Modeling and Control of an Engine Fuel Injection System

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Abstract: **Control of automotive exhaust emission has become an important research area to meet the more stringent automotive emission regulations. Beside the modification on internal combustion engine, control engineering is seen as another approach to improve and meet these requirements. This paper focuses on the design and development of a control system to reduce the harmful waste of automotive exhaust emission. The control system aims to regulate the amount of fuel injected into the combustion chamber such that the air to fuel ratio (AFR) is maintained within the allowable range. The control process in this paper is demonstrated based on an analytical engine model that clearly describes engine's air and fuel dynamic with no loss of engine system performance. Since the dynamics of the internal combustion engine and fuel injection systems are highly nonlinear, a linear model is obtained in this paper, based on a system identification approach to allow methodical application of linear control theories. The linear quadratic Gaussian (LQG) control strategy is employed in this paper. The LQG controller, designed based on the linear model of the engine system, results in good controlled output response and provides better controlled output response by reducing the transient effect occurred in LQG controller design.**

Keywords: fuel injection system, LQG controller, air to fuel ratio.

I. INTRODUCTION

In the past, the performance of the production engine control module has been sufficient to pass all government-mandated emission standards. However, recently, the need to reduce CO_2 levels has made the improvement on fuel efficiency an urgent matter, to cope with the stricter environmental regulations concerning emissions from exhaust gases. Moreover, the number of vehicles around the world has reached 50 millions in 2007 and expected to increase by 5% every year and reach approximately 60 million by the year 2010 [1], turning the internal combustion engine emission to be one of the main contributors to the environment pollution with harmful gases such as carbon monoxide (CO), hydrocarbons (HC), nitrogen oxides (NO_x) and also other particulate emissions. These situations require extensive improvements to the fuel injection controllers. The use of a catalytic converter placed in the exhaust pipeline to chemically convert harmful exhaust emissions to more environmental-friendly gases is well established. However, the catalyst efficiency is affected by the fluctuations of the AFR between lean and rich mixture, as well as with accumulated vehicle mileage. This has resulted in car manufacturers and their suppliers to start developing new engine control strategies instead of using traditional technology [2].

Stoichiometric ratio (the ideal AFR) is important to achieve high percentage of conversion efficiency of pollutants from engine exhaust [3], as shown in Fig. 1. The stoichiometric ratio can only be attained if the air and fuel mixture in the combustion chamber is in the ratio of 14.7. Therefore, to maintain the AFR at stoichiometric value of 14.7, suitable controllers for the fuel injection system must be used to regulate the amount of fuel injected into the system.

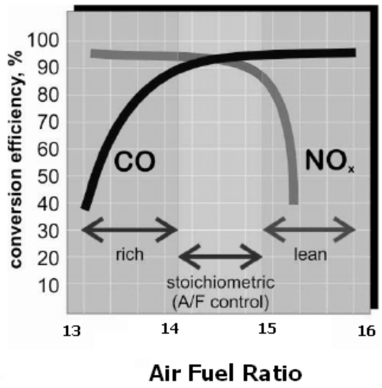


Figure 1: Relationship between conversion efficiency and air fuel ratio.

In the conventional AFR control system, the electronic control unit (ECU) of the vehicle will collect the engine data and determines the suitable amount of fuel (valve timing) to be injected through reference engine data. These reference data are saved in 3-D lookup maps produced by vehicle manufacturer factory test cell and based on a number of tests on production engines. Replacing the 3-D maps with automatic controller into engine's control system has the potential to decrease the time and effort required in the calibration of fuel valve timing using reference 3D lookup maps, thus improve the performance of the AFR control system [4].

Various researches are conducted to design engine emission controllers aiming for better AFR and exhaust quality. For example, Muske, et. al. [5] has employed a classical PI controller to the air fuel ratio control of engine. Ju-Biao [6] have worked on a simulation of AFR control based on neural network approach, where the traditional PI controller is combined with neural network estimation to estimate the air fuel ratio signal without the transportation delay. Pieper [7] performed a simulation study of AFR control from nonlinear engine model using sliding model control method to achieve perfect robust performance but there are problems due to the uncertain and nonlinear natural of the dynamic of engine. All of these methods, however, are either not performing satisfactorily or impractical. For instance, the use of simpler classical PI controller seemed to performed poorly in real time engine system whereas the neural network and sliding model control approaches are rather complex and make them impractically to be applied in real-time. Therefore, LQG control method with simple and effective control approach is proposed such to overcome those problems as stated previously.

The aim of this paper is to present the design of a linear quadratic Gaussian (LQG) controller and a fuzzy logic controller (FLC) for AFR control. The objective of the controllers is to maintain the AFR at stoichiometric level with the variation of intake air. The controllers regulate the amount of fuel into the combustion chamber by controlling the effective fuelling time constant. Before designing the controller, the nonlinear engine model is approximated by its linear version obtained via system identification technique. The linear model, which is more suitable for AFR control purpose, is then used in the design of the LQG controller. The effectiveness of the controllers is tested via computer simulation by applying them to the nonlinear engine model. The performance of the LQG controller is then compared with that of the FLC and the benefits and limitations of each controller are highlighted.

