ENHANCEMENT OF PARALLEL THINNING ALGORITHM FOR HANDWRITTEN CHARACTERS USING NEURAL NETWORK

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Especially for Mak and Aba,
Thanks alot for your love, support, sacrifices and blessings…

For Yak,
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For Didek, Bibi and Endumit,
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God bless you all!

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ABSTRACT

Thinning is a well known pre-processing step in many image analysis techniques and it has been applied in a wide variety of applications in the fields of pattern recognition and machine vision. Thinning can be applied onto various images to produce one-pixel width skeletons that represent a good abstraction of the image shape. This project aims to improve a parallel thinning algorithm that satisfies two fundamental requirements of thinning, namely the processing speed and the quality of skeletons. This work is an attempt to devise an effective thinning method by applying a neural network model, used for handwritten characters. Parker's parallel thinning algorithm is chosen in this project because it satisfies the requirements of producing good quality skeletons although it does not provide an efficient processing speed. Thus, this project presents a framework for the implementation of a multilayer perceptron neural network with backpropagation algorithm, in the Parker thinning algorithm in order to produce a fast fully parallel thinning algorithm. The proposed thinning algorithm is tested on a set of binary images of isolated handwritten characters, obtained from CEDAR and MNIST databases. Comparison is conducted on the proposed thinning algorithm, Zhang-Suen thinning algorithm and Parker thinning algorithm. The analysis of these three thinning algorithms is based on two parameters, namely topological analysis and implementation speed analysis. The experimental results show that the proposed thinning algorithm produced acceptable, one-pixel width skeletons that preserve connectivity. It also works faster than Parker thinning algorithm but it cannot deals with the necking problem. The skeletons produced also retain the general abstraction of the global shape of the handwritten characters images.

ABSTRAK

Teknik penipisan merupakan satu operasi pra-pemprosesan yang penting dalam kebanyakan teknik analisis bagi imej, dan ia telah digunakan dengan meluas dalam bidang pengecaman corak dan penglihatan mesin. Penipisan boleh digunakan bagi pelbagai jenis imej untuk menghasilkan rangka imej bersaiz satu piksel, yang dapat memberikan perwakilan bentuk imej yang baik. Projek ini bertujuan untuk memperbaiki satu algoritma penipisan selari yang dapat memenuhi keperluan utama proses penipisan iaitu kepantasan pemprosesan dan kualiti rangka imej yang terhasil. Kajian ini juga merupakan usaha untuk menghasilkan satu algoritma penipisan yang efektif, yang mengaplikasikan satu model rangkaian neural untuk aksara tulisan tangan. Algoritma penipisan Parker dipilih kerana ia memenuhi keperluan untuk menghasilkan kualiti rangka imej yang baik. Namun, ia tidak dapat menghasilkan kepantasan pemprosesan yang efisyen. Maka, melalui projek ini, satu rangka kerja diperkenalkan bagi implementasi rangkaian neural perseptron multi aras menggunakan pembelajaran terkawal rambatan balik dalam memperbaiki algoritma penipisan Parker, untuk menghasilkan satu algoritma penipisan selari yang pantas. Algoritma cadangan diuji terhadap set imej binari bagi aksara tulisan tangan, yang diperolehi dari pangkalan data CEDAR dan MNIST. Perbandingan dilakukan terhadap algoritma cadangan, algoritma penipisan Zhan-Suen dan Parker. Analisis dijalankan terhadap ketiga-tiga algoritma tersebut berdasarkan analisis topologi dan analisis kepantasan implementasi. Keputusan ujikaji menunjukkan algoritma cadangan menghasilkan rangka imej bersaiz satu pixel yang mengekalkan sifat keterkaitan. Ia juga beroperasi lebih pantas dari algoritma penipisan Parker tetapi tidak dapat menyelesaikan masalah *'necking'*. Rangka imej yang terhasil juga memberikan perwakilan umum bentuk yang baik bagi imej aksara tulisan tangan.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Pattern recognition is a set of mathematical, statistical and heuristic techniques used in executing `man-like' tasks on computers. Pattern recognition plays an important role in many applications such as document processing, robot vision, recognition of paintings, character recognition and other fields. Automation of pattern recognition helps to speed up processing time as well as to automate processes without human intervention.

Character recognition systems are a subset of pattern recognition. Characters can be in the handwritten or printed form. Handwriting recognition is defined as the task of transforming a language represented in its spatial form of graphical marks into its symbolic representation (Plamondon and Srihari, 2000). Handwritten character recognition has wide applications in office automation, cheque verification, and a large variety of banking, business and data entry applications.

