MATHEMATICAL MODELING OF EARTHQUAKE INDUCED VIBRATIONS AND ITS EFFECTS ON MULTISTORY BUILDINGS

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To my beloved father and mother,

Mahmud and Rosnah

And

My truly supporter,

Azhar

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IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

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ABSTRACT

A strong earthquake will cause sudden fatality, great economic loss and panic to community. A mathematical model of forced vibrations on multistory buildings is used as a tool to study the mechanism of vibration that caused by an earthquake. The model is used to calculate the natural frequencies, ω and period, P of the building vibrations. Furthermore, the vibration on each floor and the maximal amplitude of the building vibrations also being analyzed. This research only concentrates on three floors building and five floors building. Matrix method is used to get analytical solution of the earthquake model and the graphs of calculated frequencies against period and vibrations of the floors against time are plotted via Maple 12 package. Based on that graphs, the related discussions are being made.

ABSTRAK

Gempa bumi yang kuat akan menyebabkan malapetaka, kerugian ekonomi yang besar dan ketakutan kepada masyarakat. Suatu model matematik bagi getaran daya ke atas bangunan bertingkat digunakan sebagai alat untuk mengkaji mekanisma getaran yang disebabkan oleh gempa bumi. Model ini digunakan untuk mengira frekuensi semulajadi ω dan tempoh P bagi getaran bangunan. Seterusnya, getaran setiap aras dan amplitud maksima dari getaran bangunan juga di analisis. Penyelidikan ini tertumpu pada bangunan tiga tingkat dan lima tingkat. Kaedah matrik digunakan untuk mendapatkan penyelesaian analisis model gempa bumi dan graf-graf frekuensi yang dikira melawan tempoh dan getaran aras melawan masa dilakarkan dengan mengguna pakej Maple 12. Berdasarkan graf-graf yang dilakar, perbincangan yang berkaitan dilakukan.

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LIST OF SYMBOLS

=	Equal
+	Plus
-	Minus
m	Mass
F	Force
$x_1, x_2, \ldots x_n$	Displacement of the floors
f(t)	External Force
M	The mass matrix
K	The stiffness matrix
n	Number of floors
ω	Frequency
P	Period
λ	Eigenvalue

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CHAPTER 1

INTRODUCTION

1.1 General Introduction

Most people use models every day, even though they do not realized it. Life and nature are filled with processes for which most adults have well developed models. According to Blancard (1998), mathematical models of the real world phenomena can be formulated in term of differential equation. This research used differential equations to study about an earthquake induced vibrations on a multistory building (Marchand and McDevitt, 1999).

There are many questions on the mathematical contextual for the vibrations of an earthquake. For example, which floor will feel the biggest vibration and the smallest vibration? How long the period will produce the destructive resonance vibration on the building? Which building will collapse first compare to the other building? Why an earthquake will demolish one building but leave the one next door untouched? Hence, all the questions above can be answered by extends the study of Ordinary Differential Equations (ODE) concept to model the vibrations of a multistory building.

This research takes full advantage of analytical technique to address a large range of mathematical concept. The familiarity with Maple 12 is important because it will use as a tools in this research. The purpose of this research is to model the vibrations of a multistory building, caused by earthquake and then we will use the solution of the method to discuss about amplitude of the vibration, frequency of the vibration and resonance.

1.2 Background Of The Research

Earthquake is one of the catastrophes in the world and one of the most devastating on earth. A strong earthquake will cause sudden fatality, great economic loss and shock to community. From the United States Geological Survey (USGS) National Earthquakes Information Center, it has mentioned that the largest earthquake in the world since 1900 was happened in Chile, 1960 with magnitude of 9.5 scalar Richter. This earthquake killed 1655 people, injured 3000 and displaced two million.

According to Zappler and Long (2002), there are some causes of earthquakes. Throughout human history, the unpredictable and ruinous effects of earthquakes have provoked some explanations. Example in Japan, their residents believe that earth tremors caused by the subterranean stirrings of a giant catfish and restrained with a big mallet by a watchful deity. Contrary with the Greek philosopher Aristotle, he theorized that winds from above were drawn into hollow passangeways deep inside the earth and these winds caused quakes when agitated by fire and seeking to escape.