II. ENGINE MODEL

Modeling of an engine is important for research and development purpose. Most engine models use a combination of analytical and empirical methods to represent each of the engine components [8]. Analytical engine model such as filling and emptying model [1], CFD approach [9] and mean value model [10] describe engine models through detailed mathematical functions with the inclusion of physic laws. On the other hand, empirical engine model such as neural network [11], polynomial [12] and interpolation of steady state map [13] employ experimental data to predict most of the engine processes.

The engine model used in the work presented in this paper is based on the generic mean value engine model developed by [14]. It is obtained via an analytic approach and is suitable for exhaust emission control purposes. In this model, the engine operation is represented by a system with two input variables. The first input variable is an air mass, which is a function of throttle valve angle excited by the driver that also determines the engine speed. The second input variable is the fuel mass injected by electromagnetic nozzle as the main actuator. The air and fuel are mixed and injected into a cylinder placed in an intake manifold. After the compression and combustion processes, the combusted gas is transferred into an exhaust pipe. Here, the AFR quality will be measured and feedback for control purpose. The engine system is generally described by the air dynamic, the fuel dynamic and the torque dynamic. The following sections explain the development of the engine model used in the research. All the symbols used are described in the Appendix at the end of the paper.

A. Air Dynamic

Air flow in the engine's inlet manifold is the main part of mean value engine modeling. In an internal combustion engine, air is induced into the cylinders through inlet valve that depends on the pumping action of the engine which is affected by the volumetric efficiency.

The air mass balance in the inlet manifold is described by Eq. (1).

$$\dot{m}_a = \dot{m}_{ai} - \dot{m}_{ao} \qquad (1)$$

In this case, the air mass rate entering the intake manifold, $\dot{m}_{ai}$. is described by Eq. (2), which is closely related to the engine's throttle body.

$$\dot{m}_{ai} = MAX \cdot TC \cdot PRI \qquad (2)$$

The normalized throttle characteristic for simulation purpose is represented by a function of throttle angle α as shown in Eq. (3).

$$TC = \begin{cases} 1 - \cos(1.14459 \cdot \alpha - 1.06) & for\ \alpha \le 79.46 \\ 1 & for\ \alpha \ge 79.46 \end{cases} \qquad (3)$$

whilst the normalized pressure ratio influence (*PRI*) is represented by Eq. (4).

$$PRI = 1 - \exp\left(9\left(\frac{P_m}{P_{atm}} - 1\right)\right) \qquad (4)$$

The pressure in the manifold, P_m , is obtained from the ideal gas law, and given in Eq. (5)

$$P_m = \frac{m_a \cdot R \cdot T_m}{V_m} \qquad (5)$$

$$m_a = \frac{M_a \cdot P_m \cdot V_m}{R \cdot T_m} \qquad (6)$$

The air mass exiting the manifold, thus entering the engine body cylinder is described by Eq. (7),

$$\dot{m}_{ao} = c_1 \cdot \eta_{vol} \cdot m_a \cdot \omega_e \qquad (7)$$

and the air mass rate leaving the manifold, C_1, is represented in Eq. (8) below:

$$c_1 = \frac{V_d}{4\pi \cdot V_m} \qquad (8)$$

The volumetric efficiency, η_{vol} , from mass flow exiting the manifold is determined by a non-linear empirical relationship as shown in Eq. (9), which is a complex function of engine geometry and engine parameter obtained from experimental data.

$$\eta_{vol} = (24.5 \cdot \omega_e - 3.1 \cdot 10^4) \cdot m_a^2 + (-0.167 \cdot \omega_e + 222) \cdot m_a + (8.1 \cdot 10^{-4} \cdot \omega_e + 0.352) \qquad (9)$$

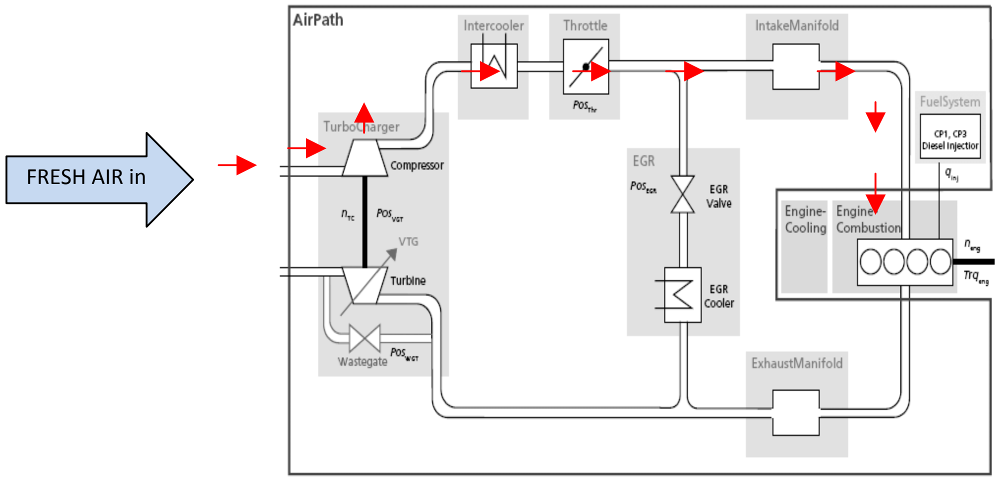


Figure 2: Schematic of air system.

