

MATHEMATICAL THINKING IN MULTIVARIABLE CALCULUS
THROUGH BLENDED LEARNING

HAMIDREZA KASHEFI

UNIVERSITI TEKNOLOGI MALAYSIA

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HAMIDREZA KASHEFI

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DEDICATION

To
My beloved Wife Fariba Mirzaei
And
My Children Melika and Vania

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ABSTRACT

Blended learning is proposed as a sufficient environment in supporting students' mathematical learning based on mathematical thinking. This research explores a blended learning model for teaching and learning of multivariable calculus in fostering students' mathematical thinking based on the data gained through the preliminary investigation, the existing model, and integrating creative problem solving. The main purpose of this research is to identify the effectiveness of the blended learning in developing and supporting students' thinking powers in the construction of mathematical knowledge, problem solving, and in reducing their difficulties in multivariable calculus course. The theoretical foundation of the three worlds of symbolic, embodied, and formal mathematics was adopted to develop strategies for mathematical knowledge construction and to enhance students' mathematical thinking. Sixty two first year engineering students participated in this study. Data were collected via think-aloud verbal protocols, students' written solutions to assessments, semi-structured interview, students' web comments, and semi-structured questionnaire. The findings revealed that the students' mathematical knowledge construction and problem solving had improved. They could overcome their difficulties in the learning of multivariable calculus. The data collected also showed that students used mathematical thinking activities and multiple representations of mathematics worlds, especially the symbolic and the embodied worlds, when solving the problems.

ABSTRAK

Pembelajaran sehati dicadangkan sebagai persekitaran yang mencukupi dalam menyokong pembelajaran matematik pelajar berdasarkan pemikiran matematik. Kajian ini meneroka model pembelajaran sehati dalam pengajaran dan pembelajaran kalkulus berbilang pembolehubah kearah memantapkan pemikiran matematik. Model ini direkabentuk berasaskan dapatan daripada kajian awal, model pembelajaran pemikiran matematik yang sedia ada serta penyelesaian masalah kreatif. Tujuan utama kajian ini adalah untuk mengenal pasti keberkesanan pembelajaran sehati membentuk dan menyokong daya pemikiran dalam pembinaan pengetahuan matematik, penyelesaian masalah dan mengurangkan kesukaran dalam kursus kalkulus berbilang pembolehubah. Tiga dunia matematik iaitu simbolik, termaktub, dan matematik formal yang diutarakan diaplikasikan untuk membangun strategi pembinaan pengetahuan matematik serta memantapkan pemikiran matematik. Enam puluh dua pelajar kejuruteraan tahun satu merupakan sampel kajian ini. Data diperolehi melalui kaedah protokol mengutarakan fikiran, hasil penyelesaian bertulis pelajar dalam penilaian, penilaian struktur bersepara serta komen atas talian dan soal selidik struktur bersepara. Dapatan kajian menunjukkan pembinaan pengetahuan matematik pelajar dan penyelesaian masalah bertambah baik. Pelajar didapati berkemampuan mengatasi kesukaran dalam pembelajaran kalkulus berbilang pembolehubah. Data yang dikumpul juga menunjukkan bahawa pelajar menggunakan aktiviti pemikiran matematik dan pelbagai perwakilan dunia matematik, terutama dunia simbolik dan termaktub, apabila menyelesaikan masalah.

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LIST OF ABBREVIATIONES

| | | |
|--------|---|--|
| APOS | - | Action-Process-Object-Schema |
| CMS | - | Course Management System |
| CPS | - | Creative Problem Solving |
| F2F | - | Face-to-Face |
| HTML | - | Hyper Text Markup Language |
| IAUKSH | - | Islamic Azad University of Kermanshah |
| Moodle | - | Modular Object-Oriented Dynamic Learning Environment |
| PASW | - | Predictive Analytics SoftWare |
| PDA | - | Personal Digital Assistant |
| PDF | - | Portable Document Format |
| RQ | - | Research Question |
| UTM | - | Universiti Teknologi Malaysia |

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Calculus is one of the most important courses for engineering students that is offered as a prerequisite course to other advanced mathematics or even engineering courses. The importance of calculus learning for engineering students is to provide them with ways of working with several mathematical ideas and various representations and also use this knowledge in their engineering fields (Roselainy, Yudariah, and Sabariah, 2007). The lack of understanding of concepts in calculus may hinder the understanding of other concepts or even subjects. In other words, mathematics, in particular calculus, enables engineering students in learning to apply a wide range of mathematical techniques and skills in their engineering classes and later in their professional work (Croft and Ward, 2001). However, for most undergraduate students, specifically engineering students, calculus is one of the most difficult courses in their fields of study (Schwarzenberger, 1980; Morgan, 1988; Cornu, 1991; Eisenberg, 1991; Artigue and Ervynck, 1993; Tall, 1993a; Yudariah and Roselainy, 2001; Willcox and Bounova, 2004; Kashefi, Zaleha, and Yudariah, 2010d, 2011a).

