

Curve Length Estimation using Vertex Chain Code

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Abstract— Most of the applications in image analysis are based on Freeman chain code. In this paper, for the first time, vertex chain code (VCC) proposed by Bribiesca is applied to improve length estimation of the 2D digitized curve. The chain code has some preferences such as stable in shifting, turning, mirroring movement of image and has normalized starting point. Due to the variety of length estimator methods, we focused on the three specific techniques. First, the way Bribiesca proposed which is based on counting links between vertices; second, based on maximum length digital straight segments (DSSs) and lastly local metrics. The results of these length estimators with the real perimeter are compared. Results thus obtained exhibits that length estimation using VCC is nearest to the actual length.

Keywords-2D curve, curve length estimation, vertex chain code.

I. INTRODUCTION

Image boundary detection is one of the features for image analysis. In a digitized image, perimeter of the image is estimated by applying the length estimator using VCC (vertex chain code). For image presentation using VCC algorithm proposed in [1] is applied. The computational difficulty in estimating the length of the digital curve is extracted the chain code from the image [2]. This is the task of offline algorithm, although the algorithm is linear.

With applying this algorithm and especially VCC, estimating the length of the curve becomes straight forward. This paper is organized into 3 sections: first an introduction about VCC, digitization and length estimators are given. Next the applied methods in this paper are presented. Finally, result of these length estimators by implementing of VCC is displayed. Our data set are Jordan curves shown in Figure 1.



Figure 1. Data Set

II. METHODOLOGY

A. Chain Code

One of the methods for representing the boundary of image is chain code which was proposed by Freeman [3] (Fig. 2). Freeman [4] states that in general, these three objectives must be fulfilled a coding layout for formation of line: (1) it must faithfully preserve the information of interest (2) it must permit compact storage and convenient for display and (3) it must facilitate any required processing. Besides the mentioned conditions, the vertex chain code satisfied the following objectives: (1) The VCC is invariant under translation and rotation, and optionally may be invariant under starting point and mirroring transformation. (2) Using the VCC it is possible to represent shapes composed of triangular, rectangular, and hexagonal cells. (3) The chain elements represent real values not symbols, they are part of the shape and indicate the number of cell vertices of contour nodes. (4) Using the VCC it is possible to obtain relations between the bounding contour and interior of the shape [5]. When pixels are used to represent shapes, we have a structural problem called the connectivity paradox. There are two ways for connecting pixels: four connectivity and eight-connectivity. In the content of this paper we use pixels with four-connectivity [5].

Chains can represent the boundaries or contours of an image. The classical (Freeman) chain (Fig. 2) is defined as the direction of the contour of object from starting point, while an element of the VCC (Fig. 3) indicates the number of cell vertices, which in touch with the bounding contour of the shape in that element position [6].

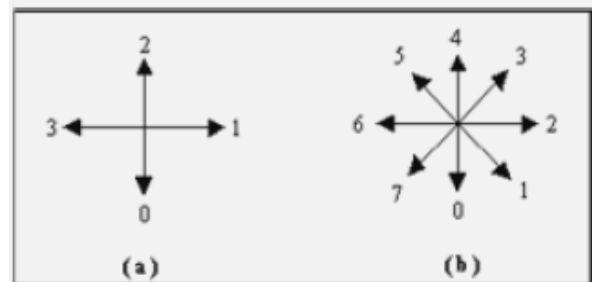


Figure 2. Freeman chain code: direction of the neighbors (a) 4-connected; (b) 8-connected [6]

In the definition of vertex chain code, an element of the vertex chain indicates the number of cell vertices, which are in touch with the bounding contour of the shape in that element position [5] (Fig. 3).

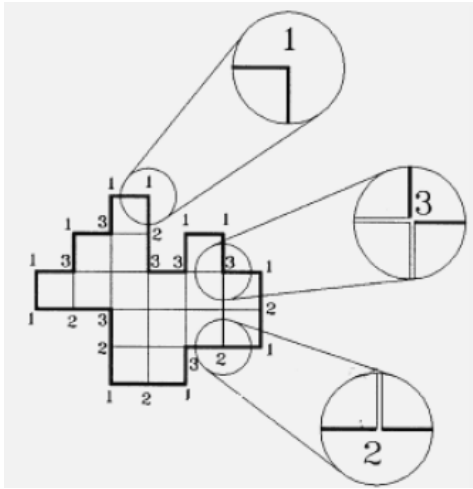


Figure 3. Fig.3: The elements of the vertex chain code [5]