B. Fuel Dynamic

In an internal combustion engine, fuel is injected directly into the engine cylinder, just before the combustion process is required to start. Load control, β which is one of the engine's model state variable, is achieved by varying the amount of fuel injected at each cycle. Therefore, β always need to maintain at certain level so to keep Air fuel ratio at stoichiometric level.

The mathematical modeling of the cam contour and helix groove depends on the specific components used in real engine work and the fuel injection system is assumed as a linear system with the signal input from the governor that depends on the load condition, as shown in Eq. (10).

$$\tau_f \cdot \dot{m}_{f_i} + m_{f_i} = m_{fc} \tag{10}$$

Eq. (10) describes the relationship between fueling commands and fuel flow rate into the cylinder, which is characterized by a combination of lag and transport delays due to the discrete nature of the intake process. For a sequential-fire port fuel-injection system, the fueling model is simplified as a first order equation in term of the actual fuel rate entering the combustion chamber m_{f_i}. The effective fueling time constant τ_f is modeled as:

$$\tau_f = 0.05 + \frac{1.2\pi}{\omega_e} \cdot \frac{m_{fc}\,\vartheta}{MAX} \tag{11}$$

C. Rotation Torque Dynamic

The piston engine model comprises the air flow through the inlet valve and the combustion torque calculation. The effect of the AFR on the combustion torque is included in the torque efficiencies measurement. Besides that, the friction torque is also taken into account for effective torque calculation.

The torque built in the engine model is a function of engine speed, fuel flow and air flow. The combustion and torque production subsystem contain delays associated with the four combustion processes as modeled in the engine's indicated torque, T_i , given in Eq. (12):

$$T_i = c_T \cdot \frac{\dot{m}_{ao}(t - \Delta t_{it})}{\omega_e(t - \Delta t_{it})} \cdot AFI(t - \Delta t_{it}) \cdot CI(t - \Delta t_{ci}) \tag{12}$$

From Eq. (12), the normalized air fuel ratio influence function is described as

$$AFI = \cos(7.3834 \cdot (A/F - 13.5)) \tag{13}$$

$$\frac{A}{F} = \frac{\dot{m}_{ao}}{m_{f_i}} \tag{14}$$

while the normalized compression influence CI is described as

$$CI = (\cos(CA - MTB))^{2.875} \tag{15}$$

Finally, the crankshaft rotation which follows the torque balance relationship about a rigid shaft is described as:

$$I_e \cdot \dot{\omega}_e = T_i - T_f - T_a \tag{16}$$

where the engine friction torque T_f is resulted from coulomb and viscous friction torque such that friction torque results to:

$$T_f = 0.1056 \cdot \omega_e + 15.10 \tag{17}$$

where 0.1056 denotes the viscous friction coefficient and 15.10 denotes the coulomb friction coefficient.

D. System Identification of a Linearised Engine Model

System identification is a process of deriving the mathematical model of dynamic systems from measured data [15]. In this paper, system identification technique is performed to determine the approximated linear engine model in state space representation, which is needed for the design of LQG controller.

The approach involves the application of random input signals that represent the injected fuel into the engine system and recording the corresponding output signal, which is the AFR. These input/output data are then used to estimate the engine model based on the deterministic stochastic subspace identification method [16]. By using *sysid* from MATLAB software, the identification process results in the estimated third order linear state-space model given in Eq. (18) and Eq. (19).

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{18}$$
$$\underline{y} = C\underline{x} + D\underline{u} \tag{19}$$

where,

$$A = \begin{bmatrix} 0.18734 & 0.13306 & 0.10468 \\ 0.08183 & 0.78614 & -0.54529 \\ -0.00054 & 0.10877 & 0.26882 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.00516 & -0.01720 \\ -0.00073 & 0.09841 \\ -0.00011 & 0.13589 \end{bmatrix}$$

$$C = [158.16 \quad 8.4277 \quad -0.44246]$$

$$D = [0 \quad 0]$$

Fig. 4 shows the output signals of the estimated linear and actual nonlinear models, which suggest that close approximation of the system model is obtained with best fit of up to 90.69 percent. This third-order state space equation is validated to represent linear engine plant model in discrete form with the smallest loss function value of 0.0641 and the smallest final prediction error of 0.0643 compared to any other order of estimated engine models.

III. LQG CONTROLLER

LQG controller is a linear quadratic regulator combined with Kalman filter, and has been widely used in the active control of various high-order dynamical systems for optimal control performance. The linear quadratic regulator normally works well with excellent stability margin but the presence of process noise and the unavailability of sensors for all state measurements required for full state feedback control makes its implementation impractical. On the other hand, LQG controller caters for the presence of noise and includes a Kalman filter to estimate all the states which are not available by measurement.