Various problem learning areas have been identified in basic and multivariable calculus. Some of these were, the difficulty of learning some specific mathematical topics, the difficulty in algebraic manipulation, assimilating complex new ideas in a limited time, recalling of factual knowledge, students' beliefs and

learning styles, poor problem solving skills and the inability to select and use appropriate mathematical representations (see Tall and Schwarzenberger, 1978; Smith, 1979; Orton, 1983a, b; Morgan, 1988; Artigue and Ervynck, 1993; Tall, 1993a; Tall and Razali, 1993; Norman and Pritchard, 1994; Yudariah and Tall, 1999; Hirst, 2002; Yudariah and Roselainy, 2004; Sabariah, Yudariah, and Roselainy, 2008; Roselainy, 2009; Kashefi, Zaleha, and Yudariah, 2010d, 2011a).

Studies on students' learning have found various methods that can support students in the learning of calculus. Promoting mathematical thinking with or without computer is one of the most important methods to support students in the learning of calculus. There is quite an extensive study on promoting mathematical thinking in calculus such as works by Dubinsky (1991), Schoenfeld (1992), Watson and Mason (1998), Yudariah and Tall (1999), Gray and Tall (2001), Mason (2002), Tall (1986, 1995, 2004), and Roselainy (2009).

Mathematical thinking is a dynamic process which expands students' understanding with highly complex activities, such as abstracting, specializing, conjecturing, generalizing, reasoning, convincing, deducting, and inducting (Mason, Burton, and Stacey, 1982; Tall, 1991; Yudariah and Roselainy, 2004). Authors like Tall and Dubinsky and their colleagues, endeavored to support students' mathematical knowledge construction and mathematical thinking in calculus especially basic calculus through the use of computers. In a study of multivariable calculus, Roselainy and her colleagues (Roselainy, 2009; Roselainy, Yudariah, and Mason, 2007; Roselainy, Yudariah, and Sabariah, 2007) presented a model of active learning in face-to-face (F2F) multivariable calculus classroom. The model was based on invoking students' mathematical thinking powers, supporting mathematical knowledge construction, and promoting generic skills such as communication, teamwork, and self-directed learning.

Generic skills such as problems solving, communication, and teamwork skills play important roles in supporting students to think mathematically. Creative Problem Solving (CPS) as a problem solving framework which invokes students' generic skills such as communication and teamwork can support students in the

learning of engineering, science, and even mathematics courses through computer tools (Lumsdaine and Voitle, 1993a; Lumsdaine and Lumsdaine, 1995b; León de la Barra et al., 1997; Pepkin, 2004; Cardellini, 2006; Wood , 2006; Gustafson, 2006; Kandemir and Gür, 2009; Williamson, 2011). CPS is a multi steps method for solving problems in various disciplines that not only use analytical, creative, and critical thinking in the most appropriate sequence but can also use the capabilities of computers (Lumsdaine and Lumsdaine, 1995b; Forster, 2008; Chen and Cheng, 2009; Robertson and Radcliffe, 2006, 2009; Maiden et al., 2010).

The literature as well as the preliminary investigation (see Chapter 3) showed that many students struggle as they encounter new mathematical ideas and objects in multivariable calculus course. There are very few researches that investigate on how to develop and support students' thinking powers in the construction of mathematical knowledge in multivariable calculus by using computers. Moreover, not many studies were done that involve effective communication, teamwork, and problem solving in mathematics courses specifically multivariable calculus by CPS and computer tools. Thus, in this study, we shall explore and propose a model of teaching and learning of multivariable calculus that enhance students' thinking powers by using computer which also involves generic skills via CPS steps based on mathematical thinking activities. Blended learning that integrates the benefits of both F2F and computer-based environment is proposed as a sufficient environment in supporting students' mathematical learning based on mathematical thinking.

1.2 Background of the Problem

The literature reviewed on students' difficulties and the various efforts to improve the situations which showed a general trend, moving away from remedial classes towards teaching to increase understanding (Roselainy, 2009). Improving students' learning through the enhancement of their problem solving and mathematical thinking skills as well as through using technological tools to support conceptual understanding and problem solving methods are now thought to be more

appropriate to enable them to cope with the mathematics needed for their engineering problems. In order to characterize engineering students' use of mathematics, it is crucial to recognize not only the mathematical content knowledge that engineering students apply in engineering contexts but also other generic skills that engineering students learn from mathematics courses (Cardella, 2006).

