EXPERIMENTAL-BASED SIMULATED ANNEALING FOR JOB SHOP SCHEDULING PROBLEMS WITH STOCHASTIC PROCESSING TIMES

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To my beloved husband and sons

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ABSTRACT

Job shop scheduling problem is widely known as one of the most difficult NP-Hard problems to solve and present efforts to solve the problems are mostly expressed in the form of heuristics. This thesis investigates the application of simulated annealing algorithm for solving job shop scheduling problem with stochastic processing times. Schedule quality is assessed based on the distribution of the schedule makespan, which is the maximum completion time of all jobs. The main idea is the integration of simulation into the simulated annealing algorithm. As such, variants of simulated annealing procedure for deterministic problems are first analyzed which are then extended to stochastic versions by incorporating simulation to evaluate schedules generated by the algorithms. Experimental results show that the stochastic variants provide an efficient tool in incorporating all the available distributional information on the processing times into the scheduling procedure. In addition, incorporating statistical tools such as the sampling methods enhance to certain extend the quality as well as the efficiency of the solutions. The performance of the simulated annealing variants is further investigated when three different temperature functions are proposed. The extensive computational tests and analysis on selected problem instances show the superiority of the proposed algorithms compared to some typical dispatching algorithms in high variability levels. Finally, the correlations between the expected makespan and the α-quantile of makespan are examined. The solutions obtained for low variability levels indicate that the two measures are perfectly correlated, and makespan distributions mostly follow the normal distributions, with few cases where they fail the normality tests. Although only stochastic processing times are considered in this thesis, the formulations and methodology can be extended to handle different objective functions as well as other kinds of uncertainties, such as uncertain arrival times, due dates and the handling of unpredictable machine breakdown and incorporation of new activities.

ABSTRAK

Masalah penjadualan bengkel kerja merupakan salah satu daripada masalah NP-Tegar yang paling sukar diselesaikan dan kebanyakan usaha penyelesaian masalah ini dinyatakan dalam bentuk heuristik. Tesis ini mengkaji penggunaan algoritma simulasi penyepuhlindapan dalam menyelesaikan masalah penjadualan bengkel kerja dengan masa pemprosesan stokastik. Kualiti jadual dinilai berdasarkan taburan *makespan*, iaitu tempoh penyudahan maksima bagi semua kerja. Idea utama adalah penggabungan simulasi ke dalam algoritma simulasi penyepuhlindapan. Dalam usaha ini, varian prosedur simulasi penyepuhlindapan bagi masalah berketentuan mulanya dianalisis dan kemudian dilanjutkan kepada versi stokastik dengan menggabungkan simulasi ke dalam algoritma simulasi penyepuhlindapan untuk menilai jadual yang dihasilkan oleh algoritma tersebut. Keputusan eksperimen menunjukkan bahawa varian stokastik ini cekap dalam menggabungkan semua maklumat berkaitan taburan masa pemprosesan ke dalam prosedur penjadualan. Di samping itu, alatan statistik seperti kaedah persampelan yang yang dimasukkan ke dalam algoritma berupaya pada tahap tertentu, meningkatkan kecekapan algoritma dan kualiti jadual. Prestasi simulasi penyepuhlindapan seterusnya dianalisis apabila tiga fungsi suhu yang berbeza dicadangkan. Hasil kajian dan analisis terhadap beberapa masalah ujian yang dipilih menunjukkan kelebihan algoritma yang dicadangkan berbanding dengan beberapa algoritma penghantaran biasa pada tahap stokastik yang tinggi. Akhirnya, korelasi antara jangkaan dan quantil-α bagi *makespan* dikaji. Penyelesaian yang diperoleh pada tahap stokastik rendah menunjukkan bahawa kedua-dua pengukur berkolerasi sempurna, manakala *makespan* didapati tertabur secara normal, kecuali beberapa kes yang berstokastik tinggi. Walaupun hanya masa pemprosesan stokastik dipertimbangkan, rumusan dan metodologi yang dibincangkan dalam tesis ini boleh dilanjutkan kepada pelbagai fungsi objektif dan jenis stokastik yang lain seperti masa ketibaan stokastik, kerosakan mesin tidak menentu serta kemasukan aktiviti baru.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Scheduling is broadly defined as a process of assigning a set of tasks to resources over time in order to meet certain objectives while respecting a set of constraints. Resources may refer to machines, equipment, labor or space while tasks may include operations in a production process, activities or customers. Scheduling problems appear in many applications including for examples, manufacturing and service industry, compiler optimization and parallel computing. In the manufacturing field, a scheduling problem involves the determination of the starting times of the jobs to be processed on some machines such that an appropriate performance measure of interest is optimized.

