationships between $(\in, \in \vee q_k)$ -fuzzy interior ideals and $(\in, \in \vee q_k)$ -fuzzy ideals in ordered semigroups.
3. To present new concepts of $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideals and characterise ordered semigroups in terms of $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideals.
4. To define $(\in, \in \vee q_k)$ -fuzzy bi-ideals and extend the study of characterisation of ordered semigroups in terms of $(\in, \in \vee q_k)$ -fuzzy bi-ideals.
5. To introduce the concept of $(\in, \in \vee q_k)$ -fuzzy quasi-ideal of ordered semigroups and classification of various classes of (left and right regular, left and right simple, regular, completely regular) ordered semigroups.

6. To investigate $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy interior ideals, $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$ -fuzzy interior ideals of ordered semigroups and to characterise ordered semigroups by the properties of fuzzy interior ideals of types $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ and $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$.

1.5 Scope of the Study

This research covers the detailed study of ordered semigroups by the properties of numerous novel types of fuzzy ideals called $(\in, \in \vee q_k)$ -fuzzy left (right) ideals, $(\in, \in \vee q_k)$ -fuzzy interior ideals, $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideals, $(\in, \in \vee q_k)$ -fuzzy bi-ideals, $(\in, \in \vee q_k)$ -fuzzy quasi-ideals, $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy interior ideals and $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$ -fuzzy interior ideals. Present study also focuses on the new classification of regular (left and right regular, completely regular, intra-regular, left and right simple, semiprime) ordered semigroups on the basis of new conceptions.

1.6 Significance of the Study

Study on this research topic and especially in this area is still very rare. The present study is more significant due to the vast applications in real world problems involving uncertainties. In recent times, less has been reported about the role of ordered semigroups and their characterisation by fuzzy ideals and needs proper attention to overcome this problem. For instance, the use of fuzzified algebraic structures in the modern days has become a powerful tool in the fields of control engineering, computer science and automata theory.

The fundamental importance of this research is to discover some new fuzzified ideal structures of ordered semigroups and to provide the characterisations of ordered semigroups and their several classes like regular (resp. completely regular, intra-regular, semiprime) ordered semigroups in terms of fuzzy ideals, fuzzy interior ideals, fuzzy generalised bi-ideals, fuzzy bi-ideals and fuzzy quasi-ideals of type $(\in, \in \vee q_k)$.

Fuzzy structures are linked with theoretical soft computing, but as algebraic structures, especially ordered semigroups and their different characterisations have numerous applications in computer arithmetics, coding theory, sequential machines, finite state machines, error-correcting codes, fuzzy automata, information sciences, theoretical physics and formal languages.

Moreover, the concept of $(\in, \in \vee q_k)$ -fuzzy subsystems is introduced and new classifications of ordered semigroups are investigated. Thus, the results obtained from this research would form the bases of new theories and be valuable contributions to Mathematics in general, and particularly to the Theory of Ordered Semigroups. Furthermore, it constitutes a platform for further development of ordered semigroups and their applications to other branches of algebra.

1.7 Research Methodology

Based on the problem statements and research objectives, the methodology adopted for this research is presented in this section.

The quest for the fuzzification of algebraic structures was long considered an unreasonable target, until Rosenfeld's fuzzy subgroups [4]. The latest advances in the investigation of fuzzy subgroup theory have drawn increasing interest to this class of algebraic structures. This knowledge of Rosenfeld's concept is also of fundamental importance in the most important generalisation, i.e. $(\in, \in \vee q)$ -fuzzy subgroup [1, 7]. With the introduction of aforementioned concept, several studies have been made in other branches of algebra using this idea. It is important to note that $(\in, \in \vee q)$ -fuzzy theory depends on belongs to (\in) or quasi-coincident with (q) relation between a fuzzy point $[x; t]$ and a fuzzy subset F , i.e. $[x; t] \in F$ ($F(x) \geq t$) or $[x; t]qF$ ($F(x) + t > 1$) where $t \in (0, 1]$. Moreover, if $[x; t]qF$, then $F(x) < t$. Therefore, $F(x) \geq 0.5$.