CPS is a framework designed to assist problem solver in invoking communication and teamwork skills and using computer tools to support students' thinking powers and overcome obstacles. The roots of CPS are found in Alex Osborn's classic book, *Applied Imagination*, (1953). The CPS are refined and extended by many researchers over the past six decades (Parnes, 1967; Isaksen and Treffinger, 1985; Isaksen and Dorval, 1993; Isaksen, Treffinger, and Dorval, 1994). Through CPS, some researches (Lumsdaine and Voitle, 1993a; León de la Barra et al., 1997; Cardellini, 2006; Gustafson, 2006; Wood , 2006; Williamson, 2011) tried to support students' learning and thinking powers in engineering, science, and even mathematics courses. However, there is very little literature reporting on the use of CPS to help engineering students in the learning of calculus and in using computer tools (Lumsdaine and Lumsdaine, 1995b; Pepkin, 2004; Gustafson, 2006; Wood , 2006; Kandemir and Gür, 2009).

Researchers, by promoting mathematical thinking with computer or without it, try to support students in understanding mathematical concepts and solving real problems in F2F classroom. Encouraging mathematical thinking and supporting students' mathematical knowledge construction can help to reduce their difficulties in calculus. Researchers like Dubinsky and Tall and their collaborators, have been trying to support students' mathematical thinking powers and overcome students' difficulties in basic calculus by using computers.

Dubinsky (1991) used Action–Process–Object–Schema theory, better known as APOS theory, to describe certain mental construction for learning mathematical concepts. In this theory, the Actions are routinized as Processes, encapsulated as Objects and embedded in a Schema of knowledge. In short, the APOS theory is used to describe what it means to understand a concept and how students can make that

construction (Breidenbach et al., 1992; Dubinsky and Yiparaki, 1996). Dubinsky and Yiparaki (1996) noted several specific pedagogical strategies for helping students to make the mathematical knowledge constructions. The main strategies used for this method are ACE (Activities, Class discussion, and Exercises) teaching cycle, cooperative learning groups to engage students in problem solving activities and the use of ISETL (Interactive SET Language) as an interactive mathematical programming language (Dubinsky and Yiparaki, 1996; Asiala et al., 1996).

Gray and Tall (see Gray and Tall, 1994, Gray et al., 1999) had introduced a similar cycle of mental construction as in APOS theory, called “procept” which is the amalgam of three components namely a process which produces a mathematical object, and a symbol which is used to represent either a process or an object. Reflecting on the theoretical development on the construction of mathematical knowledge in elementary and advanced mathematics, Gray and Tall (2001) then proposed three distinct types of mathematics worlds to describe certain mental construction for learning mathematical concepts. They suggested three different ways of constructing mathematical concepts from *perception* of objects (as occurs in geometry), *actions* on objects (as in arithmetic and algebra) and *properties* of objects that lead to formal axiomatic theories.

In a further study, Tall (2004) pointed out that there are not only three distinct types of mathematics worlds; there are in fact three significantly different worlds of mathematical thinking: *conceptual-embodied*, *proceptual-symbolic*, and *axiomatic-formal*. This theory underlies the creation of computer software which Tall called *generic organizer* and used it in his researches (Tall, 1986, 1989, 1993b, 2000, 2003) to support students’ mathematical construction and to build embodied approach to mathematical concepts. However, the generic organiser does not guarantee the understanding of the concept and Tall (1993b, 1997) reported some cognitive obstacles faced by students when using this organiser. Tall believed that learners require an external *organising agent* in the shape of guidance from a teacher, textbook, or some other agency. In this way, Tall suggested that the combination of a human teacher as guide and mentor using a computer environment for teaching, pupil exploration, and discussion can support students’ mathematical knowledge

construction and prevent misleading factors (Tall, 1986). In fact, Tall tried to support all modes of reality building (experience, communication, and creativity) and reality testing (experiment, discussion, and internal consistency) of Skemp's theory to construct mathematical concepts.

In the study of multivariable calculus, Roselainy and her colleagues (Roselainy, 2009; Yudariah and Roselainy, 2004; Roselainy, Yudariah, and Mason, 2005, 2007; Roselainy, Yudariah, and Sabariah, 2007) adopted the theoretical foundation of Tall (1995) and Gray et al. (1999) and used frameworks from Mason, Burton, and Stacey (1982) and Watson and Mason (1998) to develop the mathematical pedagogy for classroom practice. They highlighted some strategies to support students to empower themselves with their own mathematical thinking powers thus help them in constructing new mathematical knowledge and generic skills, particularly, communication, teamwork, and self-directed learning (Yudariah and Roselainy, 2004). In the classroom activities, they used themes and mathematical processes through specially designed prompts and questions to invoke students' use of their own mathematical thinking powers and to further develop these powers according to the complexity of the mathematical concepts (Roselainy, Yudariah, and Mason, 2005). In this way, students' attention was focused and directed to the prompts and questions in the beginning until the students became aware of the type of questions faced (Sabariah, Yudariah, and Roselainy, 2008).