The control objective of the LQG is to minimize a criterion, which is a quadratic function of the system states and control signals [17], when the system is subject to certain initial conditions. When designing an optimal controller, the system is assumed to be linear or a linearised system model is used, and has a state space equation such as given in Eq. (18) and Eq. (19). The control law is chosen such that it minimizes the cost function,

$$J_{LQ} = \underline{x}^T(t_f)H\underline{x}(t_f) + \int_0^{t_f}\left(\underline{x}^T(t)Q\underline{x}(t) + u^T(t)Ru(t)\right)dt \quad (20)$$

where H, Q and R are weighting matrices. H and Q are at least positive semidefinite and R is positive definite. t_f is the final time that the control is required. For a control system that is designed to operate for a long time period, the following cost is used [18].

$$J_{LQ} = \int_0^{\infty}\left(\underline{x}^T(t)Q\underline{x}(t) + u^T(t)Ru(t)\right)dt \quad (21)$$

The control law is given by Eq. (22) and Eq. (23),

$$u(t) = -K(t)\underline{\hat{x}}(t) \quad (22)$$

$$K = -R^{-1}B^T P_r \quad (23)$$

where K is the feedback gain matrix, $\underline{\hat{x}}(t)$ is the state estimated using the Kalman filter and P_r is obtained by solving the Ricatti equation given in Equation (24).

$$\dot{P}_r = -P_r A - A^T P_r - Q + P_r B R^{-1} B^T P_r \quad (24)$$

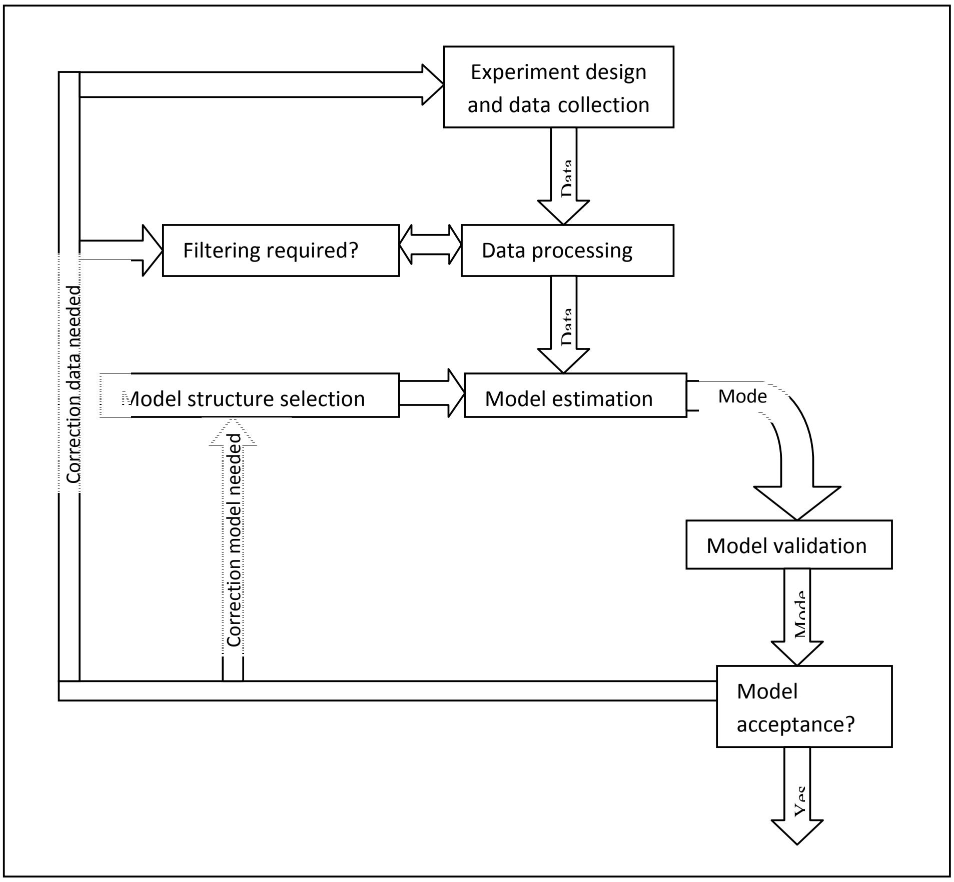


Figure 3: Cycle of system identification function.

The Ricatti equation has only final condition, $P_r(t_f) = H$, and the values of P_r corresponding to the optimal trajectory can therefore be found by solving it backward in time using any numerical integration method. In MATLAB®, P_r can be obtained using the function, `care(A,B,Q)`.