Further, Jun [2] generalised the concept of quasi-coincident with relation and introduced generalised quasi-coincident with (q_k) relation between a fuzzy point and a fuzzy subset for arbitrary $k \in [0, 1)$, i.e. $[x; t]q_kF$ ($F(x) + t + k > 1$) or equivalently $F(x) + t > 1 - k$. In this case, since $F(x) < t$, therefore $F(x) \geq \frac{1-k}{2}$.

Further details of this expressions are given in Chapter 3, Section 3.2.

After these groundbreaking discoveries, Ma *et al.* [14] reported the generalisation of Bhakat and Das concept [1] and introduced $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy ideals and $(\bar{\in}_\gamma, \bar{\in}_\gamma \vee \bar{q}_\delta)$ -fuzzy ideals in *BCI*-algebras and discussed several characterisation theorems in terms of these new notions.

Inspired by these outstanding findings, new theories, i.e. $(\in, \in \vee q_k)$ -fuzzy ideal theory and $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy ideal theory of ordered semigroups are introduced on the bases of Jun's and Ma *et al.* ideas. The present research is systematically divided into two major parts. In the first part, using Jun's idea [2] of q_k relation, new type of fuzzy ideal theory called $(\in, \in \vee q_k)$ -fuzzy ideal theory in ordered semigroups is introduced. This new fuzzy ideal theory is divided into five different types of fuzzy ideals called fuzzy left (right) ideals, fuzzy interior ideals, fuzzy generalised bi-ideals and fuzzy quasi-ideals of type $(\in, \in \vee q_k)$ in ordered semigroups and several classes of ordered semigroups such as left (right) regular, regular, intra-regular, completely regular, semiprime, semisimple and simple ordered semigroups are characterised by considering the properties of these new types of fuzzy ideals. In the second part, the research based on Ma *et al.* [14] idea, new type of fuzzy interior ideals called $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy interior ideals and $(\bar{\in}_\gamma, \bar{\in}_\gamma \vee \bar{q}_\delta)$ -fuzzy interior ideals are defined. Further, ordered semigroups are classified by the properties of these new notions. The details on how this research has been conducted is given in the following.

The research opens up with the introduction of fuzzy left (right) ideals of type $(\in, \in \vee q_k)$ in ordered semigroups, based on Jun's idea [2]. An example is constructed to support the new definition. Several fundamental theorems are discussed by the properties of $(\in, \in \vee q_k)$ -fuzzy left (right) ideals. By considering $k = 0$ in the new theorems, it is reduced to the existing literature, which shows the authenticity of the present work. Indeed, the concept of level subset is introduced to elaborate the relationships between ordinary fuzzy ideals and $(\in, \in \vee q_k)$ -fuzzy ideals. In addition, using regular ordered semigroups, the connection between generalised product and lower part of fuzzy subset $(F \wedge G)$ of S is constructed.

The notion of fuzzy interior ideal is generalised to the concept of $(\in, \in \vee q_k)$ -fuzzy interior ideals in ordered semigroups using Jun's idea [2].

With its unique attributes, this new definition is supported by an example and a variety of new theorems are developed in terms of this new notion. It is worth mentioning that every fuzzy ideal is an interior ideal but the converse is not true in general. In order to prove this statement, several results are investigated in which the concepts of $(\in, \in \vee q_k)$ -fuzzy ideals and $(\in, \in \vee q_k)$ -fuzzy interior ideals coincide. In addition, these results are determined for different classes of ordered semigroups such as semisimple, intra-regular and regular ordered semigroups. Further, the necessary and sufficient condition for an ordered semigroup to be simple is that it is $(\in, \in \vee q_k)$ -fuzzy simple. Similarly, semiprime $(\in, \in \vee q_k)$ -fuzzy ideals are introduced and using these semiprime, left regular and intra-regular ordered semigroups are discussed.

Additionally, the concept of upper and lower parts of a fuzzy subset F of S and characteristic functions χ_A of A are introduced. In particular, the interval $[0, 1]$ is divided into two sub-intervals, i.e. $[0, \frac{1-k}{2}]$ and $[\frac{1-k}{2}, 1]$ where the lower part lies in the interval $[0, \frac{1-k}{2}]$. Since $(\in, \in \vee q_k)$ -fuzzy ideal theory is related to the lower part, therefore, ordered semigroups are characterised by the properties of lower parts only.