In general, researchers used F2F classroom and technology-based in teaching and learning to enhance and support student' thinking powers and skills. According to White (2001), both of the learning environments above offer some advantages that the other cannot replace. Since both methods have their own strengths, some researchers suggest using the blended learning (Garnham and Kaleta, 2002; Osguthorpe and Graham, 2003) which will provide the optimal "mix" between computer-based, in particular online learning and traditional F2F learning (Black, 2002; Aycock, Garnam, and Kaleta, 2002). The blended learning that incorporates the best characteristics of both the traditional and online classroom settings (Reay, 2001; Black, 2002; Aycock, Garnam, and Kaleta, 2002) can be used to support students' learning in mathematics subjects.

1.3 Statement of the Problem

Both Dubinsky and Tall in their research used special mathematics software and programming language that are difficult to use in the formal calculus class. Moreover, they have not focused much on supporting students' thinking and knowledge construction in multivariable calculus. The Dubinsky method was based on some strategies like computer activities as mathematical programming language, class discussion and team working, and problem solving activities. In this approach, Dubinsky and his colleagues used visual tools after knowing the mathematics and algebraic manipulation. The most challenging aspect in their method was very few engineering students at the freshman or sophomore level that learnt a mathematical programming language can write program and use this skill in their college study. Furthermore, only students with career goals in programming need to study these languages (Lumsdaine and Lumsdaine, 1995b).

In the study of multivariable calculus, Roselainy and her colleagues (Roselainy, 2009; Roselainy, Yudariah, and Mason, 2007; Roselainy, Yudariah, and Sabariah, 2007) focused on supporting engineering students' thinking powers, mathematical knowledge construction and generic skills in multivariable calculus. Using prompts and questions as an important strategy in Roselainy method focuses more on the symbolic world of mathematical thinking. Moreover, in their model of active learning, they did not use computer tools to support students' learning and thinking powers. The findings of the preliminary investigation (see Chapter 3) indicated that although Roselainy et al.'s method helps in making students aware of their mathematical thinking processes, students still have difficulties when they encounter new mathematical ideas and concepts (Kashefi, Zaleha, and Yudariah, 2010d, 2011a).

The methods that were used by Dubinsky, Tall, and Roselainy and their colleagues try to support students' mathematical knowledge construction; however, they do not make use of robust tools to support them. For example, all methods used communication between students and teacher, but these communications were not supported by current tools such as online and offline computer facilities that can be

used as synchronous and asynchronous communication. Moreover, in the case of multivariable calculus, Roselainy and her colleagues did not use any computer facilities such as animations, web-based tools, and visual aids in lecturers to support students' visualization. There is few established literature on using computer tools and generic skills such as communication, teamwork, and problem solving via CPS steps to develop and support students' thinking powers based on mathematical thinking in multivariable calculus. Thus, the most important purpose of this study is to investigate the potential of blended learning as an approach to develop and support students' thinking powers and knowledge construction by using computer tools. It also tries to invoke generic skills via CPS steps, especially in the learning of multivariable calculus.

1.4 Objectives of the Study

This study will design and develop a model that conceptualized a framework for supporting students' mathematical learning in multivariable calculus by using computer tools and generic skills based on mathematical thinking through a blended learning environment. The main goal of this study is to determine the effectiveness of the blended learning multivariable calculus course on the students' knowledge construction and problem solving specifically in reducing students' difficulties.

The three objectives of this research are:

1. To design an alternative blended learning approach in fostering students' mathematical thinking in multivariable calculus course.
2. To construct and implement a multivariable calculus course in the blended learning environment based on the designed model.
3. To identify the effectiveness of the blended learning multivariable calculus course in developing and supporting students' thinking powers in the construction of mathematical knowledge, problem solving and in reducing their difficulties.

1.5 Theoretical Framework

It is widely believed that to make significant progress in an area of research, there is a need for a structured theory of learning (Stewart, 2008). The theory should be used to explain the specific successes and failures of students in learning mathematics. In addition, the theory must be extensible and applicable to other phenomena, different from the ones that it was developed from.

Previously most theories in mathematics education came from general educators and psychologists such as Piaget (1965), Bruner (1966) and Skemp (1979), who only had something to say that was particularly related to mathematics (Tall, 2004d). Today research in mathematics education is based on theoretical perspectives and mathematics education owns many theories which are directly related to aspects of mathematics (Stewart, 2008). This section will firstly attempt to identify the theoretical reasons for selecting blended learning to promote mathematical thinking in calculus based on the both types of theories. Secondly, Tall's theory of the three worlds of mathematical thinking will be highlighted as a theory that has formed this research.

