DYNAMIC MODELLING OF TWIN ROTOR MULTI SYSTEM IN HORIZONTAL MOTION

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ABSTRACT

This paper investigates the parametric linear approach and utilisation of neural networks (NNs) for modelling of a twin rotor multi system (TRMS) in horizontal motion. Parametric modelling using Auto Regressive Modelling (ARX) model using Recursive Least Squares (RLS) algorithm. On the other hand, non-parametric modelling, makes use of Multi Layer Perceptron-Neural Network (MLP-NN) technique. All of these techniques will be used to characterize the behaviour of TRMS. Comparative assessment between these two techniques was conducted and the MLP-NN shows better results compared to RLS for modelling the TRMS. Mean Square Error (MSE), One Step Ahead (OSA) prediction and Correlation Tests were used for verification and validation of both models. Both models are found to be within the 95% confident level.

Keywords*:   Multi layer perceptron-neural networks, system identification, twin rotor multi system.*

1.0   INTRODUCTION

System identification (SI) is one of the most fundamental requirements for many engineering and scientific applications. The objective of system identification is to find exact or approximate models of dynamic systems based on observed input and output data. These input and output data can be obtained through experimental work, simulation or directly collected from the plant. Once a model of the physical system is obtained, it can be used for solving various problems such as, to control the physical system or to predict its behaviour under different operating conditions [1].

The modelling is done by assuming no prior knowledge of model structure or parameters relating to physical phenomena, i.e. black-box modelling. This is realized by minimizing the prediction error of the actual plant output and the model output [2].

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The procedure, for identifying a dynamical system, consists of four basic steps, as shown in Figure 1. Once a model of the system is obtained, it is required to verify whether the model is good enough to represent the system. A number of validation tests are available in the literature. These include correlation tests, mean square error, estimation and test data. In this work, neural network (NN) architecture based on multi-layer perception (MLP) network as shown in Figure 2 is used to characterize the system. The network is trained using the back propagation-learning (BPL) algorithm based on one-step-ahead (OSA) prediction technique. The results are obtained in both time and frequency domains. The system will be validated using input/output mapping, mean-squared error and correlation tests. The performance of non-parametric identification using NN is compared to the parametric identification using recursive least square (RLS) technique by evaluating the mean square errors. The non-parametric model of the TRMS thus developed and validated will be used in subsequent investigations for the development of simulation of rigid- body motion, vibration suppression and control strategies for twin rotor systems [3].
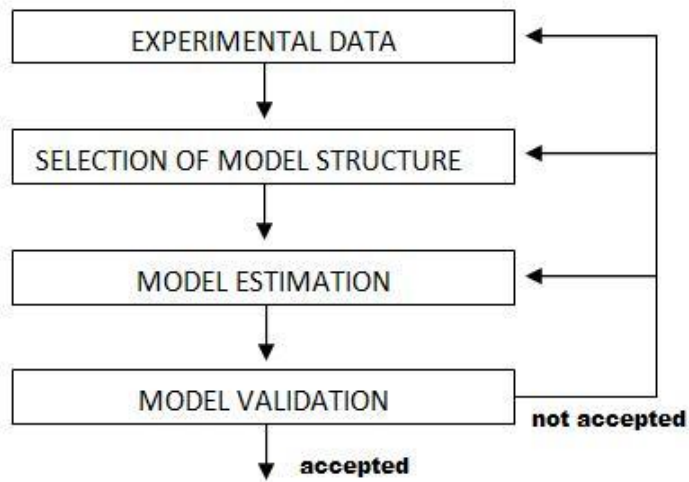

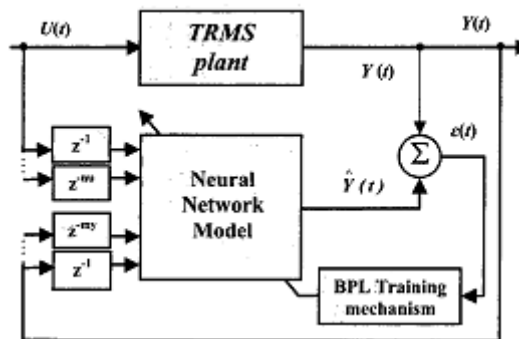Figure 1: The system identification procedure

Figure 2: Training the MLP network to model the TRMS

2.0 THE TWIN ROTOR MULTI SYSTEM

The TRMS, shown in Figure 3, is a laboratory set-up designed for control experiments. In certain aspects it behaves like a helicopter. The TRMS rig consists of a beam pivoted on its base in such a way that it can rotate freely both in the horizontal and vertical directions producing yaw and pitch movements, respectively. At both ends of the beam there are two rotors driven by two D.C. motors. The main rotor produces a lifting force allowing the beam to rise vertically making a rotation around the pitch axis (vertical angle). While, the tail rotor (smaller than the main rotor) is used to make the beam turn left or right around the yaw axis (horizontal angle) [2, 4, 5].