For the scalar case of Q and R, the cost function, J_{LQ}, can be interpreted as the weighted sum of the state and control. The choices of Q and R matrices allow the respective weighting of the energies of different signals and through that, increases the importance of keeping certain signals small in expense of the others. Generally speaking, selecting Q large means that, to keep cost function J_{LQ}, small, the state $x(t)$ must be smaller. On the other hand selecting R large means that the control input $u(t)$ must be smaller to keep J_{LQ} small. Choices of these matrices follow no particular rules. They depend on the designer's understanding of the behaviour of the system to be controlled, followed by some tuning by trial and error, until satisfactory performance is achieved. However, as a guideline, Q can be chosen such that it results in the contribution of each state being roughly equal [18]. To solve the LQR problem described so far, the system in Eq. (18) must be completely controllable so that J_{LQ} in Eq. (21) is finite. For the engine system to be controlled, the following weighting matrices are selected:

$$Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$R = 500.$$

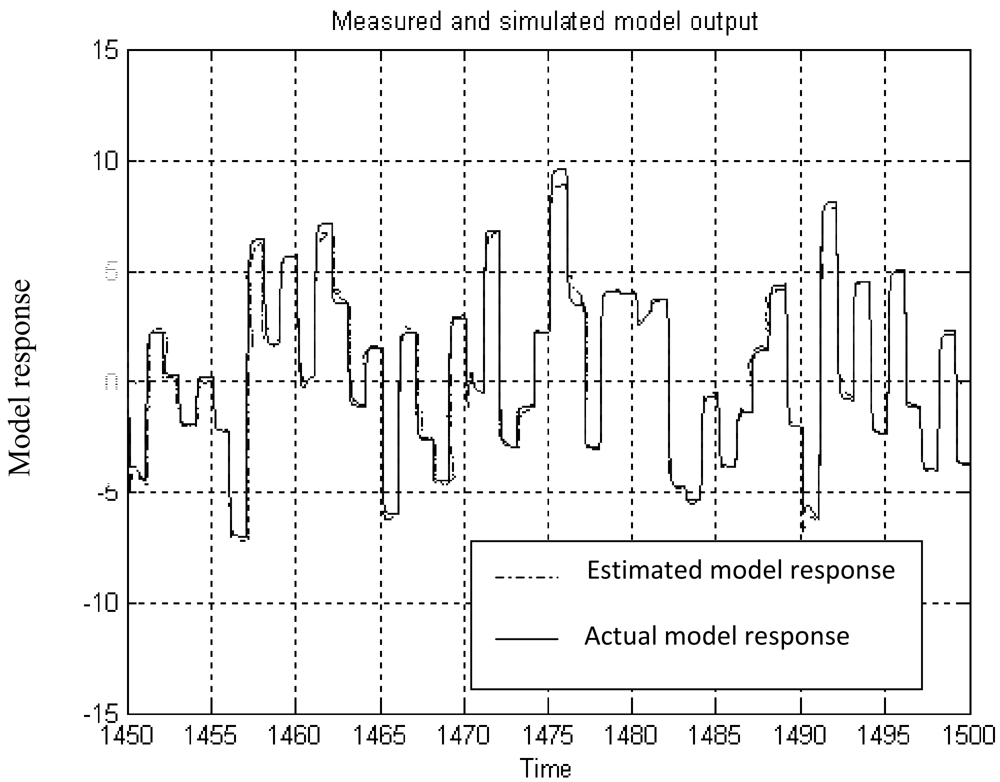


Figure 4. Actual and estimated plant output response.

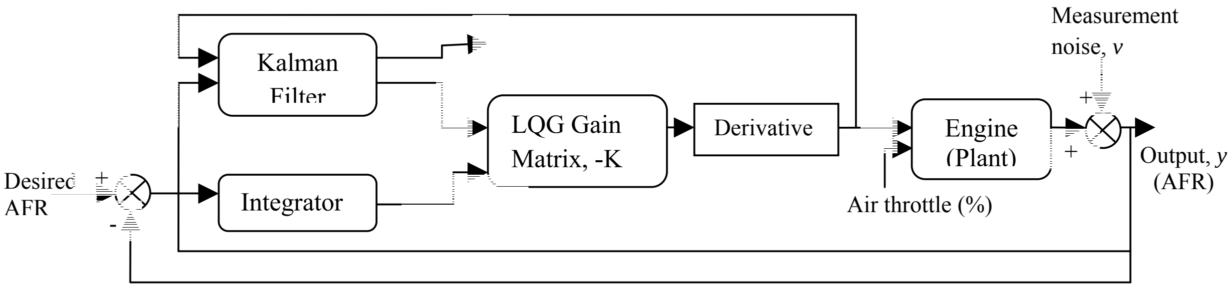
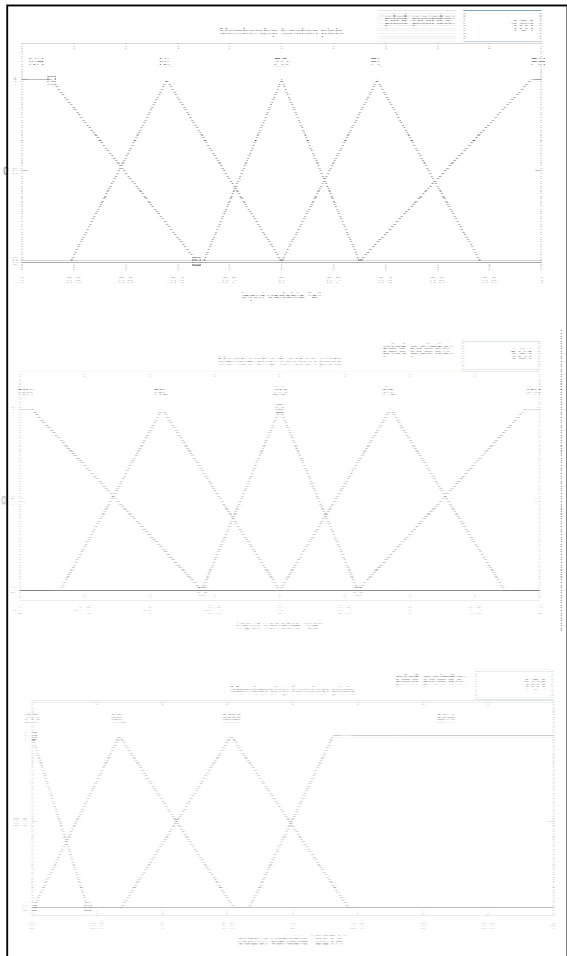