B. Digitization of Curve

The digitization of curves or boundaries has been studied in image analysis for about forty years [2]. We start with parametric curves. A planar Jordan curve γ is given by a parameterized path. The digitization of curves or boundaries has been studied in image analysis for about forty years [2]. We start with parametric curves. A planar Jordan curve γ is given by a parameterized path

$$\gamma : [a, b] \rightarrow \mathbb{R}^2 \quad (1)$$

With $a \neq b$; $\varphi(a) = \varphi(b)$; and $\varphi(a) \neq \varphi(b)$; for all $a \leq s < t < b$: It holds that

$$\gamma = \{(x, y) : \varphi(t) = (x, y) \wedge a \leq t \leq b\} \quad (2)$$

A planar Jordan arc is defined by a subinterval $[c, d]$ with $a \leq c < d \leq b$. A Jordan curve is also called a closed Jordan arc. A rectifiable Jordan arc γ has bounded arc length. In 1883, initially C. Jordan defined a curve γ in parametric form as

$$\gamma = \{(x, y) : x = \alpha(t) \wedge y = \beta(t) \wedge a \leq t \leq b\} \quad (3)$$

The three methods for getting 2D digitized curve are:

1. The cyclic 4-path arc, $\sigma_{r,4}(\gamma)$ or 8-path $\sigma_{r,8}(\gamma)$ of r-grid points following an r-grid-intersection digitization of γ .
2. The cyclic 4- or 8-path following vertices of r-grid squares in the frontier of the Gauss digitization $G_r(S)$ of set S in the r-grid, where $G_r(S)$ is the union of all r-grid squares having their centroid in the given set S, or 3. The closed difference set between outer and inner Jordan digitization.

$$J_r^+(S) \text{ and } J_r^-(S), \text{ i.e. } c1(J_r^+(S) / J_r^-(S)) \quad (4)$$

Where, $J_r^+(S)$ is the union of all r-grid squares having a nonempty intersection with the given set S, and $J_r^-(S)$ is the

union of all r-grid squares contain in the topological interior of the given set S [2]. In this paper for presenting the image by means of VCC, the first method is implemented by a slight modification to adjust the digitization for our purpose. Due to the fact that to obtain VCC we have to move just in orthogonal direction, so avoidance from the diagonal steps between the vertices is an obligation. When this situation is encountered we find three points. Two of them are the start and end point of the diagonal link, $p = (x_1, y_1)$ and $r = (x_3, y_3)$ respectively. The third point is the point between them that is one of the closest start or end point of the adjacent diagonal which is achieved by counting close distance between these two possible points and the curve which is $q = (x_2, y_2)$. By this process three points are obtained. To determine VCC for q point we used following matrix.

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} = x_1 y_2 + x_3 y_1 + x_2 y_3 - x_3 y_2 - x_2 y_1 - x_1 y_3$$

The result may be a positive or negative value. However, if it is positive, the VCC element is set to 1 otherwise, it is 3.

C. Length estimator

Since first studies of digitization, many algorithms have been proposed to estimate the length of a digitized curve. Some approaches are based on local metrics, such as the weighted metrics, other approaches are based on polygonalizations of digital curves, e.g., directed on subsequent calculations of maximum-length digital straight segments (DSSs), or of minimum length polygons (MLPs). Two main category of length estimator are local and global estimators. Local estimators are local metrics which are based on shortest paths in a weighted adjacency grid of pixels. The weights are chosen to approximate Euclidean distance. For example, critical steps in the 2D grid can be weighted by θ and diagonal steps by $\sqrt{2}\theta$. An advantage of such estimates is that they have linear online algorithm.

On the other hand, the global length estimators are classified into four categories [7]; DSS (digital straight segments)-based polygonalization, MLP (minimum-length polygon), tangent based estimation, and Angle-based estimators. These pixel-based methods use estimates of line orientation. The discussion of the last three categories is beyond the scope of this paper. DSS (digital straight segments)-based polygonalization is a popular method in image analysis, allowing us to transform digital boundaries into polygonal objects (see Figure 4). The sum of the length of the DSSs defines a DSS-based length estimator. Two type of implementations of DSS-based length estimators are: If the digital curve is defined as an 8-curve, the Debled-Reveille's algorithm is used and it is called the \mathcal{E}_{DR_DSS} estimator. If the digital curve is defined as a 4-curve a length estimator is based on Kovalevsky's algorithm and is called the VK_DSS

estimator. These two DSS-based length estimators are known to be multi grid-convergent for convex Jordan curves γ .