There is a variety of scheduling problems in the scheduling literature. Project scheduling and machines scheduling are two main applications that have motivated researchers in the scheduling area. In machine scheduling, a large number of specific applications depending on the machine environment and specific job characteristics have been considered. In project scheduling, there are variants of the resource-constraint project scheduling problem (RCPSP). Furthermore, applications like timetabling, rostering or industrial scheduling are connected to both areas, making them much closer to each other [1]. A scheduling problem can be *deterministic* where all problem parameters are assumed to be known with certainty or *stochastic* when at least some parameters are not known with certainty. A scheduling problem is called *static* when all the information is available at time zero and remains unchanged over time. On the other hand, when jobs arrive on a continuous basis and vary over time, the scheduling problem is called *dynamic* scheduling problem. In this thesis, a fairly general scheduling model that has a numerous applications and contains many other models as a special case is considered. The scheduling problem is called Job Shop Scheduling Problem (JSSP). JSSP is defined as problem of allocating resources to tasks over times, subject to precedence and resource constraints so that some measure of performance achieve its optimal values. The area of applications for the scheduling theory is wide, including computers and manufacturing, transportation as well as services. Assembling cars and scheduling airplane maintenance crews are examples of industrial operations that can be modeled as job shop scheduling problems.

1.2 Background of Problems

JSSP is well known for being one of the most difficult NP-Hard combinatorial optimization problems to solve in practice. The terminology of JSSP originates from the problems arising in manufacturing, where the resources are called machines and the tasks are called jobs. JSSP in general, consists of concurrent and conflicting goals to be satisfied using a finite set of machines and jobs. Each job consists of a set of operations that must be processed in a predetermined processing order through the machines which specify the precedence restrictions. Since the sequence of operations in a job is fixed, the sequence of the executions on each machine must be decided to obtain a complete schedule. The objective of JSSP is therefore, to find the sequence of the operations to be processed on each machine such that some functions of the performance measure are optimized. The general JSSP with n jobs and m machines has an infinite number of feasible schedules. This is because the idle times between operations can be varied.

The deterministic Job Shop Scheduling Problems (DJSSP) where each job's processing time is specific and known in advance have attracted considerable attention for several years [2-9]. Researchers have focused on the generation of good schedules in the presence of complex constraints and conflicting objectives, which assume fixed processing times, known jobs' arrival times and/or unbreakable machines. Unfortunately, most of the real world scheduling problem is subject to many sources of uncertainty or randomness. Uncertainty has to do with a situation where there are more than one possible outcomes and it is not possible to exactly describe the future as well as the existing state. Machine breakdowns, unexpected release of high priority jobs and the randomness in the processing times are some common examples of sources of uncertainty. For instance, in a stochastic scheduling problem, the duration of processing of an activity at certain time may change, because of an unexpected event. The processing time information is among the most critical inputs in solving the scheduling problems. Any change in processing times is likely to affect the solution and its corresponding objective function value. Luh in [10] mentioned that in the manufacturing industry, some of the ill-effect of uncertainties include system instability, excess inventory, customer dissatisfaction by not meeting the due dates, and more importantly, loss of revenue and, therefore has stressed the importance of developing systematic methods to address the problems of scheduling under uncertainty, in order to create efficient and reliable schedules. In general, when schedule under uncertainty, all the complexities of the deterministic counterparts are preserved, but with an extra challenge, that is, the performance measures become random themselves and cannot normally be obtained analytically as functions of parameters in a closed form [11]. This simple difference between stochastic and deterministic problems leads to many complexities in stochastic problems, making the scheduling problems more difficult.

One of the most studied performance measures of the stochastic JSSP is the makespan or the schedule's length, denoted by $C_{\max}$, which relates directly to the completion time of a project in Project Evaluation and Review Technique (PERT) environments. As stated by Jaime in [12], both addition and maximum of random variables are involved in the recursive representation of the makespan which has the similar structures to PERT problems, where the exact analysis is unavailable. There is considerable number of approaches in the PERT literature. Approaches which are based on approximating or bounding the distributions of the completion times of the activities are common in the PERT literature [13-16]. Another natural and flexible way to approximate the distribution function of performance measure is the Monte Carlo simulation [12]. However, simulation alone is only able to evaluate one specific solution to the SJSSP at a time, and incapable of performing a search of the entire solution space for an optimal or good solution. Due to the hard theoretical limitation of the stochastic counterparts, only in some special scheduling problems, heuristics such as the priority dispatching rules have an elegant solution [7]. In many applications classical approaches that guarantee to find the optimal solution require a lot of computational effort and are limited only for small size instances.

