ELEMENT FREE GALERKIN METHOD OF COMPOSITE BEAMS WITH PARTIAL INTERACTION

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To my thoughtful father and mother, lovely wife and kids, and family.

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ABSTRACT

Composite beam with partial interaction behaviour has ignited many studies, not just on its mechanics but also on solutions of its one-dimensional partial differential equation. Inadequate solution by available analytical methods for this high order differential equation has demanded for numerical approach and therefore Element Free Galerkin (EFG) method is applied for the first time in this present work. The work consists of three parts; first is the formulation of Galerkin weak form and assemblage of the EFG discrete equilibrium equation. One-dimensional formulation of the weak form is performed by adopting the variational approach and the discrete equation, which is in matrix form and written using the Matlab programming code. Subsequently in second part, the EFG formulation is developed for both the slip and uplift models, where the former adopted equal curvature deflection assumption while the latter considered the unequal curvature. The proposed EFG formulation gives comparable results in both models, after been validated by established analytical solutions, thus signify its application in partial interaction problems. The third part provides numerical tests result on EFG numerical parameters such as size of support domain, polynomial basis and quadrature points with seven different types of weight functions for this composite beams behaviour. Conclusively, Cubic Spline and Quartic Spline weight functions yield better accuracy for the EFG formulation results, compares to other weight functions. The capability of the EFG formulation was also studied in terms of its application on free vibration problem and various composite beam cross-sections. Results from the numerical tests deduced the demand for optimised parameters value as the parameters are highly reliant on user-defined value. Additionally, the research supports the need for more efficient EFG code’s algorithm, stiffness matrix, shape function formulation and background integration methods, in approximating the higher order differential equation which refers to dynamics analysis.

ABSTRAK

Kelakuan rasuk komposit dalam keadaan interaksi separa telah mencetuskan pelbagai kajian, dan tidak hanya melibatkan perihal mekanik tetapi juga untuk penyelesaian satu matra bagi persamaan pembezaan separanya. Ketidakupayaan yang wujud pada kaedah analitikal sediada di dalam menyelesaikan persamaan pembezaan tertib tinggi telah menuntut kepada penggunaan kaedah berangka, yang dengan itu kaedah *Element Free Galerkin* (EFG) diaplikasi, buat pertama kalinya. Kajian ini terbahagi kepada tiga bahagian, bahagian pertama menyentuh tentang formulasi bentuk lemah Galerkin dan persamaan keseimbangan diskrit EFG. Bentuk lemah satu matra diterbitkan mengikut kaedah perubahan manakala persamaan diskrit di dalam bentuk matrik ditulis menggunakan kod program Matlab. Di bahagian kedua, formulasi EFG dibangunkan untuk pertamanya bagi keadaan gelincir dengan anggapan pesongan adalah sama bagi kedua-dua komponen rasuk dan model kedua melibatkan bersama kesan angkat-naik yang pesongannya adalah berasingan. Formulasi EFG telah memberikan keputusan yang setara apabila dibandingkan dengan penyelesaian analitikal sediada, dengan itu memungkinkan aplikasinya di masalah interaksi separa. Bahagian ketiga memberikan keputusan ujian berangka terhadap beberapa parameter EFG seperti saiz sokong domain, asas polinomial dan titik kamiran yang melibatkan tujuh pemberat fungsi yang berbeza bagi kelakuan rasuk komposit tersebut. Kesimpulannya, pemberat fungsi *Cubic Spline* dan *Quartic Spline* memberikan keputusan ketepatan yang lebih baik berbanding yang lain. Kemampuan formulasi EFG juga diaplikasi terhadap frekuensi tabii untuk masalah getaran bebas dan pelbagai keratan-rentas rasuk komposit. Bersandar kepada keputusan ujian berangka yang dijalankan, didapati nilai parameter yang optimum adalah perlu memandangkan parameter yang diuji amat bergantung kepada nilai cadangan daripada penganalisis. Lanjutan itu, kajian ini menyokong kepada perlunya algoritma kod EFG, matrik kekukuhan, formulasi rangkap bentuk dan kaedah kamiran latarbelakang yang lebih efisen bagi penghampiran persamaan pembezaan tertib yang lebih tinggi, yang merujuk kepada analisis dinamik.

