HIROTA BILINEAR COMPUTATION OF MULTI SOLITON SOLUTIONS KORTEWEG de VRIES EQUATION

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To my beloved family

A very understanding mother, Jamaliah Hassan

A supportive husband, Mohammad Misrul Ain

and my child, Muhammad Ian Shaheem

To all my dearest friends

and

To my thoughtful supervisor

Assoc. Prof. Dr. Ong Chee Tiong

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ABSTRACT

Soliton is the solution of nonlinear partial differential equation that exists due to the balance between nonlinearity and dispersive effects. The existence of these two effects in Korteweg de Vries (KdV) equation enables us to obtain solitons solutions. The purpose of this research is to obtain the multi soliton solutions of KdV equation by using Hirota bilinear method. This method can produce the explicit expression for soliton solutions of KdV equation. From these solutions, a general pattern of F function in Hirota bilinear method is revealed. The amplitude of interacting soliton will determine the phase shift pattern. Various interactive graphical outputs produced by MAPLE computer programming can illustrate the solutions of these multi soliton up to eight-soliton solutions of KdV equation.

ABSTRAK

Soliton adalah penyelesaian bagi persamaan pembezaan separa tak linear yang wujud akibat keseimbangan di antara kesan tak linear dan penyelerakan. Kewujudan dua kesan ini dalam persamaan Korteweg de Vries (KdV) membolehkan kita untuk mendapat penyelesaian soliton. Tujuan kajian ini adalah untuk mendapatkan penyelesaian multi soliton bagi persamaan KdV dengan menggunakan kaedah Hirota bilinear. Kaedah ini boleh menghasilkan ungkapan eksplisit bagi penyelesaian soliton dalam persamaan KdV. Daripada penyelesaian ini, corak umum bagi fungsi F didedahkan. Amplitud bagi soliton yang berinteraksi akan menentukan corak anjakan fasa. Pelbagai paparan grafik yang interaktif dihasilkan melalui pengatucaraan komputer MAPLE dapat memberi ilustrasi penyelesaian multi soliton sehingga lapan soliton dalam persamaan KdV.

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LIST OF ABBREVIATIONS/SYMBOLS

DNA	-	Deoxyribo Nucleic Acid.
KdV	-	Korteweg de Vries.
PDE	-	Partial Differential Equation.
2D	-	Two dimensional.
3D	-	Three dimensional.
$U(x,t)$	-	The elongation of the wave function with variable x and t.
x	-	Space domain.
t	-	Time domain.
η	-	The height of the peak.
c	-	Speed of the wave.
δ	-	Phase shift of soliton.
l	-	The depth of water.
g	-	The gravitational acceleration.
k	-	Amplitude of soliton.
μ	-	Existence of soliton wave

A_{ij}	-	Interaction of two solitons

CHAPTER 1

INTRODUCTION

1.0 Introduction

This chapter introduces the topic chosen for the study. It presents the background of study followed by the problem statement. The objectives of the study are listed and the scope of the study is also discussed. The chapter ends with the significance of the study.

1.1 Preface

The existence of solitary waves was first discovered by Scottish engineer, John Scott Russell in year 1834 (Miin, 2012). It is called as solitary waves as it often occurs as this single entity and is localized. He was observed this solitary wave phenomenon on the Edinburgh Glasgow canal and called it ‘great wave translation’. Throughout this phenomenon, he had performed some laboratory experiments and found that taller waves travel faster and are narrower (Drazin and Johnson, 1989).

A breakthrough came later in the 19^{th} century, when Diederik Korteweg and Gustav de Vries in year 1895 starting a hydrodynamic description, derived from a nonlinear partial differential equation which had solutions in which the nonlinearity is counterbalanced by a dispersive term which stabilizes the shape of solitary wave, thereby vindicating John Scott Russell's observations. The partial differential equation arrived by Korteweg and de Vries's reasoning had the same form as one previously studied by Joseph Boussinesq in year 1871 (Riseborough, 2010).

In addition, Riseborough (2010) also stated that Zabusky and Kuskal had found an amazing discovery in year 1965 by showing that, in continuum limit, the experiment conducted by Fermi-Pasta-Ulam in year 1955 yielded the Korteweg de-Vries (KdV) equation. They had found that although the equation was nonlinear, the solitary waves described by Korteweg de Vries (KdV) equation appeared as if they did not interact with each other. That is, after two solitons collide, they emerge with their shape and velocities unchanged. The only signature of the collision was phase shift. Due to this particle-like attributes of the wave pulse excitations of the Korteweg de Vries (KdV) equation, Zabusky and Kuskal first introduced the term "soliton" to describe them.