Instead of concentrating on the classical algorithmic approaches that are based on mathematical and dynamic programming, the attention of the operations research community over the past few decades has turned towards more flexible and powerful search methods that can provide good and reasonable response time though these solutions may not necessarily optimal. Local improvement methods, such as the beam search, the shifting bottleneck and in recent years, metaheuristics such as Tabu Search (TS), Simulated Annealing (SA), Genetic Algorithms (GAs), Ant Colony Optimization (ACO) and Greedy Randomized Adaptive Search Procedure (GRASP) are becoming successful alternative to classical algorithmic approaches that based on mathematical and dynamic programming for solving stochastic combinatorial optimization problems. These methods not only have been proven effective and efficient in solving many practical problems but they also manage to accommodate variations in problem structures. Among the various search methodologies used for the scheduling problems, Simulated Annealing has been recognized as general search strategy and optimization method which is useful in attacking both deterministic and stochastic combinatorial optimization problems. Given the complexity and difficulty of the stochastic job shop scheduling problem, the field is wide open for more work especially in the areas of modeling and solution methods. This research considers JSSP under stochastic environment and develops SA to address it.

1.3 Problem Statement

A job shop scheduling problem consists of a set J of n jobs $J = \{J_1, J_2, ..., J_n\}$ and a set M of m machines $M = \{M_1, M_2, ..., M_m\}$. For each job J_j, a sequence of operations $\{O_{ij}\}$ is to be processed on a specific machine M_i in a predetermined order and has a processing time P_{ij}. The processing orders of the jobs are also known as the technological constraints. The release time r_j of each job J_j indicates that no processing of the job can take place before the release time. Each machine can process only one operation at a time. Also only one operation from each job can be processed at a time and once an operation has started on a particular machine, it must complete processing without interruption. The processing times P_{ij}, are positive random variables described by known probability distributions function $\text{PDF}(P_{ij})$. The objective of the stochastic problem is to find an off-line schedule, denoted by s, of the operations to be processed on each machine such that the objective function value is optimized. In this research, two makespan related objective functions, namely the expected makespan, formally denoted by $E(C_{\max})$ and α-quantile makespan, denoted by $q_\alpha(C_{\max})$ will be examined. The SJSSP for the minimum expected makespan is formulated as a Disjunctive Programming formulation [17] as follows:

$$\text{Minimize } E(C_{\max}(\text{s})) = E(\max_{i,j}\{S_{ij} + P_{ij}\}) \tag{1.1}$$

Subject to:

$$S_{kj} \geq S_{ij} + P_{ij} \qquad \forall j \in J, \quad (i,k) \in O_j \tag{1.2}$$

$$C_{\max} \geq S_{ij} + P_{ij} \qquad \forall j \in J, i \in M \tag{1.3}$$

$$S_{ij} \geq S_{ir} + P_{ir} \vee S_{ir} \geq S_{ij} + P_{ij} \qquad \forall j, r \in J, i \in M \tag{1.4}$$

$$P_{ij} \sim \text{PDF}(P_{ij}) \qquad \forall j \in J, i \in M \tag{1.5}$$

In this formulation, Equation (1.1) provides the non linear objective function where S_{ij} is the earliest possible starting time of an operation O_{ij}. The first set of constraints (1.2) ensures that the processing sequence of tasks or operations in each job corresponds to the predetermined order. The third set of constraints (1.3) demands that there is one job of each machine at a time. The fourth constraint (1.4) defines the stochastic nature of the processing times P_{ij}. For the α-quantile makespan objective function, the objective function (1.1) is replaced with

$$\text{Minimize } q_\alpha(C_{\max}(\text{s})) = \inf\{\delta : \Pr(C_{\max}(s) \le \delta) \ge \alpha\} \tag{1.6}$$

for a given probability $\alpha \in (0.5,1)$ and δ is a time value, called the due date. Equation (1.6) seeks for as small value of δ as possible such that there is a solution whose random makespan is, with high probability, less than δ.

1.4 Research Objectives

The main objectives of the research are given as follows:

1. To solve the SJSSP with SA procedure by treating the problem as DJSSP in which the adaptability and robustness of the deterministic optimal solution for the stochastic environment are the major concerns.
2. To develop variants of simulation-based simulated annealing algorithm based on stochastic and statistical techniques to find a good solution to the SJSSP in which the influence of stochastic levels is of major importance.
3. To analyze the trade-off between the two performance measures discussed in this thesis, namely the expected makespan and the α-makespan.
4. To determine known distributions that will reasonably fit the makespan realizations for different plans (sequences).