Next, the concept of fuzzy generalised bi-ideals is further extended to $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideals in ordered semigroups using a generalised quasi-coincident with (q_k) relation. Herein, level subsets and characteristic functions are used to connect fuzzy generalised bi-ideals of type $(\in, \in \vee q_k)$ and ordinary generalised bi-ideals. An example is constructed in support of this new definition along with several characterisation theorems of ordered semigroups in terms of $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideals. The authenticity of these theorems are evaluated by considering $k = 0$, which is reduced to the existing literature [96]. By using aforementioned ideals, various characterisation theorems of regular, left (resp. right) regular, completely regular and weakly regular ordered semigroups are studied. Further, the lower part $(\overline{\chi}_A^{-k})$ of characteristic function χ_A of A is defined, and shown to be an $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideal of S , whereas the condition for an ordered semigroup S to be completely regular is provided.

In the next step, the notion of fuzzy bi-ideals [11] is further generalised into $(\in, \in \vee q_k)$ -fuzzy bi-ideals in ordered semigroups. This new notion is supported by an example.

In order to explore for $F(x) \geq \frac{1-k}{2}$, two theorems are exemplified and it is shown that by considering $k = 0$, these theorems are reduced to the results reported before in [11]. Further, level subset and characteristic functions are used to link ordinary bi-ideals and $(\in, \in \vee q_k)$ -fuzzy bi-ideals. It is known that every fuzzy bi-ideal is fuzzy generalised bi-ideal but the converse is not true in general. Therefore, an extra condition is provided that if ordered semigroup is regular or left weakly regular, then both the concepts of $(\in, \in \vee q_k)$ -fuzzy bi-ideals and $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideals coincide. Additionally, regular and intra-regular ordered semigroups are characterised by the properties of $(\in, \in \vee q_k)$ -fuzzy bi-ideals. The necessary and sufficient condition for S to be regular, left and right simple is that every lower part of $(\in, \in \vee q_k)$ -fuzzy bi-ideals is a constant function.

Using unique attributes of Jun's idea [2] of q_k relation, new types of fuzzy quasi-ideals called $(\in, \in \vee q_k)$ -fuzzy quasi-ideals, semiprime $(\in, \in \vee q_k)$ -fuzzy quasi-ideals of ordered semigroups are developed which are the generalisations of fuzzy quasi-ideals of ordered semigroups. Besides, a level subset $U(F; t)$ of F and a characteristic function χ_A of A are used to link the ordinary quasi-ideals and fuzzy quasi-ideals of type $(\in, \in \vee q_k)$ of ordered semigroup S . Additionally, the classification of completely regular ordered semigroups by the notion of lower part of $(\in, \in \vee q_k)$ -fuzzy quasi-ideal are discussed in this penultimate step.

As discussed earlier, the last part of the present research covers extended work of Ma *et al.* [14]. By considering this pioneering idea, the concept of $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy interior ideals and $(\bar{\in}_\gamma, \bar{\in}_\gamma \vee \bar{q}_\delta)$ -fuzzy interior ideals of ordered semigroups are introduced. The new concepts are supported by suitable examples and the relationship among fuzzy interior ideals of type $(\in_\gamma, \in_\gamma \vee q_\delta)$ and $(q_\delta, \in_\gamma \vee q_\delta)$ and $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy ideals are established. Further, ordinary interior ideals and fuzzy interior ideals of the type $(\in_\gamma, \in_\gamma \vee q_\delta)$ are linked, and ordered semigroups are classified by the properties of $(\bar{\in}_\gamma, \bar{\in}_\gamma \vee \bar{q}_\delta)$ -fuzzy interior ideals and $(\bar{\in}_\gamma, \bar{\in}_\gamma \vee \bar{q}_\delta)$ -fuzzy left (right) ideals.

It can be summarised that this research is mostly based on two pioneering ideas, i.e. Jun's [2] of $(\in, \in \vee q_k)$ -fuzzy sub-algebra and Ma *et al.* [14] of $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy ideals. It is worth mentioning that the present research opens new fascinating directions in the field of ordered semigroups.