In a typical helicopter, the aerodynamic force is controlled by changing the angle of attack of the blades. The laboratory set-up is constructed so that the angle of attack of the blades is fixed and the aerodynamic force is controlled by varying the speed of the motors. Therefore, the control inputs are supply voltages of the D.C. motors. A change in the voltage value results in a change in the rotational speed of the propeller, which results in a change in the corresponding position of the beam [2, 4, 5].

Although the TRMS system permits MIMO experiments, this project addresses the problem of modelling and control of the system in a single-input single-output (SISO) mode in the vertical axis (i.e. horizontal movement). The vertical movement caused by the main rotor was physically locked and as a result there is no cross-coupling effect between the two channels of the TRMS. The problem of MIMO modelling and control is an interesting issue, and will be looked into future studies.
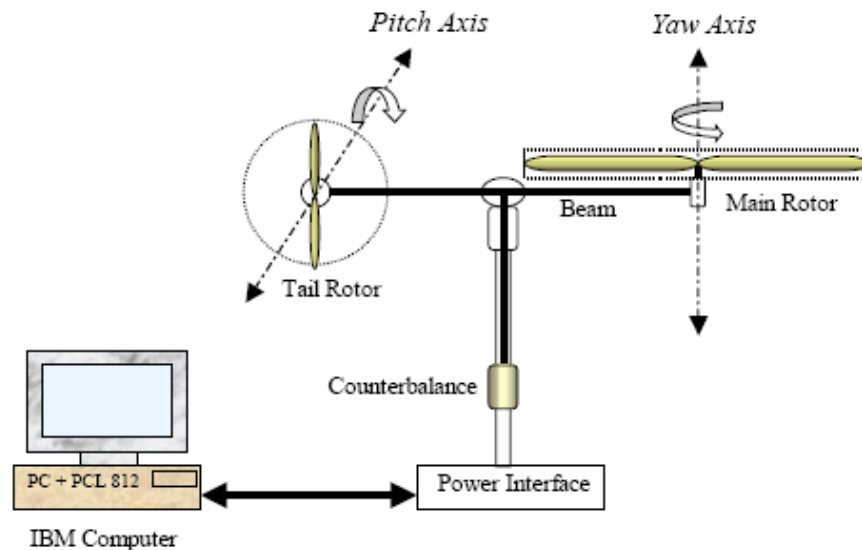

Figure 3: The schematic diagram of the TRMS

2.1 One Degree of Freedom (DOF) of TRMS modelling in horizontal plane

The TRMS possesses two permanent magnet DC motors; one for the main and the other for the tail propelling. The motors are identical with different mechanical loads. The circuit diagram of a DC motor is shown in Figure 4.

The mathematical model of the main and tail motors, as shown in Figure 5, is presented in equations (1) - (5) [6].

$$U_{h/v} = E_{ah/v} + R_{ah/v}i_{ah/v} + L_{ah/v}\frac{di_{ah/v}}{dt} \tag{1}$$

$$E_{ah/v} = k_{ah/v}\varphi_{h/v}\omega_{h/v} \tag{2}$$

$$T_{eh/v} = T_{Lh/v} + J_{tr/mr}\frac{d\omega_{h/v}}{dt} + B_{tr/mr}\omega_{h/v} \tag{3}$$