1.5 Scope of the Study

This research focuses on a priori or offline planning procedure in a classical job shop under uncertainty based on integrating a well-known metaheuristics, namely SA and simulation. The SJSSP will only consider randomness that stems from uncertainty in the durations of the jobs or processing times. To model uncertainty associated with the random processing times in the shop, probability theory is used. Other sources of uncertainty (for examples, machine breakdown and urgent arrival of new jobs) are ignored. In this model, jobs are available for processing at time zero and the objective function is to optimize some characteristics of the random makespan (the maximum completion time which is equivalent to the completion time of the last operation) distribution. In other words, this research deals with a *static* stochastic job shop scheduling problem as opposed to the *dynamic* problem when jobs arrive randomly into the system. In the static and stochastic job shop scheduling problems, the identification of an optimal solution is done before the actual realization of the random variables so that the solution may be applied with no modifications (or very small ones) once the actual realization of the random variables are known. This type of problems is known as ‘a-priori’ or off-line optimization. The static problems may serve as a heuristic basis for dynamic decisions by providing a base plan that can be dynamically updated later. We can find examples where schedules are published in advance so they are static, as the airport schedules where the actual sequence of arrivals and departures is subject to dynamic decisions [11].

In this research, no special assumptions on the distributions of the processing times, except that for each processing time, the expected value and variance are known. Makespan is chosen as the performance measure because it is a multi-objective criterion: an optimum schedule with minimum makespan value is also minimum idle time on machines, maximum machine utilization, minimum work in process and minimum number of jobs in progress. Further, makespan minimization problem is well defined and able to capture the fundamental computational difficulty which exists implicitly in determining an optimal schedule.

1.6 Significance of Findings

Most research reported in the literature of JSSP focuses on optimizing certain objective function under idealized conditions and thus do not take into consideration sources of uncertainty. This thesis contributes toward better understanding and solving SJSSP subject to uncertainty via simulation optimization technique. It is hoped that this work will lead to application in the real environments which can be modeled as a job shop. The study of this simplified model may provide an insight on the techniques to be used for more general formulations although the real world applications may have other elements to consider such as sequence dependent set-up times, machines breakdowns and random arrivals of jobs.

1.7 Major Contributions of the Research

The major contributions are:

1. The development of an efficient simulation-based SA algorithm to solve the SJSSP with random processing times and the minimum expected makespan as the criteria. The algorithm performs well against pure dispatching heuristics at all level of variability which require a moderate amount of running time, making them feasible tools for off-line scheduling. Additionally, the proposed algorithm is extended to analyze an α-quantile makespan producing similar good results.
2. The incorporation of confidence interval and a variance reduction technique called Descriptive Sampling into the basic simulation-based SA algorithm and the benefit is empirically assessed.
3. The introduction of three cooling schedules that improves the quality of the solution found by the simulation-based SA algorithm and a comparative analysis is conducted to assess the gains.
4. The identification of the correlation between the two performance measures discussed in this thesis and fitting distributions of the random makespan. The

experiment reveals perfect correlations between expected makespan and α-quantile makespan and that the optimal sequences are normally distributed in most cases.

1.8 Conceptual Framework

Figure 1.1 shows the conceptual framework describing the knowledge areas related to each component of SJSSP under study. Some major components of the framework will be briefly described.