Figure 5. Block diagram engine system controlled by the LQG controller.

Fig. 5 shows the block diagram of the engine system controlled by the LQG controller. The integrator is used to reduce the steady-state error whereas the derivative term functions to speed up the controller signal and reduce the transient effect.

IV. FUZZY LOGIC CONTROLLER

In general, fuzzy logic controller involves three main processes: fuzzification, rule base design and defuzzification [19]. Fuzzification is a process of converting each piece of input data to degrees of membership based on certain membership functions. The fuzzification block thus matches the input data with the conditions of the rules, to determine how well the condition of each rule matches that particular input instance. Fig. 6 shows two inputs fuzzification membership function for the engine model. The two fuzzy input variables are the engine AFR error, e, and the change of this error value, Δe.



Five input fuzzy set:

NH(negative high) -large negative error value

NL(negative low)-small negative error value

ZO(zero)-zero error value

PH(positive high)- large positive error value

PL(positive low)-small positive error value.

Five output fuzzy set:

ZO(zero)-zero gain value

ML(medium low)-small gain value

MM(medium medium)-medium gain value

MH(medium high)-medium large gain value

Figure 6. Inputs and Output membership function contain in fuzzification and defuzzification process.

Table1. Fuzzy rules.

Error

Change in error

ce \ e	NH	NL	ZO	PL	PH
NH	MH	MM	ML	ML	MM
NL	MH	MM	ML	ML	MM
ZO	MH	ML	ZO	ML	MH
PL	MM	ML	ML	MM	MH
PH	MM	ML	ML	MM	MH

After fuzzification process, these membership function values are ready to be processed by the rule base through conditional “if-then” statements. The rule base is used in the decision-making process in regulating engine AFR around a prescribed setpoint. Table 1 shows a total of 25 rules implemented for the AFR control purpose. There is no specific theorem available for designing a complete fuzzy rule.

Therefore, a full understanding of plant behavior is needed, in order to result a suitable fuzzy rule base. Error value which is the difference of air fuel stoischiometric ratio and actual air fuel ratio has to be regulated around zero value. So, whenever percentage of input air throttle, which influencing the flow rate of air mass, is increasing, input fuel flow rate, which is control by fuzzy output, need to increase too so that error is always close to zero.

The fuzzified result must be converted to a number that can be sent to the process as a control signal. This process is called defuzzification. The membership from the fuzzy output in Fig. 4 is used to sum and defuzzify the control signal into a crisp analogue output value from the 25 rules. Controller crisp output value is the abscissa under the center of gravity of the fuzzy set,

$$u = \frac{\Sigma_i \mu(x_i) x_i}{\Sigma_i \mu(x_i)} \qquad (19)$$

x_i, is a running point in a discrete universe, and $\mu(x_i)$ is its membership value in the membership function [6]. Following the evaluation of rules, the defuzzification process that transforms the fuzzy membership values into a crisp output value used to control the fuel pulse width or fuel injection valve opening. Fig. 7 shows the simulation model of engine system with fuzzy logic controller. In this case, the 3-D maps for AFR control from the ECU has been replaced by the FLC.

V. RESULTS AND DISCUSSIONS

In this paper, the engine is assumed to run at steady state condition with percentage variation of engine's input air throttle angle shown in Fig. 8.

Fig. 9 shows the AFR response when only the LQG controller with an integrator is used. Figure 9 reveals that the performance of the controller is not as good as expected since it takes longer for the controller to bring the AFR to stoichiometric value compared to the ECU. This is because, although the integral action reduces the steady-state error, it results in the system becoming more oscillatory since the integrator causes the system to accelerate quickly in the direction to reduce the error.

When a modification on LQG controller is made by adding a derivative block, the performance of the LQG controller is improved, as shown in Fig. 10. The derivative block minimises the overheat and the oscillations that acts as an anticipatory mode of the controller i.e. it anticipates which direction the process is heading by looking at the rate of change of the error and hence, speed up the controller gains and regulate AFR in shorter period of time.