$$T_{eh/v} = k_{ah/v}\varphi_{h/v}i_{ah/v} \tag{4}$$

$$T_{Lh/v} = k_{th/v\cdot}|\omega_{h/v\cdot}|\omega_{h/v} \tag{5}$$

where,

$U_{h/v}$: Horizontal / vertical voltage control input

$E_{ah/v}$: Electro motive force of tail / main motor

$R_{ah/v}$: Armature resistance of tail / main motor

$L_{ah/v}$: Armature inductance of tail / main rotor

$i_{ah/v}$: Armature current of tail / main motor

$k_{ah/v}$, $k_{th/v}$: Constants

$\varphi_{h/v}$: Magnetic flux of tail / main rotor

$\omega_{h/v}$: Rotational speed of tail / main rotor

$T_{eh/v}$: Electro-magnetic torque of tail / main rotor

$T_{Lh/v}$: Load torque of tail / main rotor

$J_{tr/mr}$: Moment of inertia in tail / main DC motor

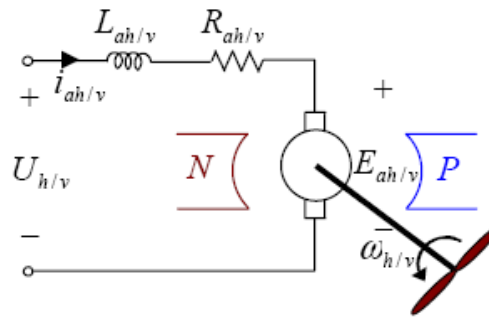$B_{tr/mr}$: Damping coefficient of tail / main DC motor

Figure 4: Circuit diagram of a DC motor

The mathematical model of the remaining parts of the system in horizontal plane is described in equations (6) - (8) (Figure 5 represents the propulsive force in horizontal plane). In equation (6) the first term is the torque of propulsive force due to the tail rotor, the second term implies the torque of the friction force, and the third term refers to the torque of the flat cable force that is completely nonlinear and can be obtained by point by point measurement.

$$\frac{d\Omega_h}{dt} = \frac{l_t F_h(\omega_h)\cos\alpha_v - T_{fric,h} - T_{cable}(\alpha_h)}{D\cos^2\alpha_v + E\sin^2\alpha_v + F}$$

(6)

where,

$$\alpha_v = cte, D = \left(\frac{m_m}{3} + m_{mr} + m_{ms}\right)l_m^2 + \left(\frac{m_t}{3} + m_{tr} + m_{ts}\right)l_t^2,$$

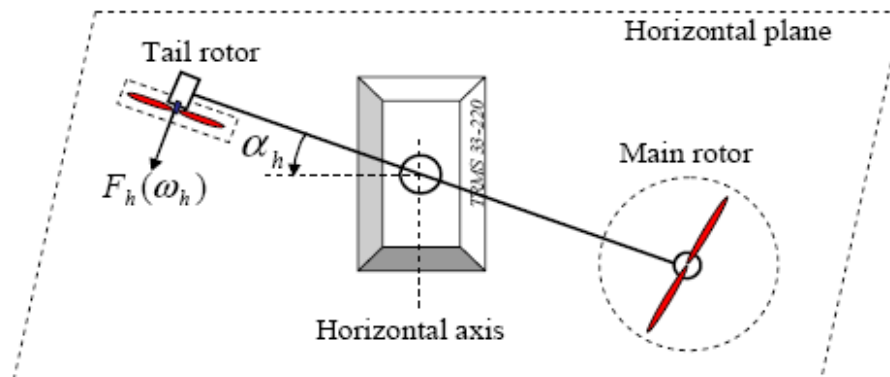$$E = \frac{m_b}{3}l_b^2 + m_{cb}l_{cb}^2, \quad F = m_{ms}r_{ms}^2 + \frac{m_{ts}}{2}r_{ts}^2$$

Figure 5: Propulsive force in horizontal plane

where,

$$F_h(\omega_h) = \begin{cases} k_{fhp} \times |\omega_h| \times \omega_h & for \quad \omega_h \geq 0 \\ k_{fhn} \times |\omega_h| \times \omega_h & for \quad \omega_h < 0 \end{cases} \tag{7}$$

$$\frac{d\alpha_h}{dt} = \Omega_h \tag{8}$$

where,

$F_{h/v}$: Function of aerodynamic force from tail / main rotor

m_t : Mass of tail part of the beam

m_{tr} : Mass of the tail DC motor

m_{ts} : Mass of the tail shield

m_m : Mass of the main part of the beam

m_{mr} : Mass of the main DC motor

m_{ms} : Mass of the main shield

m_b : Mass of the counter-weight beam

m_{cb} : Mass of the counter-weight

l_t : Length of tail part of the beam

l_m : Length of main part of the beam

l_b : Length of counter-weight beam

l_{cb} : Distance between the counter weight and the joint

r_{ms} : Radius of the main shield

r_{ts} : Radius of the tail shield

α_h : Horizontal position of TRMS beam

α_v : Vertical position of TRMS beam

Ω_h : Angular velocity of TRMS beam in horizontal plane

Ω_v : Angular velocity of TRMS beam in vertical plane

J_v : Moment of inertia about horizontal axis

$T_{fric,v}$: Torque of the friction force in vertical plane

$T_{fric,h}$: Torque of the friction force in horizontal plane

$T_{cable}(\alpha_h)$: Torque of the flat cable force

$k_{fhp}, k_{fhn}, k_{fvp}, k_{fvn}$: Positive constants

Since the analytical modelling is very complex, system identification (SI) will be used in this research to model the TRMS.