According to the theory of three modes of representation of human knowledge (Bruner, 1966), enactive, iconic and symbolic are three forms of representation in mathematics. Tall (1995) noted that the various forms of symbolic representation are: verbal (language, description), formal (logic, definition), and proceptual (numeric, algebraic etc). In further studies, Tall (2004, 2007) based on Bruner's theory stated that there are not only three distinct types of mathematics worlds; there are actually three significantly different worlds of mathematical thinking namely conceptual-embodied, proceptual-symbolic, and axiomatic-formal (see Figure 1.1). The three distinct types of mathematical thinking: embodied, symbolic, formal are also particularly appropriate in calculus (Tall, 2007).

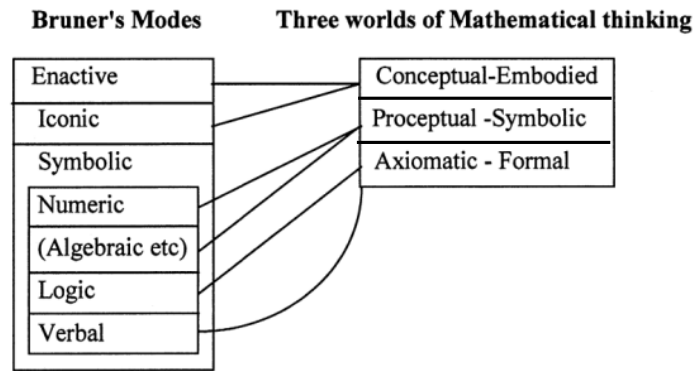


Figure 1.1: The relation between three Bruner's modes
and three worlds of mathematical thinking

Skemp (1979) identifies three modes of building and testing conceptual structures as shown in Table 1.1. Skemp's theory notes that in the process of mathematical knowledge construction, one, two, or three modes of reality building can be used in combination with one, two, or three modes of reality testing (Skemp, 1979).

Table 1.1 Reality construction

| Mode | Reality Building | Reality Testing |
|------|------------------|----------------------|
| 1 | Experience | Experiment |
| 2 | Communication | Discussion |
| 3 | Creativity | Internal Consistency |

According to Skemp, pure mathematics relies on Mode 2 and 3, but it is not at all based only on Mode 1 (Tall, 1986). Tall showed that how computer environment brings a new refinement to the theory of Skemp and extended Skemp's theory to four modes: Inanimate, Cybernetic, Interpersonal, and Personal. The last of these corresponds to Skemp's Mode 3. The interpersonal mode of building and testing concept also corresponds to Skemp's Mode 2, whilst the first two are a

modification of Skemp's Mode 1 (Tall, 1989, 1993b). In fact, the computer provide an environment and that give a new way for building and testing mathematical concept by supporting all modes. Therefore, computer environment can be used in all these modes and learners also may build mathematical concepts by considering examples (and non-examples) of process in interaction with this environment especially in the embodied world of mathematics (Tall, 1986).

In other words, computer environment provides not only a numeric computation and graphical representation; it also allows manipulation of objects by an enactive interface (Tall, 1986) that by using them we can support students' knowledge construction and help them to overcome their difficulties in the embodied world of mathematics. Tall (1989) by combination of a human teacher as guide (organizing agent) and using a computer environment (generic organiser) for teaching tried to support students' mathematical knowledge construction. In Tall's method, teachers as organizing agent do not have a directive role and they only answer questions which may arise in the course of the student investigations through a Socratic dialogue with them (Skemp's Mode 2) which is enhanced by the presence of computers (Tall, 1986, 2004). See Figure 1.2.

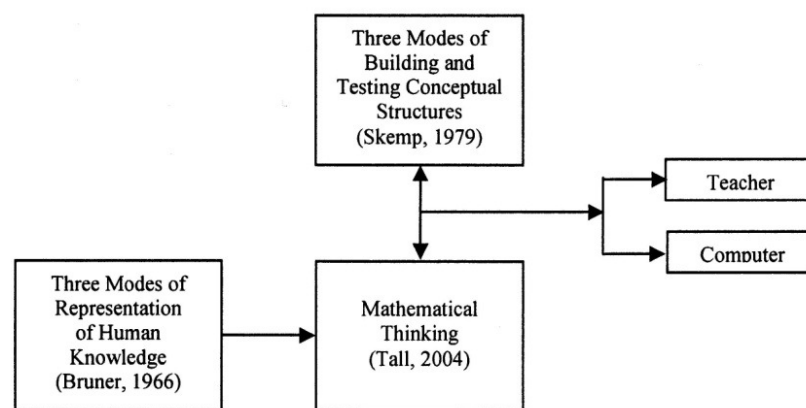


Figure 1.2: The relation between the theories of Bruner, Tall, and Skemp

According to Chew, Jones, and Turner (2008), blended learning researchers today seem to have an emphasis toward practices without a clear understanding of or

underpinned educational theories. Hence, the need to explore educational theory and its relationship with technology is essential. There is growing agreement that there is not, and probably never will be, one great unified General Theory of Adult Learning that will solve all our problems (Carman, 2005). Rather, blended learning offerings should be based on an appropriate blend of learning theories, such as those put forward by Piaget (1969), Bloom (1956), Vygosky (1978), Gagné (1985), Keller (1987), Merrill (1994), Clark (2002), and Gery (1991) . See Figure 1.3.