Fig. 11 shows the AFR control result when the FLC is used. It demonstrates the effectiveness of the FLC when the AFR overshoot effect is completely reduced without causing a large transient effect to the system when the engine's input air throttle angle changes. Compared to the LQG controller, the overall performance of engine's AFR response with fuzzy logic controller is better in terms of the time it takes to reach stoichiometric value and overshoot during transient. However, design procedure and steps to result a good FLC performance are difference based on behavior of plant model which is probably not as straightforward as the LQG controller that is using only one type of controller model with different control parameter.

VI. CONCLUSIONS

In this paper, the linear engine model has been obtained from the simulation of the nonlinear engine model through system identification technique. Once the linearised model is obtained, the LQG controller is designed to regulate the AFR of the engine at stoichiometric value when the engine's intake air vary. It has been shown that the proposed strategy provides improved performance in terms of generating control effort and following the desired trajectory. The simulation results suggest that the LQG controller provides better control in allows stability of the closed-loop system to be determined.

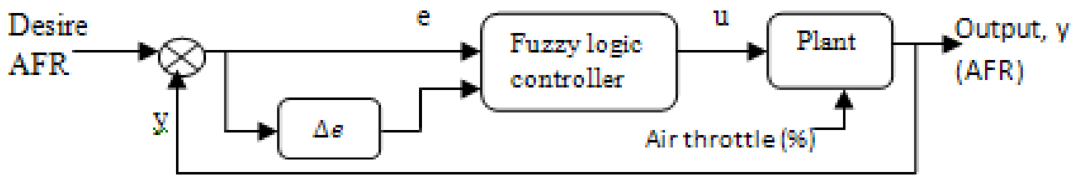


Figure 7. The Fuzzy Logic controller and engine model in MATLAB-SIMULINK.

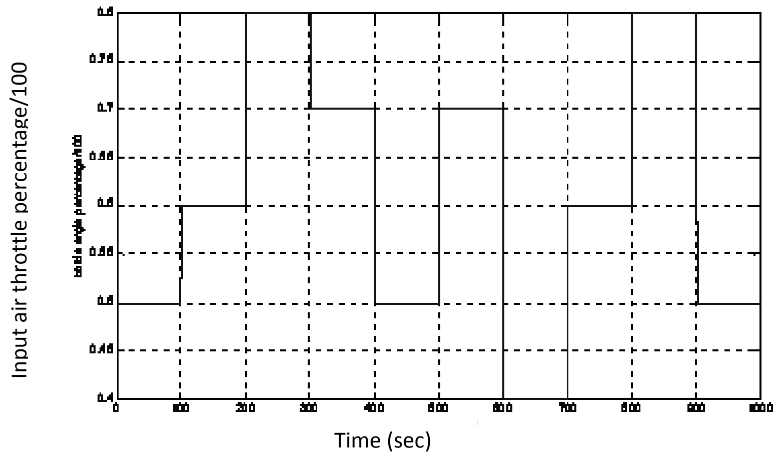


Figure 8. Throttle angle percentage versus time.

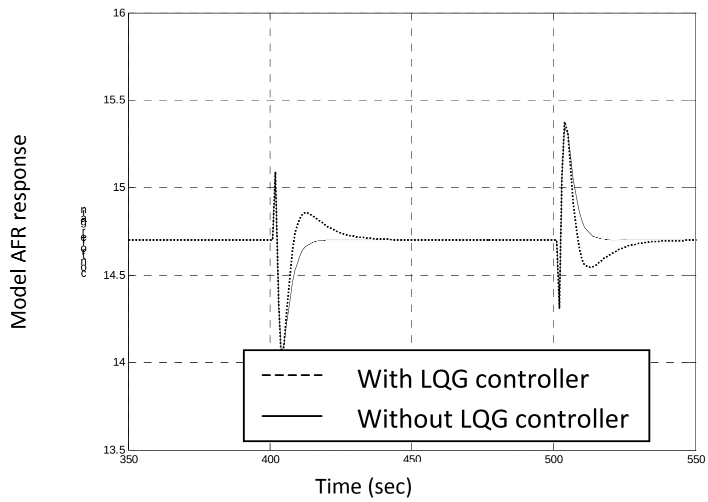


Figure 9. AFR response with LQG controller (dashed) and without LQG controller (solid).