As solitons can be interacting with each other, thus the study on these interacting solitons had been conducted extensively by researchers since 1970s. According to Zhao (2012), the sign and absolute value of velocity determine the propagation direction and speed of soliton. He found that the propagation direction and behavior of these interacting solitons followed the asymptotic forms.

1.2 Background of study

As mentioned above, the concept of the soliton was initiated by Zabusky and Kuskal in year 1965. As the result, theory of the inverse scattering transform was developed. Apart from that, many numerical studies were used to indicate the soliton behavior towards nonlinear wave solutions. According to Drazin and Johnson (1989), here are three main numerical methods which are finite difference method, finite element method and spectral method.

The Korteweg de Vries equation can model the dynamics of solitary waves. The KdV equation can be expressed as

$$u_t + 6uu_x + u_{xxx} = 0.$$

This is a nonlinear partial differential equation where $u(x,t)$ is the amplitude of the wave, x is the spatial term and t, is the time evolution. The u_{xxx} term is a dispersive term and the uu_x term is a nonlinear term (Druitt, 2005).

Therefore, this research will investigate the analytical solution of KdV equation by using Hirota bilinear method for multi soliton solutions. Hirota bilinear method was introduced by Japanese researcher, Hirota in year 1971. This new direct method was developed to construct multi soliton solutions to integrable nonlinear evolution equation. The Hirota bilinear method turned out to be the fastest and easiest way to obtain the results of KdV equation solution for multi soliton (Hietarinta, 1997).

Nowadays, most of the researchers applied Hirota bilinear method to further revise on the multi soliton solutions. Yang and Mao (2008) had conducted a study on soliton solution and interaction property by using the Hirota bilinear method. They had found that in two solitons interactions, both solitons display the states before collision, exhibit the impacted and superposed states in the interactive area and after collision the solitons represent the identical states before collision. They also argued that from the changes of amplitude and velocity, it shows that solitons interactions do not exchange their physical quantities but undergo a phase shift.

1.3 Problem Statement

The Korteweg de Vries equation is a nonlinear partial differential equation that can be solved numerically and analytically. However, to obtain the analytical solutions of KdV equation is not easy. Several methods had been used earlier to obtain the solutions of KdV equation. In this research, we will observe the soliton ladder of solutions. We have to produce the permutation parameters of solitons interactions in order to obtain all the eight soliton solutions of KdV equation. As these solutions are difficult to calculate manually, thus we need a computer programming tools to derive the F function and produce the various graphical outputs for up to eight-soliton solutions of KdV equation.

1.4 Objective of Study

The objectives of this research are:

1) to solve KdV equation analytically by using Hirota bilinear method.
2) to obtain the phase shift pattern in multi solitons solution of KdV equation.
3) to obtain the general pattern of F function for multi solitons solution of KdV equation.

1.5 Scope of Study

This study focuses on the multi soliton solutions of KdV equation by Hirota bilinear method that transforms the nonlinear partial differential equation into bilinear equation via the transformation of dependent variable. Meanwhile, the explicit expression of multi soliton solutions for KdV equation is obtained. The solutions will be investigated up to eight-solitons and computer programming; MAPLE will be used to obtain the graphical outputs of multi soliton solutions for KdV equation. The solution is discussed until eight-soliton as KdV equation mostly applied for shallow water waves which consist of ripple. Eight-soliton solutions already give a long calculation but it is sufficient to be applied for multi soliton solutions. Besides that, the phase shift pattern of two-soliton interaction is discussed in order to observe the relationship of amplitude towards soliton phase shift. The general pattern of F function is explored as well as to help us in determining the accurate function applied in solving multi soliton solutions of KdV equation.

1.6 Significance of Study

In detail, this study will discuss the multi soliton solutions of KdV equation up to eight-solitons. The Hirota bilinear method will be used to obtain these solutions of KdV equation which developed from the concept of soliton. Thus, it can be applied in many areas such as shallow water and deep water waves, fibre optics, DNA and protein, and biological model.

The characteristics of KdV equation itself which are nonlinear and dispersive could gives important thoughts in solving a critical phenomenon such as tsunami. The balancing of the nonlinearity and dispersion effects in tsunami phenomenon is important to understand the occurrence of wave's dispersion near beaches as it travels in a long distance. The travelling of tsunami waves which behaves like soliton can be modelled as KdV equation.

Through this research, we will able to obtain the solution of multi soliton up to eight-solitons which shows the soliton ladders of KdV equation by using Hirota bilinear method. The phase shift pattern and general pattern of F function will gives another thoughts which convenience others to solve multi soliton solutions.

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