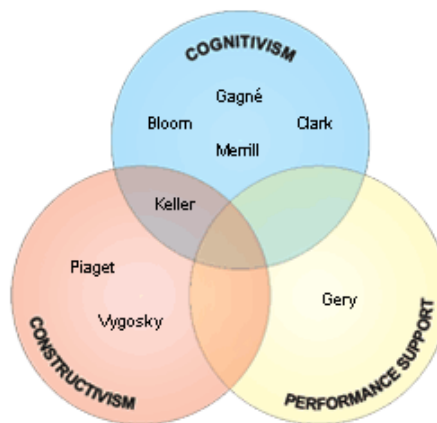


Figure 1.3: A blend of learning theories

There are many definitions of blended learning in the literature review; however, the term is still vague (Oliver and Trigwell, 2005; Graham, 2006; Hisham Dzakiria et al., 2006). The three common definitions of blended learning are: the integrated combination of instructional delivery media, the combination of various pedagogical approaches, and the combination of F2F and online instruction (Oliver and Trigwell, 2005; Graham, 2006; Huang, Ma, and Zhang, 2008). In this study, the blended learning is defined as the integration of traditional learning activities with some technological aids which is familiar with the third one (Reey, 2001). The definition of blended learning as the combination of F2F formats and web-based formats identified an environment that includes two important components of Tall's method: generic organizer (computer) and organizing agent (teacher) (Figure 1.4) (Kashefi, Zaleha, and Yudariah, 2010a). In fact, blended learning by supporting all Skemp's modes can support students' mathematical knowledge construction.

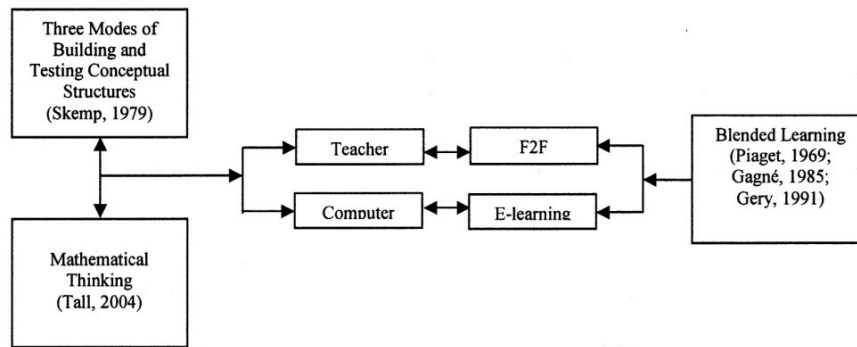


Figure 1.4: The relation between the theories of Tall, Skemp and blended learning

Current learning theories support the notion that learning occurs through an individual's interaction with others in the context of a real world event. These theories support the teacher's role as one of facilitator, not lecturer or director. In other words, learning occurs as students develop knowledge, construct meanings, and test out their theories in their community and social environments (Giddens and Stasz, 1999). Blended learning by employing online and offline computer has the potential as a social environment to support students' learning by invoking generic skills. Fahlberg-Stojanovska and Stojanovski (2007) noted that the best learning can take place when all three primary senses of seeing (visual), hearing (audio) and doing (enactive) are involved in an interactive environment. They proposed links between these senses and two components of blended learning as shown in the following figure (see Figure 1.5):

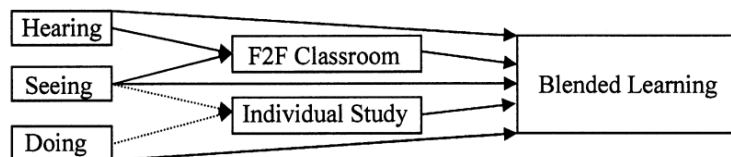


Figure 1.5: The relation between three primary senses and blended learning

Moreover, based on repeated studies Muir (2001) reported that students learn in different ways such as reading, hearing, seeing and doing, but the best learning

occurs when students learn through the combination of these senses. Blended learning has the potential to involve all these senses better than using computer or lecture separately. Therefore, due to the relation between Bruner's modes and primary senses on one hand and also the relation between primary senses and blended learning on the other hand we can see a link between Bruner's theory and the components of blended learning (Kashefi, Zaleha, and Yudariah, 2010a). See Figure 1.6.