3.0 MODEL VALIDATION

3.1 One Step-Ahead Prediction (OSA)

One Step Ahead prediction measures of accuracy. This can be expressed as follows:

$$\hat{y}(i) = f\{u(t), u(t-1), \dots, u(t-n_u), y(t-1) \dots, y(t-n_y)\} \tag{9}$$

where $f(.)$ is a non linear function, u and y are the inputs and output respectively. The residual or prediction is given as:

$$(t) = y(t) - \hat{y}(i) \tag{10}$$

Often $\hat{y}(t)$ will be relatively good prediction of $y(t)$ over the estimation set, even if the model is biased. The model is estimated by minimizing the prediction errors.

3.2 Mean Squared Error

The most common methods of validation is to utilize the mean-squared error between the actual output, $y(n)$, of the system and the predicted output, $\hat{y}(n)$, produced from the input to the system and the optimized parameters;

$$f(e) = \frac{1}{n}\sum_{i=1}^{n}(|y(n) - \hat{y}(n)|)^2 \tag{11}$$

where n is the number of input/output samples.

3.3 Correlation Tests

A more convincing method of model validation is to use correlation test. In the theory of linear systems, the usual statistical approach to validating identified linear models consists of computing the autocorrelation function of the residuals and the cross-correlation function between the residuals and the input. It has been shown that acceptable predictions over different data sets are produced only if the model is unbiased. If the model structure and the estimated parameters are correct then the prediction error sequence $\varepsilon(t)$ should be unpredicted from all linear and nonlinear combinations of past inputs and outputs and this will hold if and only if the following conditions are satisfied [7]:

$$\phi_{\varepsilon\varepsilon}(\tau) = E[\varepsilon(t-\tau)\varepsilon(t) = \delta(t)$$

$$\phi_{u\varepsilon}(\tau) = E[u(t-\tau)\varepsilon(t) = 0 \quad \forall \tau$$

$$\phi_{u^2\varepsilon}(\tau) = E[(u^2(t-\tau) - \bar{u}^2(t))\varepsilon(t) = 0 \quad \forall \tau \tag{12}$$

$$\phi_{u^2\varepsilon^2}(\tau) = E[(u^2(t-\tau) - \bar{u}^2(t))\varepsilon^2(t) = 0 \quad \forall \tau$$

$$\phi_{\varepsilon(\varepsilon u)}(\tau) = E[\varepsilon(t)\varepsilon(t-1-\tau)u(t-1-\tau) = 0 \quad \tau \geq 0$$

where $\phi_{u\varepsilon}(\tau)$ indicates the cross-correlation function between $u(t)$ and $\varepsilon(t)$, $\varepsilon u(t) = \varepsilon(t+1)u(t+1), \delta(\tau)$ = an impulse function.

Ideally the model validity tests should detect all the deficiencies in algorithm performance including bias due to internal noise. Consequently the full five tests defined by equation (12) should be satisfied. In practice normalized correlations are computed. The sampled correlation function between two sequences $\varphi_1(t)$ and $\varphi_2(t)$ is given by:

$$\hat{\phi}_{\psi_1\psi_2}(\tau) = \frac{\displaystyle\sum_{t=1}^{N-\tau} \psi_1(t)\psi_2(t+\tau)}{\sqrt{\displaystyle\sum_{t=1}^{N} \psi_1^{2}(t)\sum_{t=1}^{N} \psi_2^{2}(t)}} \tag{13}$$

Normalization ensures that all the correlation functions lie in the range $-1 \leq \ddot{\phi}_{\varphi_1\varphi_2}(\tau) \leq 1$ irrespective of the signal strengths. The correlations will never be exactly zero for all lags and the 95% confidence bands defined as $96.1/\sqrt{N}$ are used to indicate if the estimated correlations are significant or not, where N is the data length. Therefore, if the correlation functions are within the confidence intervals the model is regarded as adequate [8].