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$\mathbf{A}(\mathrm{x})$	-	weighted moment matrix
$\mathrm{A},_{\mathrm{x}}$	-	first derivative of the weighted moment
$\mathrm{A},_{\mathrm{xx}}$	-	second derivative of the weighted moment
A_b	-	cross-sectional areas (of bottom component)
$a_i(\mathbf{x})$	-	non-constant coefficients
A_t	-	cross-sectional areas (of top component)
$\mathbf{b}$	-	matrix of distributed load
$\mathbf{B}(x)$	-	non-symmetry matrix
C	-	constant value of shear connector
c	-	order of consistency
$\mathbf{c}$	-	stiffness matrix
C_b	-	distance between the contact surface of the lower components from the centroidal axis
C_t	-	distance between the contact surface of the top components from the centroidal axis
d	-	dimension of the problem
d_c	-	nodal spacing
d_s	-	support domain
E_a	-	adhesive’s Modulus
E_b	-	modulus elasticity (of bottom component)
Er_u	-	displacement norm
E_t	-	modulus elasticity (of top component)
F	-	applied forces acting at the centroid of top and lower components
$f(x)$	-	distributed load
$\mathbf{F}$	-	global nodal force vector

$\mathbf{F}_b$	-	global force vector of body force
$\mathbf{f}_\mathrm{I}$	-	nodal force vector of body force
$\mathbf{F}_Q$	-	global force vector of traction force
I_b	-	moment inertia (of bottom component)
i_c	-	connectors spacing
I_t	-	moment inertia (of top component)
J	-	weighted least-square discrete error norm
$\mathbf{K}$	-	global nodal stiffness matrix
k_c	-	slip modulus of connectors
$\mathbf{K}_{IJ}$	-	nodal stiffness matrix
k_s	-	shear connector modulus
k_t	-	foundation modulus
k_y	-	vertical connection stiffness
L	-	Length
$\mathbf{L}$, $\mathbf{H}$	-	matrix of differential operators
m	-	number of terms in basis function
M_b	-	resistant moments acting at the surface (of bottom component)
M_t	-	resistant moments acting at the surface (of top component)
n	-	number of nodes in support domain
$n_{\lambda t}$	-	total number of nodes on the essential boundary
N_I	-	Langrage interpolants
n_r	-	number of rows of connectors
P	-	concentrated load
$p_i(\mathbf{x})$	-	basis functions
$\mathbf{Q}$	-	matrix of concentrated forces
q_c	-	shear flow at interface
q_y	-	vertical force per unit length at interface
r	-	normalised distance
r_u	-	displacement relative discrepancy
sgn	-	*signum function*
T	-	uplift force per unit length

t_a	-	adhesive thickness
$\mathbf{t}_b$	-	nodal force vector of traction force
U	-	strain energy
$\mathbf{u}$	-	column vector of approximate displacement
$\mathbf{U}$	-	global displacement parameter vector
$\bar{\mathbf{u}}$	-	boundary condition matrix
$u(\mathbf{x})$	-	unknown scalar function of a field variable
u_B	-	longitudinal slip of bottom component
$u^h(\mathbf{x})$	-	approximate unknown scalar function of a field variable
u_I	-	nodal parameter
u_T	-	longitudinal slip of top component
v	-	vertical displacement
v_B	-	bottom component deflection
v_{full}	-	vertical displacement due correspond tofull interaction
v_{slip}	-	vertical displacement correspond to slip
v_T	-	top component deflection
W	-	potential load energy
$w(\mathbf{x}\text{-}\mathbf{x}_I)$	-	moving weight function
$\mathbf{x}$	-	quadrature point/ spatial coordinates
y	-	distance of mass centre of components from barycentre
z	-	distance between the centroidal axis of the top and lower components
α_s	-	dimensionless size of support domain
β	-	shape control parameter
Δ_s	-	interface slip
Δ_u	-	vertical uplift
δ	-	variational term
δu	-	test function
ε	-	Strain
λ	-	Langrange multiplier