These new directions are further discussed in the form of suggestions for future establishments which will emerge as outstanding findings. To conclude this section, Figure 1.1 shows the methodology of this research in the form of a flow chart.

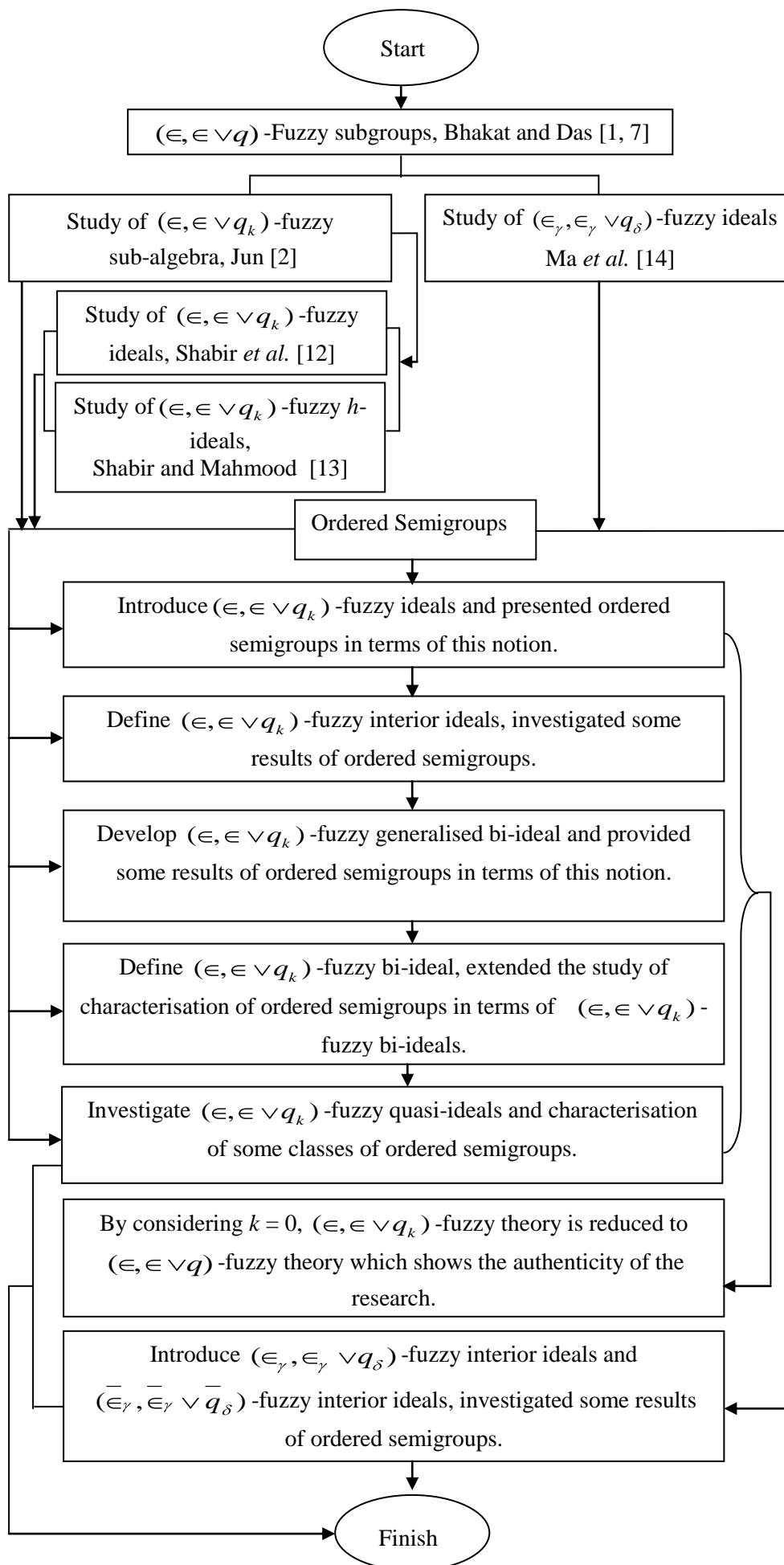


Figure 1.1. Research Methodology Flow Chart

1.8 Thesis Outlines

The thesis is organised in seven main chapters. Chapter 1 covers the introduction to the work on fuzzy ideals of types $(\in, \in \vee q_k)$ of ordered semigroups. This chapter lays down research background which outlines the general introduction followed by statement of the problem, research objectives, scope of the study, significance of the study and research methodology.