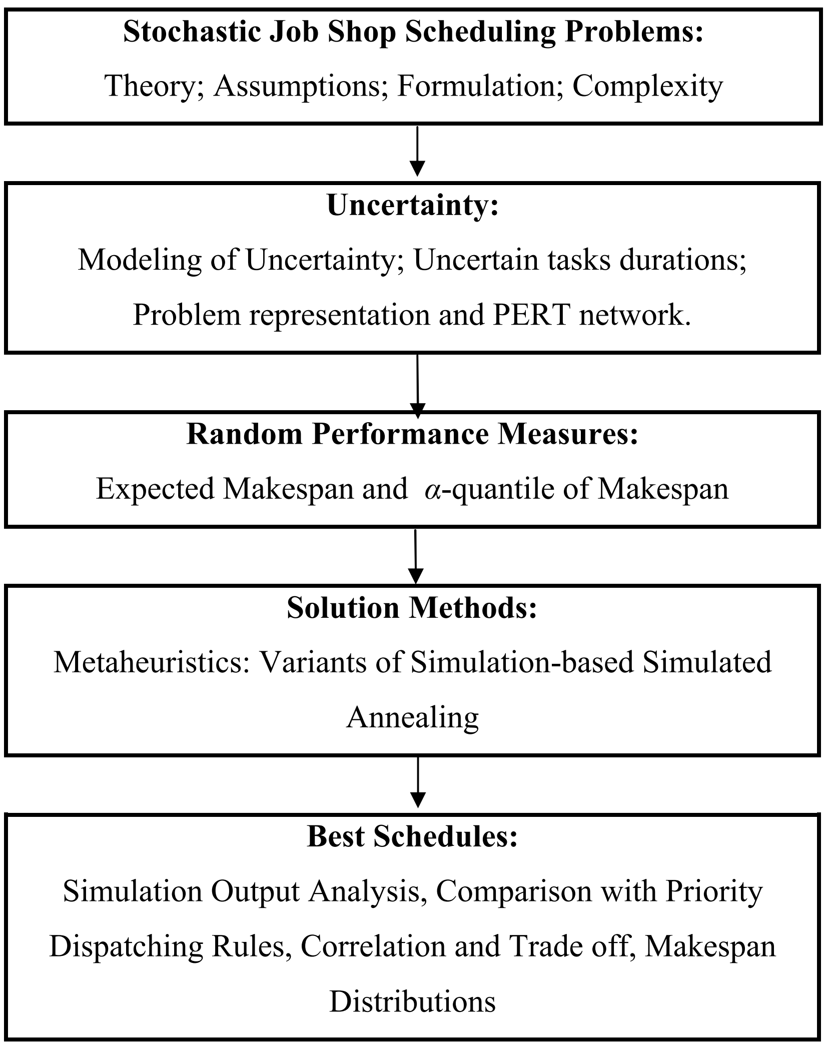

Figure 1.1 Conceptual Frameworks of SJSSP

1.8.1 Stochastic Job Shop Scheduling Problems: Assumptions

The SJSSP is a natural extension of the standard DJSSP. In the stochastic context, the following three assumptions may take place.

(i) Job release dates are not known in advance.
(ii) Machines can break down.
(iii) Processing times are modeled by random variables.

The job data, such as processing times, due dates and release dates, may not be exactly known in advance; only their distributions are known in advance. The above forms of randomness can be modeled in several ways. For instance, one may model the possibility of machine breakdown as an integral part of the processing times. This is done by modifying the distribution of the processing times to take the possibility of break down into account. In principle, to construct an instance of a stochastic job shop scheduling problem, sufficient information to derive a complete probability distribution is required. However, no special assumptions on the distributions of processing times, P_{ij}, except that for each machine M_i and job J_j the expected value $E(P_{ij})$ and variance $\sigma^2(P_{ij})$ exist. These processing times are assumed to be statistically independent of each other. Though it is rather unrealistic assumption from practical point of view, based on availability of historic data about processing times and other random parameters, probability distributions associated to those parameters are determined. Together with some typical types of distribution, such as a uniform, normal, or exponential distribution, one can state the desired probability distribution.

1.8.2 Problem Representation and Stochastic Precedence Networks

Many of the heuristics methods that have been successfully applied to deterministic JSSPs are based on the disjunctive graph approach. This graph represents the scheduling problem and provides mathematical structure for both search and evaluation. In the scheduling with the stochastic processing times, the nodes in the activity network do not have fixed duration, but the precedence relations described by the disjunctive graph still define the feasibility of a sequence of operations. A feasible solution can then be represented by a stochastic precedence network. Given the processing order of machines, the graph representing the schedule can easily be constructed by connecting nodes representing operations on the same machine with arcs reflecting the processing sequence. A job shop problem can lead to a countless number of feasible solutions. Note that, for a JSSP instance with m machines and n jobs, there are at most $(n!)^m$ possible feasible solutions. Therefore, with large problems, the computational time to obtain an optimal solution would be extremely long if all solutions were to be examined. Disjunctive graphs and the related notations will be described in detail in Chapter 2.

1.8.3 Modeling of Uncertainty in Scheduling

A number of different models have been proposed and used to represent uncertainty in scheduling. In stochastic scheduling, uncertain information can be described by means of random variables of known probability distributions. Under this assumption, the objective function strongly depends on the probabilistic structure of the model. The objective function is usually a function of random variables which include, for examples, the expected makespan, expected tardiness and the variance measure. The schedules of static and stochastic problems with random variables can be generated off-line, where decisions are taken in advance of their executions. This type of schedules is applicable in a situation where no or small modifications are done once the actual realization of the random variable are known. On the other hand, if the stochastic scheduling problem is dynamic, the generation of a schedule or plan is incremental; that is, the schedule is completed as long as execution goes on. Decision usually needs to meet real-time requirements where it is also possible to change decisions during execution. The schedules generated are then called on-line or reactive schedules when there are changes in decisions during execution [18].

Uncertainty in a Robust Optimization approach assumes that uncertain information is known in the form of interval values [19, 20]. The robustness approach aims at finding solutions that hedge against the worst contingency that may arise, given that no knowledge about the probability distribution of random data is known. In a robust approach, a solution is chosen using a particular robust criterion, such as the min-max criterion or the min-max regret criterion. In the min-max criterion, one determines a solution that minimizes the largest cost over all scenarios. The min-max regret is a less conservative criterion, where one determines a solution minimizing the largest deviation from the optimum over all scenarios. This criterion was applied to several scheduling problems within the last decade [18], where the deterministic versions are polynomially solvable.