σ	-	Stress
σ_{normal}	-	interfacial normal stress
$\prod$	-	total potential energy
μ	-	coefficient of friction
$\mathbf{\Phi}$	-	shape function vector
$\Phi,_{x}$	-	first derivative of the shape function
$\Phi,_{xx}$	-	second derivative of the shape function
ω	-	Frequency
Ω	-	problem domain

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Composite members consisting of reinforced concrete and steel sections as well as retrofitted members are widely used in modern building constructions as well as strengthening and rehabilitation purposes. Since this type of member provides the beneficial effect of higher bending strength due to its composite behaviour, the interaction between two materials in the composite conditions should be assumed as partially interacting, if actual behaviour is required to be considered and analysed. This behaviour is known as partial interaction at the interface surfaces. Typical behaviour of partial interaction composite beam is as shown in Figure 1.1.

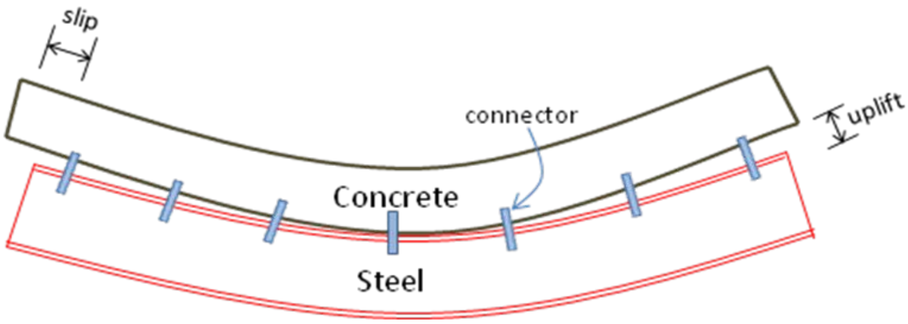

Figure 1.1 Partial interaction in composite beams

Composite member behaviour is mathematically represented by partial differential equation and can result in higher order when longitudinal slip, uplift and dynamic effects on the beam are considered. Generally, an analytical solution for such effect is difficult to obtain due to the high order. Therefore, numerical techniques such as the Finite Element method (FEM) are widely used, in solving this type of difficulty.

The Finite Element method (FEM) has been used as one of the methods in solving differential equations, numerically, for almost five decades circa the 1960's. This approximate method is well established and among the most popular choices of analysis tools, either by engineers or scientists. Due to its vast and deep researches, the FEM is well accepted throughout the engineering committees, due to its flexibility in analysing complex geometry and capability to simulate nonlinear behaviour. Nevertheless the FEM still suffers from two shortcomings; discontinuity of meshed elements and computing cost of re-meshing.

In recent years, several developments in providing solutions for those shortcomings have unfolded, hence the initiation of an alternative to the FEM have been initiated. New methods have been introduced with promising procedures, where the approximate solution is constructed solely by a set of nodes instead of element. These new types of methods are usually known as the Meshfree or Meshless methods. In this study the 'Meshless' term is used, as it is favourably used in most academic papers as compared to the former term.

1.2 Research Background and Problem Statement

The meshless method obtained its name from its mathematical algorithm ability to discretise the problem domain by simply adding or deleting nodes where desired. No element mesh is needed to connect those nodes during the discretisation process which is contrary to the FEM's procedure. This is possible due to formulation of the shape functions that is based on nodes in local support domains thus eliminating the needs for pre-defined elements.

The major difference between the meshless method and the FEM is that the problem domain is discretised only by nodes and the ability to use higher order continuous shape functions. Liu and Gu (2005) has provided a good overview on the procedure and differences between both methods as shown in Figure 1.2 and tabulated in Table 1.1, respectively. Those advantages and the infant status of the meshless method are factors that motivate this study, as they provide the first insights for the application of the present method onto partial interaction of composite beams.