In Chapter 2, a literature review of ordered semigroups characterised by the properties of different types of fuzzy ideals is provided. Further, some important concepts and fundamental results that are essential for this research are also included.

Chapter 3 illustrates the concepts of $(\in, \in \vee q_k)$ -fuzzy left (right) ideals and $(\in, \in \vee q_k)$ -fuzzy interior ideals in ordered semigroups with examples. However, regular ordered semigroups are characterised on the basis of these ideals. Further, the relationship between ordinary fuzzy ideals and $(\in, \in \vee q_k)$ -fuzzy ideals is constructed. Whereas, simple, semisimple, intra-regular and regular ordered semigroups are characterised by the properties of $(\in, \in \vee q_k)$ -fuzzy interior ideals. In particular, the concepts of $(\in, \in \vee q_k)$ -fuzzy interior ideals and $(\in, \in \vee q_k)$ -fuzzy ideals are shown to coincide in various classes such as regular, intra-regular and semisimple ordered semigroups. Finally, the upper/lower parts of $(\in, \in \vee q_k)$ -fuzzy interior ideals along with their respective characterisation theorems are explained in detail.

Chapter 4 contains detailed study of $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideals of ordered semigroups supported by examples. The relation between generalised bi-ideals and $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideals of ordered semigroups is discussed. Besides, it is also investigated that how regular, left (resp. right) regular, completely regular and weakly regular ordered semigroups can be characterised by the properties of lower part of $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideals. In addition, the conditions for the lower part of $(\in, \in \vee q_k)$ -fuzzy generalised bi-ideal to be a constant function are provided, while the characterisation of completely regular ordered semigroups in terms of fuzzy generalised bi-ideal of type $(\in, \in \vee q_k)$ is also the milestone of this chapter.

In Chapter 5, the concepts of $(\in, \in \vee q_k)$ -fuzzy bi-ideals and $(\in, \in \vee q_k)$ -fuzzy quasi-ideals in ordered semigroups are initiated. The notion of semiprime fuzzy quasi-ideals of type $(\in, \in \vee q_k)$, lower parts of $(\in, \in \vee q_k)$ -fuzzy quasi-ideals and $(\in, \in \vee q_k)$ -fuzzy bi-ideals are introduced in the subsections of this chapter. Regular and intra-regular ordered semigroups in terms of $(\in, \in \vee q_k)$ -fuzzy bi-ideals are studied in detail. Further, basic fundamental results of ordered semigroups in terms of semiprime $(\in, \in \vee q_k)$ -fuzzy quasi-ideals are presented.

Further, numerous new types of fuzzy interior ideals called (α, β) -fuzzy interior ideals and $(\bar{\beta}, \bar{\alpha})$ -fuzzy interior ideals of ordered semigroup S , where $\alpha, \beta \in \{\in_\gamma, q_\delta, \in_\gamma \vee q_\delta, \in_\gamma \wedge q_\delta\}$ and $\bar{\alpha}, \bar{\beta} \in \{\bar{\in}_\gamma, \bar{q}_\delta, \bar{\in}_\gamma \vee \bar{q}_\delta, \bar{\in}_\gamma \wedge \bar{q}_\delta\}$, $\alpha \neq \in_\gamma \wedge q_\delta$ and $\bar{\beta} \neq \bar{\in}_\gamma \wedge \bar{q}_\delta$ are analysed in Chapter 6. The concept of $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy left (right) ideals and $(\bar{\in}_\gamma, \bar{\in}_\gamma \vee \bar{q}_\delta)$ -fuzzy left (right) ideals is also covered and several characterisation theorems in terms of $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy interior ideals and $(\bar{\in}_\gamma, \bar{\in}_\gamma \vee \bar{q}_\delta)$ -fuzzy interior ideals are established.