In this research, the uncertainty is represented by probability distributions with known means and variances.

1.8.4 Stochastic Performance Measure and Objective Functions

Performance measures in scheduling are numerous, complex and often conflicting [11]. These include criteria based upon completion times, criteria based upon due dates and criteria based upon inventory and utilization costs. A popular performance measures for job shop problems based upon completion times of the jobs is the makespan, $C_{\max} = \max(C_1, C_2, ..., C_n)$ where C_j denote the time that J_j completes its last operation. It represents the total time required to complete all the jobs in a schedule

In a stochastic network, all quantities that depend on the activity durations are random variables. Consequently, the objective function which is generally a function of job completion times is a random variable with unknown distribution. It occurs often in stochastic scheduling that these random variables need to be compared to one another which will generally require knowledge of its CDF or its moments.

Instead of determination of the starting times of the operations to be processed, the goal of the stochastic analysis now is to find a strategy or solution (production sequence) whose performance measure has the best statistical distribution. Performance measure associated to each stochastic precedence network can be expressed as some functions of the completion time of jobs, such as the flow time, tardiness and lateness, which are random variables. For example, there are several scalar performance measures to characterize the random makespan, $C_{\max}$. For a risk-neutral decision maker, the objective is usually in the form of *expectation*; that is to find a solution such that the expected makespan, $E(C_{\max})$ is minimized. If there may be a substantial probability that the makespan of the solution will be much higher than its expected value, then a solution *s* can also be found with minimum *α*-quantile of the makespan, denoted by $q_\alpha(C_{\max}) = \inf\{\delta : \Pr(C_{\max}(s) \le \delta) \ge \alpha\}$, for a given probability *α* and due date *δ* as an approximate representation of risk-averseness. The probability of missing a dead line, T_0, denoted as $prob(C_{\max} \ge T_0)$ and expected tardiness: $E[max(C_{\max} - T_0,\ 0]$ are also common performance measures in stochastic scheduling. In this thesis, we analyze both $E(C_{\max})$ and $q_\alpha(C_{\max})$.

1.8.5 Stochastic Scheduling Complexity and Optimization

The complexity of the job shop scheduling problem has been studied intensively. In [21] Sotskov and Shakhlevich proved that the minimum makespan job shop scheduling problem with three jobs and three machines is NP-hard. In [22] Garey *et al*. proved that the minimum makespan of DJSSP with two jobs is NP-hard. Slight modifications of these DJSSP turn out to be difficult. In particular, job shop problems with *m* machines ($m \ge 2$) using the makespan performance criteria are NP-hard in the strong sense [7]. The stochastic scheduling problem considered in this thesis is at least NP-hard since their deterministic variants are NP-hard [7]. Several stochastic optimization problems related to ours are known to be NP-hard. PERT problems are a special case of the stochastic scheduling problems considered here. Since there are no machine restrictions, the unique optimal policy schedules each job as early as possible with respect to the precedence constraints. In other words, the computation of a single value of the distribution function of the makespan is already NP-complete, and the computation of the expected makespan is no easier [16].

1.8.6 Metaheuristics

Glover and Laguna [23] define a metaheuristic as a master strategy that guides and modifies other heuristics to produce solutions beyond those that are normally generated in a quest for local optimality. The use of adaptive memory, neighborhood search methodologies, the ability to carry multiple solutions from iteration to iteration, and acceptance criterion to create efficient and intelligent searches are all incorporated in a metaheuristics. A well-known class of metaheuristics is the local or neighborhood search. A local search metaheuristics starts from a given initial solution and iteratively generates new solutions, each of which is obtained from the previous one by performing a move on it. The procedure involves the computation and comparisons of one or more solutions by means of the objective function values and move and from one neighbor to another as long as possible while decreasing the objective value. The evaluation of the objective function is essentially a "black box" operation but it must be performed efficiently, as in many applications; evaluations are the most computationally intensive activity. In general, these strategies require a mathematical model to provide structure and guidance when applied to a combinatorial problem. The most valuable model for the JSSP is the disjunctive graph of Roy and Sussman [24] which is capable of providing a mathematical structure to the scheduling problem in term of the search and evaluation. The graph can be used to evaluate all possible feasible solutions of the scheduling problem by reversing arcs from a predefined set.