Mathematical modeling is important in the study of mechanical resonance because it may provide understanding of the underlying mechanism which influence the vibration and may suggest the prevent strategies. Marchand and McDevitt (1999) model

the vibrations of a single-story building using ODE. From this model, the ODE can be used to build the mathematical modeling of the vibration on multistory building.

Edwards and Penny (2008) also contribute to the mathematical modeling which they applied the second order differential system in a case of earthquake induced vibration on seven floors building. The calculation from the model showed that different buildings have different natural frequency of vibration.

The modeling of forced vibration on multistory building is the tool which has been used to study the mechanisms of vibration that caused by an earthquake. Analysis of frequencies, periods and amplitude of natural oscillations will give some ideas related to earthquake induced vibrations. Furthermore, the vibration of each floor and the maximal amplitude of the building vibrations also can be analyzed. This research used the existing model which is done by Edwards and Penny (2008). This research only concentrates on three floors building and five floors building. This study used eigenvalues method to solve analytical solution and plot graphs via Maple 12 package.

1.3 Statement Of Problem

The problem of an earthquake induced vibrations on a multistory building is to study the response of the tall building due to horizontal seismic motion at the foundation generated by an earthquake. For this purpose, a mathematical model related to this effect need to formulate and interpret the model. The mathematical model is used as a tool in analyzing the forced frequency, periods of natural oscillations and the amplitude as a function of period.

1.4 Objective Of The Study

The main objectives of the study are:

- To reformulate mathematical model of the effect of an earthquake on three story building without damping.
- To show that analytical solution for the vibration of each floor of the three story building is equal to the Maple 12's solution.
- To analyze the frequencies and periods of natural oscillations of three story building and five story building.
- To analyzed the vibrations of earthquake for each floor of three floors building and five floors building.
- To analyze and discuss the maximal amplitude as a function of period for three story building and five story building.

1.5 Scope Of The Study

This research will discuss about the application of differential equation for earthquake induced vibration on a multistory building. This study focuses on one model which is forced vibrations of a multistory building without damping. This study will concentrate on three floors building and five floors building.

Data used in this study were taken from the Module For Earthquake Induced Vibrations. This module is provided by California State University of Fullerton (2012). Meanwhile, the concept of the earthquake induced vibration on multistory building is taken from Edwards and Penney (2008).

1.6 Significance Of The Study

This research can help engineers and mathematicians to understand the effect of earthquake to the tall buildings. This is due to many cases of earthquake occurred in the world. In other way, it will provide some ideas for mathematicians to conduct further research about this field.

1.7 Management Of The Research

Mathematicians have proved that mathematics as a tool that can be used to model and understand the phenomena of mechanical resonance such as vibrations and so on. In general, this study actually discuss about some knowledge about vibration caused by earthquake. Other than that, the basic mathematical modeling of force vibration without damping of a multistory building model has been discuss to make sure there are some contribution in earthquake knowledge which is related to mathematics.

In this chapter, the introductory about this research are reviewed, including the important tools such as background and problem statement, some objectives, scope and significant of this research also have been discussed. Chapter 2 presents the foundation of earthquake, including definition and information of the field. Next, the chapter reviews on the earthquake induced vibration on multistory building.

Theoretical implementation is discussed in Chapter 3. This chapter discussed about the basic concept of vibration caused by an earthquake proposed by Edwards and Penny (2008). Analytical solution also stated in this chapter. The chapter discussed a

method on solving nonhomogeneous second order differential equation using eigenvalues method.

In applying the vibration model, the data analyzing study is devoted on Chapter 4. The model is used to calculate the natural frequencies, ω and period, P of the building vibrations. Furthermore, the vibration of each floor and the maximal amplitude of the building vibrations also can be analyzed. Maple 12 is used to plot the behavior of the vibration models.

Conclusion of this research and suggestion to minimize the damage of the earthquake are discussed in Chapter 5.

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