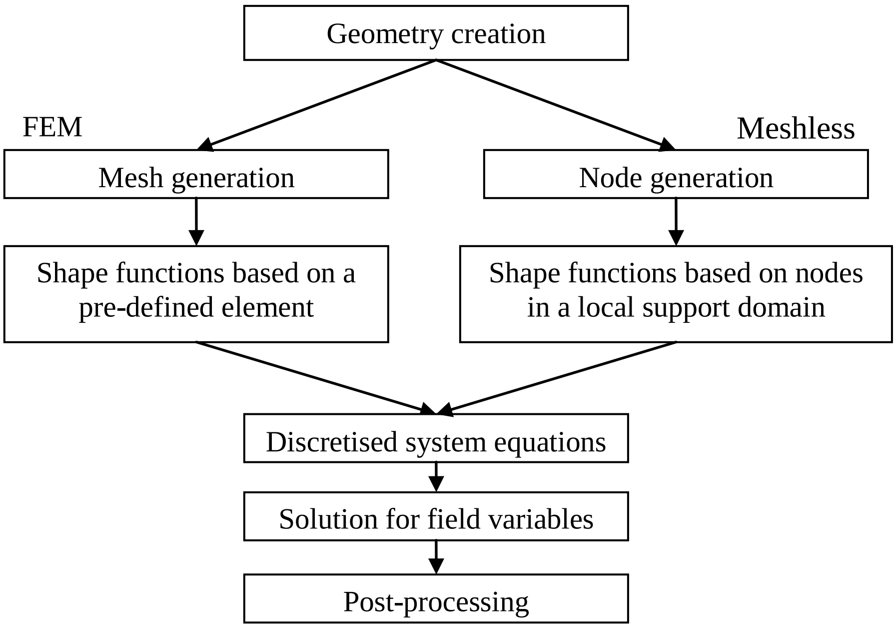

Figure 1.2 Flowchart for the FEM and the meshless methods (Liu and Gu, 2005).

Table 1.1: Differences between FEM and meshless method (Liu and Gu, 2005)

Items	FEM	Meshless method
Mesh	Yes	No
Shape function creation	Based on pre-defined elements	Based on local support domains
Discretised system stiffness matrix	Banded, symmetric	Banded, may or may not be symmetric depending on the method used
Imposition of essential boundary conditions	Easy and standard	Special treatments may be required, depending on the method used
Computation speed	Fast	Slower compared to the FEM depending on the method used
Accuracy	Accurate compared to FDM	More accurate than FEM
Adaptive analysis	Difficulty for 3D cases	Easier
Stage of development	Well developed	Infant, with many challenging problems
Commercial software packages availability	Many	Few

In regards to the partial interaction analysis of composite beam, as an alternative method to the Finite Element Method, the meshless method, with abovementioned advantages can give a new perspective on the research subject. Proposing a new element formulation for FEM in partial interaction problem is literally cumbersome, due to problems of shear locking, element remeshing and computing cost. Thus, new FEM's element with partial interaction feature will not be possible in future development of FEM software. Regards to that, meshless method formulation which apparently formulated to overcome the last two shortcomings, has also open-up an opportunity for the first shortcoming solution, unintentionally. However, before the feature can be realised in meshless software, a one-dimensional formulation of the partial interaction behaviour has to be formulated. This will be the important step in meshless formulation of partial interaction behaviour, before it can be extended to higher dimensional problems.

Element Free Galerkin (EFG) is one of the meshless methods that have been used widely among researchers to model solid mechanic problems. Its formulation is closer to FEM compares to others meshless method and easily to be extended to others meshless formulation, such as; meshless local Petrov-Galerkin (MLPG), Reproducing kernel particle method (RKPM) and hp-clouds. The Element EFG method was developed by Belytschko *et al.* (1994). An extensive review (as in

Chapter 2) by the study, found that, there are no attempts yet to be found on the application of EFG method on composite beam with slip and uplift effects. It is therefore the main interest of this study to establish the EFG formulation for such an engineering problem.

1.3 Purpose and Objectives of the Study

The purpose of this study is to establish new formulation, namely the Element Free Galerkin (EFG) method, for the analysis of the partial interaction behaviour of composite beams. The objectives of this study are as follows:-

a) To derive one-dimensional formulation and algorithm of EFG method for composite beams with longitudinal slip effect, and validate the result with established analytical solutions.

b) To conduct numerical tests on various numerical parameters of the developed EFG formulation in assessing the accuracy of its result.

c) To extend derivation for an additional effect; the vertical uplift at interfacial faces and subsequently assess the effect of the numerical parameters on its convergence result.

d) To verify the viability of the EFG formulation by conducting analysis on various typical composite beam cross-sections with various boundary conditions.

e) To derive and evaluate the capability of developed EFG formulation in finding natural frequency of free vibration problem and conduct its numerical parameters study.