The main applications to scheduling problems of metaheuristics, include Simulated Annealing, Tabu Search, Ant Colony Optimization and Genetic Algorithm. The nice thing about metaheuristics is that they can be easily modified to tackle the stochastic combinatorial optimization problems [25].

1.8.7 Simulation

Monte Carlo simulation is a widely used technique to heuristically approximate the makespan distribution, along with other performance measures or other quantities of interest in a stochastic network. The basic methodology for makespan distribution (adapted to this research) is as follows. Iteratively draw *M* (sufficiently large) independent samples $x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, \ldots\ldots\ldots, x_{ij}^{(M)}$ from the distribution of the random processing time X_{ij}, for job *j* on machine *i* and the makespan for each scenario can be efficiently calculated using a standard longest path computation to obtain a sample $c_{\max}^{(1)}, c_{\max}^{(2)}, c_{\max}^{(3)}, \ldots\ldots, c_{\max}^{(M)}$. The sampled makespans are of course independent samples from the makespan distribution. Figure 1.2 provides the histogram for makespan realizations of a test problem, (ABZ06) for a sample size of 5,000 simulation replicates given the jobs processing times are uniformly distributed with coefficient of variation of 0.0144.

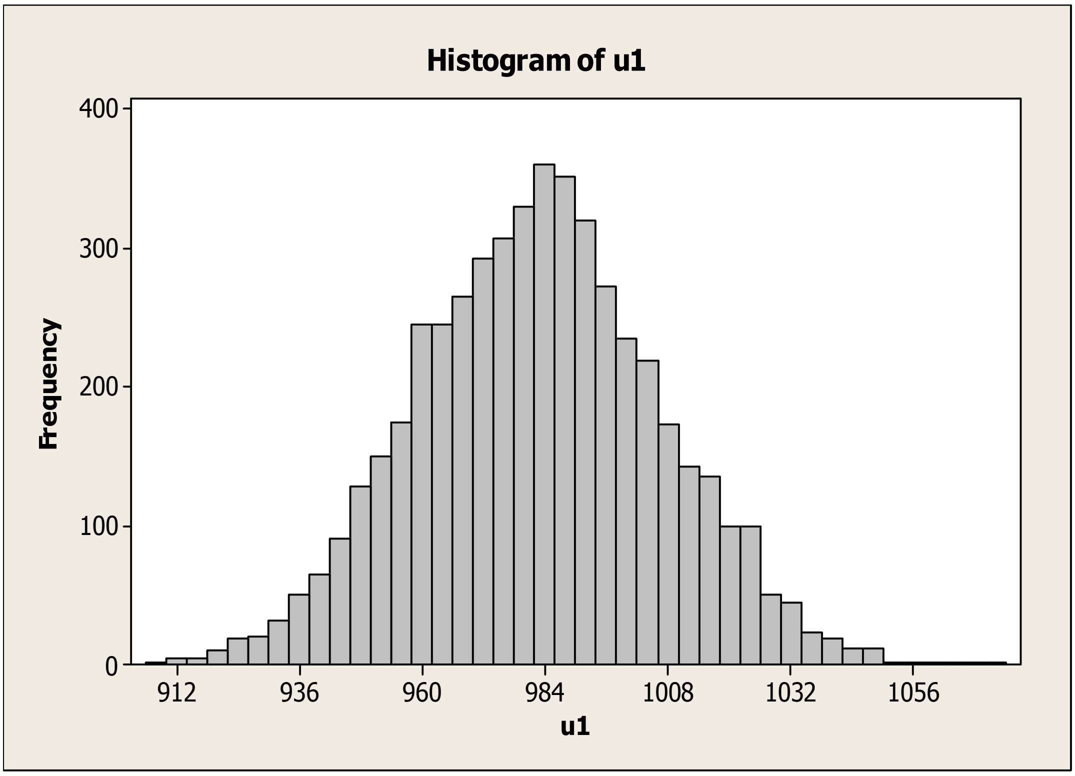

Figure 1.2: Histogram for Makespan of ABZ06

1.9 Outline of the Thesis

The rest of the thesis is organized as follows.

- Chapter 2 provides background and related works required to understand the entire thesis. This chapter introduces the formulation of the job shop scheduling problem and the disjunctive graph formulation of the problem. Basic concepts and definitions of the stochastic job shop scheduling problem are introduced in this chapter in order to understand the complexity and the nature of the problem tackled by this thesis. A review of related research on JSSP especially using simulated annealing is presented.

- Chapter 3 is exclusively dedicated to the description SA and the stochastic versions. The technique incorporates simulation to estimate the value of the objective function of the stochastic problem. The basic concepts of the procedure in the deterministic environment and the stochastic extension are dealt in details. The structural elements, the implementation and computational procedures of this solution technique in context of job shop scheduling problem with random processing times are discussed.