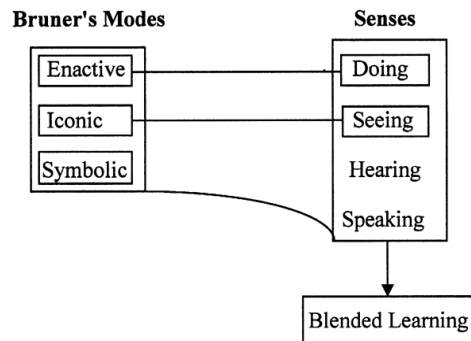


Figure 1.6: The relation between Bruner's modes, primary senses, and blended learning

The theoretical framework constructed provide a comprehensive representation of relationships between mathematical thinking (Tall's theory), three modes of representation of human knowledge (Bruner's theory), and three modes of building and testing conceptual structure (Skemp's theory) to justify the use of blended learning to support students' mathematical learning development (Kashefi, Zaleha, and Yudariah, 2012a). This theoretical model takes the form as shown in diagram Figure 1.7.

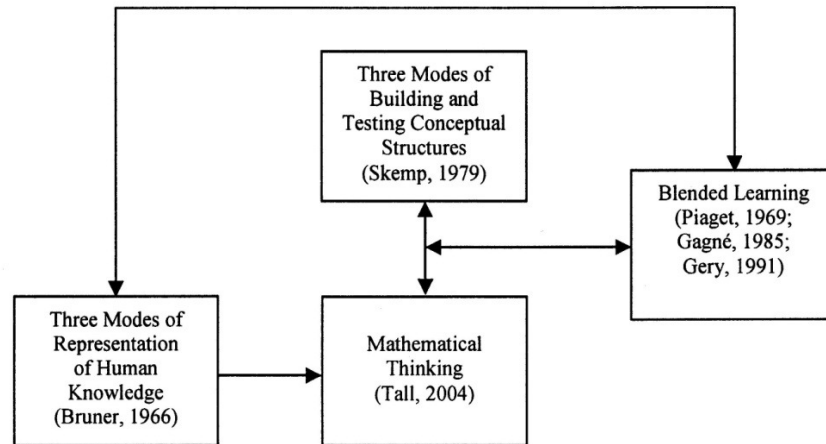


Figure 1.7: Theoretical framework of the Study

Tall's theory of the three worlds of mathematical thinking suggested a framework for the development of mathematics from childhood to the research mathematicians (Stewart, 2008). As many multivariable calculus concepts have embodied, symbolic and formal representations, based on Tall's point of view for university students it would be helpful to present them with the embodied aspects of concepts, before focusing on the formal ideas. However, the preliminary investigation revealed that the embodied ideas were often missing in lectures and students' course books. Thus, in this study it was decided to apply the theory of Tall, by firstly examining students' thinking in the preliminary investigation, and secondly applying it in teaching multivariable calculus concepts.

1.6 Conceptual Framework

According to Lester (2005), a research framework is "a basic structure of the ideas that serves as the basis of phenomenon that is to be investigated." The research framework of this study is constructed based on the purpose and research questions of the study, as shown in Figure 1.8.

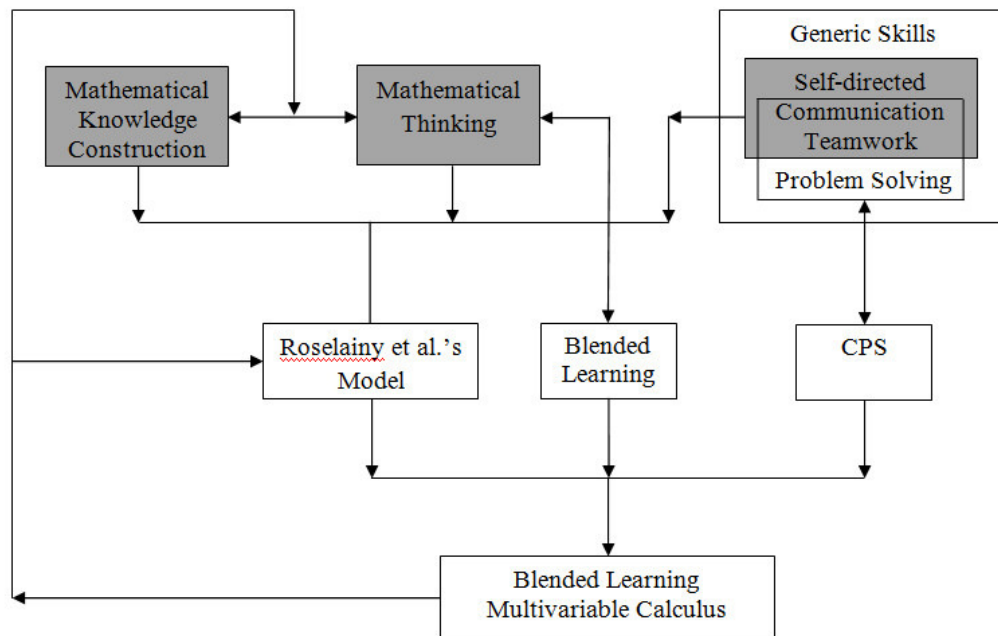


Figure 1.8: Conceptual framework of the study

The conceptual framework shows the six variables identified in this study, namely mathematical knowledge construction, mathematical thinking, students' difficulties in the learning of multivariable calculus through Roselainy model, generic skills, CPS, and blended learning and how they are interconnected.