1.4 Scope and Limitation of the Study

Formulation of Element Free Galerkin method becomes the main purpose of this study. The formulation is applied on partial interaction analysis of composite beams that follows Euler-Bernoulli's assumptions. Verification of the formulation is conducted by comparing its result with available analytical solution. The study is limited to static analysis. Even so initial effort is taken to formulate the method on free vibration behaviour.

1.5 Outline of Thesis

This present chapter gives a brief introduction on the study which consists of meshless development and its differences with a well known numerical method, the finite element. The Element Free Galerkin (EFG) method is highlighted for this research work and objectives of the study are presented at the end of the chapter. The flow of the thesis contents is depicted as in Figure 1.3, for ease review.

In Chapter 2, an extensive review on the development of partial interaction study in composite beam behavior is reviewed. The review starts with an early development of longitudinal slip effect derivation of composite beam in early 50's. Till then, the study area has been extended due to the various application of composite beam technology in construction industry. This development includes the derivation of analytical solution and numerical solution, particularly finite element method. The second part of the review touches on the development of EFG method in various attempts in solving mechanics problems.

The shape function of numerical solution is the main perimeter that gives FEM and EFG methods their own numerical characteristic. Therefore, in Chapter 3, the fundamental of EFG shape function known as Moving Least Square (MLS) method is derived in step-by-step manner. The effects of MLS numerical parameters are reviewed based on previous researches and general formulation of the method is reviewed from the weak form perspective. The numerical EFG solution for Euler-Bernoulli beam problem is presented for introductory reason.

Chapter 4 provides the new developed EFG formulation with its essential formulation procedure for the application of composite beams with longitudinal slip at interface surfaces. The code is developed using Matlab programming language and the results were verified against established analytical solutions.

Selected numerical parameter values are studied in Chapter 5 through various numerical tests. Several weight functions are used in each numerical test and their effects on the result are discussed. The convergence rates are plotted for comparison purposes.

The developed EFG formulation is extended to consider the vertical uplift effects and it is the main discussion in Chapter 6. Results are compared with analytical solution in terms of deflection, for validation purposes. EFG formulation is presented and similar numerical parameters from preceding chapter are reconsidered in present numerical tests. Suggestions on suitable numerical parameters for the EFG formulation are made available.

In Chapter 7, the capabilities of developed EFG formulation code are studied and discussed. Four different cross-sections of composite beam are used in this study and three types of boundary conditions for each cross-section have been chosen to verify the code application. Further application on these various section and boundary condition is extended to partial interaction EFG formulation, where previously, longitudinal slip EFG formulation was involved.

Chapter 8 comprises the formulation, of the linear dynamic algebraic equation and numerical study for free vibration problem. This study involves the finding of the natural frequency of the partial interaction composite beam.

Chapter 9 summarises the development of EFG formulation and its capability in terms of applications. Effects of numerical parameters on the formulation accuracy are concluded and several recommendations for future works are suggested.

Chapter 1 Problem Statement and Objectives of the Study	**Chapter 2** Review on Partial Interaction Study, Analytical Solution and EFG	**Chapter 3** Basic Formulation of EFG's Shape Functions and EFG of Euler-Bernoulli's Beam	**Chapter 4** EFG Formulation of Composite Beam with Longitudinal Slip effect and its Algorithm
Chapter 8 EFG Formulation of Free Vibration of Partial Interaction Composite Beams	**Chapter 7** Application of EFG for Common Composite Cross-sections and Boundary Conditions	**Chapter 6** EFG Formulation of Composite Beam with Partial Interaction effect and its Numerical Tests	**Chapter 5** Numerical Test of the Longitudinal Slip Algorithm
	Chapter 9 Summary, Conclusion and Suggestion for Future Works		

Figure 1.3 Flow of the thesis contents.

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