- In Chapter 4, a simulation-based SA framework is developed to address the stochastic job shop scheduling problem. This chapter contains the contribution of the proposed methods. It includes the explanation of the variants for the proposed approach with detail discussion relating to the stochastic problem. It starts with the framework of the proposed simulation-based optimization and deals with the structural elements of the algorithm and describes the computational procedures to improve the solution found by the proposed method.

- Chapter 5 presents some computational experiments and results. The experimental framework and the criteria for analyzing the results of the proposed approach are described in the first section of this chapter. The second section presents the results with interpretation. The results are compared with some priority rules.

- Chapter 6 presents the experimental results of SJSSP with the α-quantile of makespan objective. The primary goal of this chapter is to gain insight into the trade-off and correlation between two performance measures discussed in this thesis: the expected makespan and the α-quantile of makespan of SJSSP, and to see whether the two stochastic performance measures necessarily lead to the same optimal sequence. The makespan distributions of some good sequences of SJSSP are also investigated and discussed.

- Chapter 7 is the summary and conclusion. Directions for future research are also discussed.

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LIST OF ABBREVIATIONS

ACO	-	Ant Colony Optimization
BB	-	Branch and Bound
CDF	-	Cumulative Distribution Function
CI	-	Confidence Interval
CRN	-	Common Random Numbers
CV	-	coefficient of variation
DJSSP	-	Deterministic Job Shop Scheduling Problem
DP	-	Dispatching Rules
DS	-	Descriptive Sampling
GA	-	Genetic Algorithm
GT	-	Giffler and Thompson
JSSP	-	Job Shop Scheduling Problem
LL	-	Lower Limit $(1-\alpha)100\%$ confident limit
LPT	-	Longest Processing Time
LR	-	Lagrangian Relaxation
$N1$	-	Neighborhood structure of VanLaarhoven
$N2$	-	Neighborhood structure of Dell'Amico and Trubian
$N3$	-	Neighborhood structure of Novicki and Smutnicki
PD	-	Priority Dispatching
PDF	-	Probability density function
PDSA	-	Pseudo-deterministic simulated annealing
PERT	-	Project Evaluation and Review Technique
RANDOM	-	Random rule

SA	-	Simulated Annealing
SJSSP	-	Stochastic Job Shop Scheduling Problem
SPT	-	Shortest Processing Time
SSA	-	Simulation-based simulated annealing
SSA_ADPT	-	Simulation-based simulated annealing with adaptive Temperature
SSA_ADPT_CI	-	Simulation-based simulated annealing with adaptive Temperature and confidence interval
SSA_CONT_CI	-	Simulation-based simulated annealing with constant And Confidence Interval
SSA_CI	-	Simulation-based simulated annealing with constant with confidence interval
TS	-	Tabu Search
UL	-	Upper Limit $(1-\alpha)100\%$ confident limit

LIST OF SYMBOLS

s	-	A solution of JSSP
J_j	-	Job j
M_i	-	Machine i
O_{ij}	-	An operation of J_j on M_i
C_j	-	Completion time of J_j
S_{ij}	-	The length of the longest path from the start to the completion of O_{ij}
T_{ij}	-	The length of the longest path from the completion of O_{ij} to the sink
$\{O_{ij}\}$	-	A sequence of operation of J_j on M_i
T_k	-	Temperature at k^{th} iteration
T_f	-	Final temperature
T_0	-	Initial temperature
δ	-	Date line
r_j	-	Release time of J_j
F_j	-	Flow time of J_j
L_j	-	Lateness of J_j
d_j	-	Due date of J_j
T_j	-	Tardiness of J_j
E_j	-	Earliness of J_j

F_{Σ}	-	Total flow time
L_{Σ}	-	Total lateness
E_{Σ}	-	Total earliness
T_{Σ}	-	Total tardiness
$L_{\max}$	-	Maximum lateness
$T_{\max}$	-	Maximum tardiness
$SM(O_{ij})$	-	Machine successor of O_{ij}
$SJ(O_{ij})$	-	Job successor of O_{ij}
$PM(O_{ij})$	-	Machine predecessor O_{ij}
$PJ(O_{ij})$	-	Job predecessor O_{ij}
$N(s)$	-	Neighborhood structure of a solution s
$N1$	-	Neighborhood structure of VanLaarhoven
$N2$	-	Neighborhood structure of Dell'Amico and Trubian
$N3$	-	Neighborhood structure of Novicki and Smutnicki
$C_{\max}$	-	Makespan
$\hat{F}(C_{\max}(s))$	-	An estimation of statistic of makespan for s
$q_{\alpha}(C_{\max})$	-	α-quantile of makespan

LIST OF APPENDICES

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