The preliminary investigation of this research was implemented to determine the effectiveness of Roselainy et al.'s method in supporting students' learning based on mathematical thinking approach. Moreover, knowing students' difficulties in the learning of multivariable calculus through this method and looking for ways of improving them from the students and lecturers' perspectives were among the aims of the preliminary investigation.

Based on the findings in the preliminary investigation, Roselainy et al.'s model with its mathematical thinking strategies, blended learning, and CPS, a blended learning multivariable calculus course is designed and created as an

alternative learning approach that considers the advantages of both e-learning and F2F environments.

Then, how the blended learning multivariable calculus course can support students' mathematical thinking powers in the construction of mathematical knowledge, doing problem solving and in reducing students' mathematical difficulties are investigated.

1.7 Research Questions

In particular, the research would answer questions that include:

1. How an alternative approach in fostering students' mathematical thinking and generic skills in multivariable calculus can be designed based on the preliminary investigation, the existing model, integrating CPS, and blended learning?
2. How appropriate mathematical experiences for multivariable calculus course can we adapted, modified and/or created in a blended learning environment?
3. How effective is the blended learning multivariable calculus course in developing and supporting students' thinking powers in the construction of mathematical knowledge, problem solving, and in reducing their difficulties.

1.8 Importance of the Study

The findings of this study will verify the modeling and designing of a blended learning multivariable calculus course based on mathematical thinking approach and CPS strategies. This model refines and enhances the Roselainy et al.'s model by supporting mathematical thinking by using computer tools and invoking generic skills via CPS steps in blended learning environment. The preliminary investigation work on the ways of improving students' difficulties in multivariable calculus based on students and lecturers' perspectives was considered in designing the model for multivariable calculus course.

This study may evoke awareness among mathematics educators that CPS can be integrated in mathematics problem solving with the potential of using computer and invoking generic skills in its processes. Through CPS, different generic skills such as communication, teamwork, and problem solving skills can be invoked to develop and support students' mathematical knowledge construction and problem solving ability through technology tools. Moreover, applying this model in calculus provides opportunities for students to familiarize themselves with the CPS and to be able to use it in their engineering subjects or even later in their professional works.

1.9 Chapter Summary

Section 1.1 of this chapter gave the introduction of this study where the importance of calculus was briefly mentioned. This is followed by the ways researchers try to help students to overcome their difficulties in multivariable calculus by supporting mathematical thinking with or without computer in F2F classroom. This section ends with a discussion on the implementation of CPS to support students' mathematical knowledge construction in their mathematics class.

Section 1.2 briefly clarified the background of this study that was started by three studies concerned using mathematical thinking for supporting students in mathematical knowledge construction and also overcoming their difficulties in calculus were quoted. The blended learning considered as environment that has tools and potentials in supporting learning and teaching of mathematics and students' generic skills.

Section 1.3 provided the statement of problem. It initially focused on the strength and weakness of studies by Dubinsky, Tall, and Roselainy that used mathematical thinking to help students in calculus with or without computer in F2F classroom. This section also highlighted an emphasis on the lack of literature on the use of computers in multivariable calculus to develop and support students' mathematical knowledge construction based on mathematical thinking and CPS.

Following from the statement of the problem was Section 1.4 where the objectives for this study were stated.

Section 1.5 discussed the components in the theoretical framework of this study. The discussion began with the relation between the theory of the three worlds of mathematical thinking by Tall (2004) and Bruner's theory (1966). This led to the way of empowering students' mathematical thinking by using computer based on Skemp's theory (1979) and introducing blended learning (Piaget, 1969; Gagné, 1985; Gery, 1991) as a relevant environment to support students' mathematical thinking.

Section 1.6 presented the construction of conceptual framework and the relation between all possible variables in this study. Following this are the formulations of the research questions of this study (Section 1.7).

Section 1.8 highlighted the importance of the study by discussing how the findings of this study could be used to develop and support students' mathematical thinking powers by promoting mathematical thinking and CPS in blended learning environment. This chapter ends with Section 1.9 which gives a summarized outline for the whole chapter. The next chapter will present and discuss related literatures pertaining to the identified variables and methodology of this study.

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