Nowadays, handwritten character recognition still posed a great challenge to researchers because of the feature of the handwritten characters themselves. Some of the difficult issues arises in the handwritten character recognition are the wide variety of writing style, handwritten characters shapes and the noise existed in the characters.

There are two types of handwriting recognition system, online recognition and off-line recognition. In online character recognition, the computer recognizes the symbols as they are drawn. The most common applications of online recognition are digitising tablet and PDAs. Off-line recognition is performed after writing is completed. It is mostly used in specialized domains such as interpreting handwritten postal address and reading bank checks.

Basically, a handwritten character recognition system consists of three major steps, pre-processing, feature extraction and pattern classification. Generally, there are two kinds of handwritten characters, connected and isolated. The preliminary step to recognize handwritten characters is the pre-processing, which involves operations on the digitized image intended to reduce noise and increase the ease of extracting structural features. Some of the common pre-processing operations performed include thresholding, noise filtering, thinning and segmentation.

This project gives emphasize only on the pre-processing step of the handwritten character recognition. The pre-processing step chosen is the thinning process, as it is a fundamental pre-processing operation in image analysis. It is defined as the process of reducing the width of a line-like object from several pixels to a single pixel. The resultant image is called the skeleton. Skeletons have been a useful aid in handwritten character recognition because most of the relevant information in characters is not related to the thickness of the character images. Using a thinned character images helps to simplify the classification and recognition process and also producing more accurate recognition. Therefore, the performance of the recognition system depends on the thinning algorithm used in the pre-processing step. Misclassifications also may exist in the recognition system caused by the thinning process, if it produces distortions in the skeletons (Lam and Suen, 1993). Therefore, a good algorithm for thinning is needed in order to reduce the errors in classification. This has become the motivation to do a research on finding a better thinning algorithm from the handwritten character recognition perspective.

1.2 Problem Background

Skeletonization is a way to represent binary shapes with a limited amount of information. Skeletons are usually obtained through an iterative reduction operator called thinning, whereby in this process, certain types of border points are iteratively removed until no more points can be deleted. The iterative thinning algorithms can be classified as either parallel or sequential.

Generally, an effective thinning algorithm should ideally meet the following requirements: sensitivity to noise, preservation of topological and geometric properties, isotropy, reconstructability and efficiency. Over the years, many thinning algorithms have been proposed, and a comprehensive survey of these methods is contained in (Lam *et al.*, 1992). Some have obtained good results, but there are still some deficiencies. In the thinning algorithm research, there exist two main problems where the actual study focuses, namely, the algorithm execution time and the resulting thinned image shape. Many of the more recent thinning algorithms were designed to improve the speed of the algorithm. Some only focused on the quality of the skeleton produced. There is no single thinning algorithm that does all of the above criteria, many of the selection of a thinning algorithm usually involves trade-offs between one, or more of the above criteria (Parker, 1997).

Most of the research in thinning algorithms focused on the implementation of a variation on an existing thinning method, where the novel aspects are related to the performance of the algorithm. The thinning algorithms produced may also generate good skeletons for some shapes but can produce poor skeletons for others. It is very difficult to develop a generalized thinning algorithm which can produce satisfactory results for all varieties of pattern shapes (Saeed *et al.*, 2001). Although, there are many new thinning algorithms produced, they are used in other applications such as fingerprint, medical images, object images, geometrical images. Other thinning algorithms focused on the hardware implementation architecture in order to speed up the thinning process. However, the hardware implementation is not very practical as it can be quite costly.

This project attempts to improve a parallel thinning algorithm that can satisfy the two fundamental requirements of thinning, namely the speed of the thinning process and the quality of thinned results. Two existing thinning algorithms chosen for this project are Zhang-Suen thinning algorithm (Zhang and Suen, 1984) and Parker thinning algorithm (Parker, 1997).

Zhang-Suen algorithm is used as the benchmark in this project as it has been used as a basis of comparison for thinning methods for many years, and it is fast and easy to implement. Unfortunately, the algorithm has some disadvantages, such as, it enlarged the noise that should be eliminated, structures that should be preserved are destroyed and the loss of some digital patterns.