Finally, the thesis is concluded in Chapter 7 with the summary of the research along with some recommendations for future research in this field. Figure 1.2 summarises all chapters in this thesis.

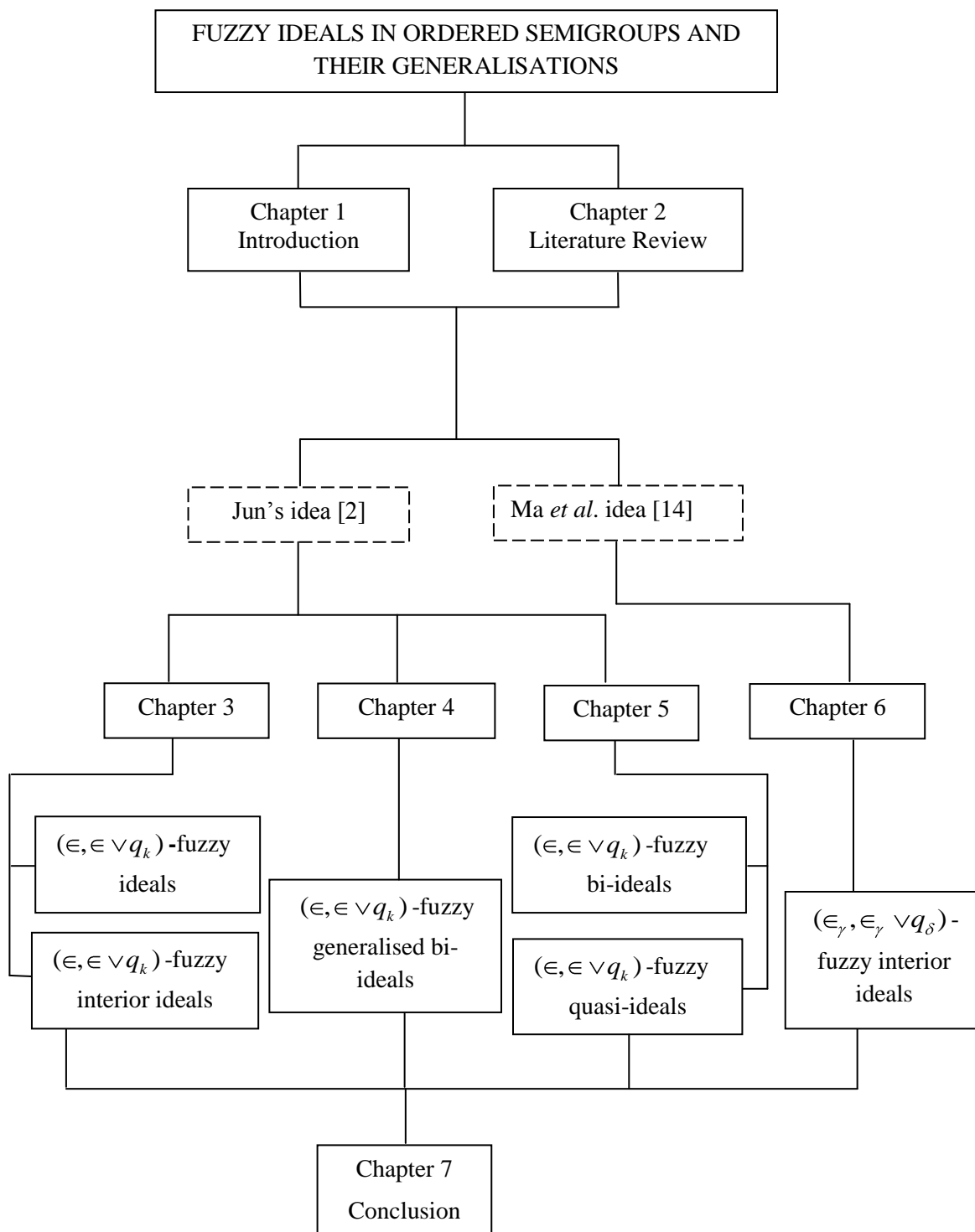


Figure 1.2. Thesis Outlines

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