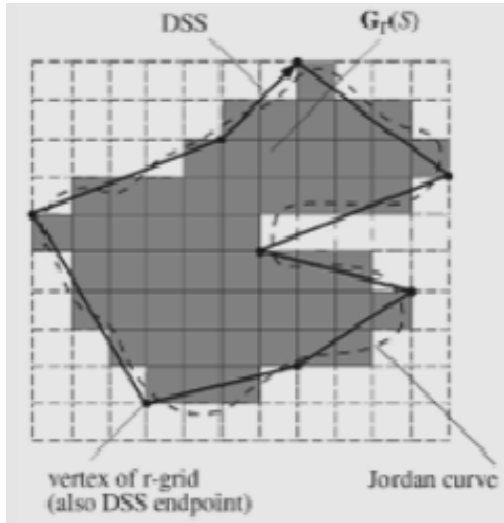


Figure 4. Segmentation of a 4-path into a sequence of maximum-length DSSs.[2]

III. LENGTH ESTIMATION BY MEANS OF VCC

A. Length estimation based on VCC

After digitization and getting VCC, we can estimate the counter of the image, E by means of VCC. So $E = r.n$

$$(6)$$

Where, r is the grid resolution and n is the number of element in VCC. In this paper value of r set to 1 experimentally.

B. Maximum length DSS

In the previous DSS estimator, following digitization, an offline algorithm applied to acquire chain code. If the link between them is DSS or not, instead we introduced new way for this method. First, we get the VCC from the binary chain code and then we convert this VCC to the Cartesian coordinate system (Bribiesca proposed this method in [5]). The polygon is gained from these coordinate is the digitized image. Finally, algorithm proposed in [7] is applied to estimate the length of polygon and the result is E_{4SS} . K1990 has the run time complexity $O(n)$.

C. Chessboard metrics

In image analysis, local metrics were historically the first attempts towards the length estimation problem. These algorithms applied to digital curves defined by options 1 or 2 as defined in Section 2-Digitization of Curve, and could be viewed as shortest path calculations in weighted adjacency graphs of pixel locations. Weights were designed with the intention of approximating the Euclidean distance. In this work we applied chessboard metrics.

$$\varepsilon_{CM}(\sigma_{r,s}(\gamma)) = \frac{1}{r}(0.948n_r + 1.343n_d) \quad (7)$$

Where n_i is the number of iteration steps and n_d of diagonal steps in the r -grid. However, it is known that this method is only of limited use if multi grid convergence is applied as a selection criterion.

IV. EXPERIMENTAL RESULTS

In this paper the data set is constructed using a circle and half moon. The function is $y = \sin(16\pi .x) / (64\pi .x)$ in a bounded distance symmetric to the y -axis [10]. By applying these length estimators with combination of the VCC, on the data set (Fig. 1), the result is as shown in Table 1. It is mentionable that by changing the resolution of the image the VCC estimator does not have any effect; however DSS is closer to the real value.

TABLE I. A COMPARISON BETWEEN LENGTH ESTIMATORS BY MEANS OF VCC

VCC		
Data set	Half moon	Cycle
length estimators		
Real perimeter	17.42	21.98
VCC Estimator	18	28
DSS Estimator	18.94	21.49
Chessboard metrics	17.064	26.544

V. CONCLUSION

This paper has presented an improved linear algorithm [1] to detect the vertices using VCC. Moreover, it is exhibited that vertex chain code is better than chain code for curve length estimation. Additionally, it makes the length estimation process faster and transparent. However, image skeletonization is mandatory to obtain vertex VCC, Finally, Euclidean coordinate according to each vertex is calculated [5]. The results show that all of the estimators are worked under boundary of error and the error is between 0.4 up to 7. The VCC estimator is effective and simple to estimate the boundary of the image by extracting the VCC and counting the number of elements in vertex chain code. It is worth to apply the VCC on other length estimators like MLP and in 8-connectivity instead of 4-connectivity.

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