Parker had overcome the problems above by merging three methods in order: Stentiford's pre-processing scheme (Stentiford and Mortimer, 1983), feeding images into Zhang-Suen's basic algorithm (Zhang and Suen, 1984), with Holt's staircase removal (Holt *et al.*, 1987) as a post-processor. Parker claimed that this new combined algorithm succeeded in producing a better quality skeletons but it does not provide an efficient processing time. Based on the review done on previous researches in thinning algorithm, there is no research done on addressing the issue of implementation speed in Parker thinning algorithm.

Consequently, in this project, we want to investigate the use of neural network for improving the combined algorithm produced by Parker (1997). In the last few years, there has been a great interest in applying neural networks technology in various fields of conventional computing. This is due to the facts that neural network provides parallelism, adaptive and it sometime simplifies problem. This motivates the implementation of neural network in order to produce a fast parallel thinning algorithm and producing high-quality skeletons.

There exists a few neural-network-based thinning algorithms that have been developed, such as Krishnapuram and Chen (1993), Lee and Park (1995), Shenshu *et al.* (2000) and Datta *et al.* (2002). These thinning algorithms proved to be much

faster but they are still slacking in producing a high quality of skeletons. Furthermore, these algorithms are applied to various image patterns. Only Ahmed (1995), Altuwaijri and Bayoumi (1998), Sasamura and Saito (2003), and Gu *et al.* (2004) applied the neural-network-based thinning algorithms in handwritten numerals, Arabic characters, Japanese characters and, English and Chinese words. Consequently, there are still much improvement can be made in the techniques of applying neural network in thinning algorithm for handwritten characters.

This project examines the application of the neural-network based thinning algorithm for handwritten character recognition. The binary images of handwritten characters consist of letters and digits are used. An analysis and comparison study is conducted in order to evaluate the efficiency and effectiveness of the thinning algorithm. Two main analyses are done, namely, topological analysis and implementation speed analysis.

1.3 Problem Statements

This project concentrates on the implementation of neural network in improving the Parker thinning algorithm. The results of the improved algorithm are then compared with the original Zhang-Suen thinning algorithm and the Parker thinning algorithm. Therefore, this project explores the questions of:

"Whether neural network can be successfully implemented in the Parker thinning algorithm?"

"Whether the proposed thinning algorithm can exhibit better or similar results with the original algorithm in term of topological analysis and implementation speed analysis?"

1.4 Objectives

The following are the objectives for this project:

(i) To develop a framework on applying neural network in thinning algorithm.

(ii) To develop a fast fully parallel thinning algorithm by applying neural network in Parker thinning algorithm.

(iii) To evaluate the performance of the proposed thinning algorithm in comparison with Zhang-Suen thinning algorithm and Parker thinning algorithm.

1.5 Scopes

The scopes of this project are as follows:

(i) This project only focuses on the thinning pre-processing step of the handwritten character recognition system

(ii) The Parker thinning algorithm is improved using neural network in this project. A multilayer perceptron neural network with standard back-propagation algorithm is used.

(iii) The dataset used in this project consists of isolated, noiseless handwritten characters, consists of letters and digits. All the handwritten characters are in the form of binary images. Two databases used in this project are CEDAR's CDROM database and MNIST database.

(iv) Two evaluation parameters that are going to be used in the comparison analysis are processing time and skeleton topology.

1.6 Significance of the Project

The Zhang-Suen thinning algorithm is a well-known thinning method that has been widely used for years, as it is fast and simple to implement. But it has few disadvantages regarding the quality of the skeletons produced. Parker (1997) addressed this problem by producing a hybrid thinning algorithm. But, it is known that a good thinning algorithm must be fast and able to produce quality thinned results. Therefore, this project analyzes the use of neural network in improving Parker thinning algorithm and evaluate whether it exhibits a similar or better results in terms of processing time and topology quality, when being compared with the previous research result. The result of this project may become a new variation of robust parallel thinning algorithm that preserves the fundamental properties. Hopefully, this project will be a motivation for future studies in producing better thinning algorithm for handwritten characters as there is yet to be a thinning algorithm that does not produce any artefacts in the skeletons.

1.7 Organization of the Report

This report consists of six chapters. Chapter 1 presents the introduction of the project and the problem background, which explains the motivation of the project. Chapter 1 also describes the objectives and scope of the study. Chapter 2 gives overview on the handwritten character recognition system, thinning methodologies, neural network and its application in thinning algorithm. Chapter 3 discusses on the project methodology and the proposed thinning algorithm development methodology used in the project. Chapter 4 discusses in details the experimental design used in developing the proposed thinning algorithm using neural network. The experiment results and analyses are outlined in chapter 5. Lastly, chapter 6 discusses in general the conclusion and future works of the project.

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