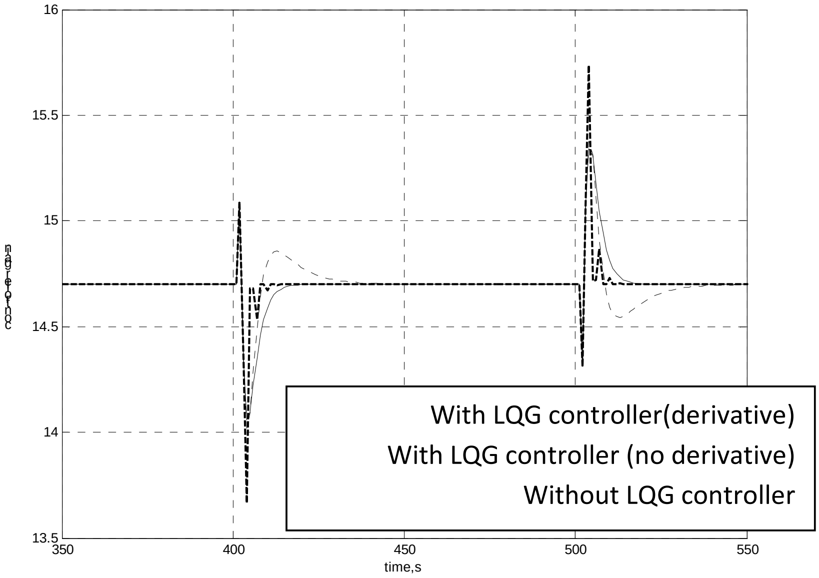


Figure 10. AFR response with LQG controller (with and without the derivative block).

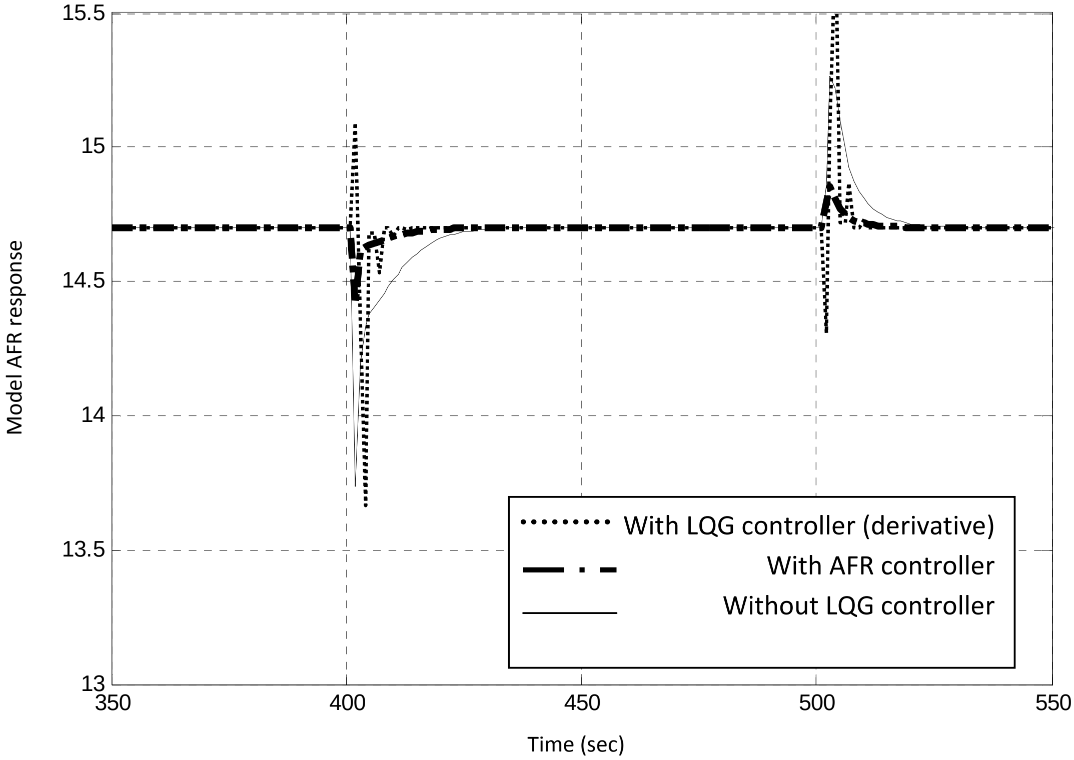


Figure 11. AFR response from engine model with fuzzy logic controller and LQG controller crop from time between 350s to 550s.

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APPENDIX
Nomenclature

R	gas constant
T_m	gas temperature
V_m	intake manifold volume
ω_e	engine angular velocity
η_{vol}	volumetric efficiency
m'_{fi}	fuel rate entering the combustion chamber
m'_{fc}	command fuel rate
τ_f	effective fueling time constant
β	desired air fuel ratio
Δt_{it}	intake to torque production delay
Δt_{ct}	compression to torque production delay
AFI	normalized air fuel ratio influence function
CI	normalized compression influence function
c_T	the maximum torque production capacity of an engine given that *AFI*=*CI*=1
A/F	actual air fuel ratio of the mixture in the combustion chamber
CA	tuning parameter of cylinder advance at the Top Dead Center
MTB	minimum tuning such that best torque acquire
I_e	effective inertia of the engine
T_i	engine indicated torque

T_f engine friction torque
T_a accessories torque
AFR Air Fuel Ratio
CO carbon monoxide
HC Hydrocarbons
NOx Nitrogen Oxides
CFD Computational Fluid Dynamic
PI Proportional Integral
LQG Linear Quadratic Gaussian
LQR Linear Quadratic Regulator
A an n-by-n matrix, where n is the number of states.
B an n-by-m matrix, where m is the number of inputs.
C an r-by-n matrix, where r is the number of outputs.
D an r-by-m matrix.
Q weighting matrix
R weighting matrix
v measurement noise
w process noise