4.0 IMPLEMENTATION AND RESULTS

Results of modelling the TRMS in horizontal motion with parametric and non-parametric techniques are presented in this section. To investigate variations in the detected vibration modes, modelling are carried out with the TRMS simulated algorithm responses to sine inputs.

4.1 Modelling with RLS
The TRMS has been modelled with RLS algorithm. The vibration model was observed with different orders. The best result was achieved with model order, $n_u = n_y = 10$. The simulated output of the system, in time domain, thus modelled is shown in Figure 6. The RLS algorithm achieved the best mean-square error level of 0.000016016. The correlation tests for the RLS based model (shown in Figure 7) were also found to be within the 95% confidence intervals.

4.2 MLP NN Modelling
Investigations were carried out using the MLP NN based on OSA prediction with different number of neurons in the layers. Various MLP NN structure with different number of hidden layersand neuron have been tested to obtained the smallest mean-squared error level. The best result was found when an MLP network with two hidden layers, each with 6 tansigmoid neurons, and one output layer with linear neuron and model orders, $n_u = n_y = 20$, was trained to characterize the TRMS. The data set, comprising 100000 data points, was divided into two sets of 30000 data points and 70000 data points.The first set was used to train the network and the model was validated with the whole 100000 points including the 70000 points that had not been used in the training process. The model reached the mean-squared error level of

0.000000485 in 1000 training passes. The performance of the MLP neural network thustrained, the algorithm convergence, the simulated output in time domain is shown in Figure 8. Comparing these with the corresponding results of RLS modelling reveals that the identification using NN has performed far better than the linear model. The correlation tests for the MLP NN based model (shown in Figure 9) were also found to be within the 95% confidence intervals.
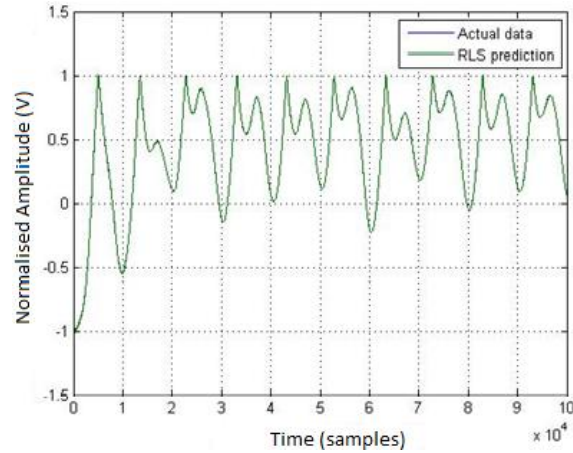
(a) Actual and RLS predicted output versus number of samples
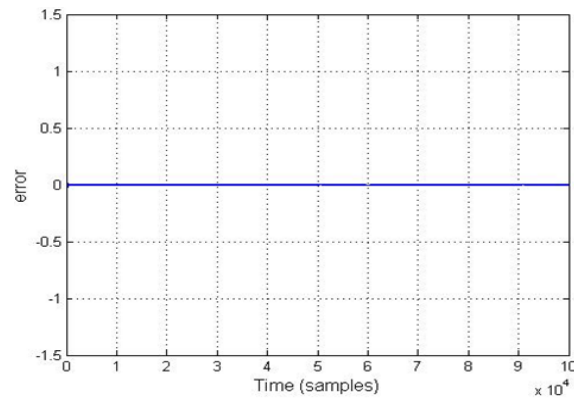
(b) Error between actual and predicted output

Figure 6: RLS prediction
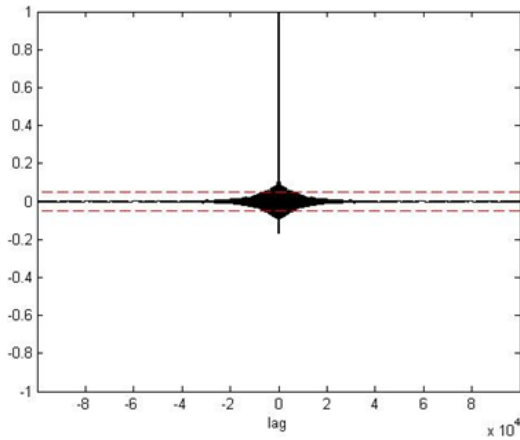
(a) Auto-correlation of residual
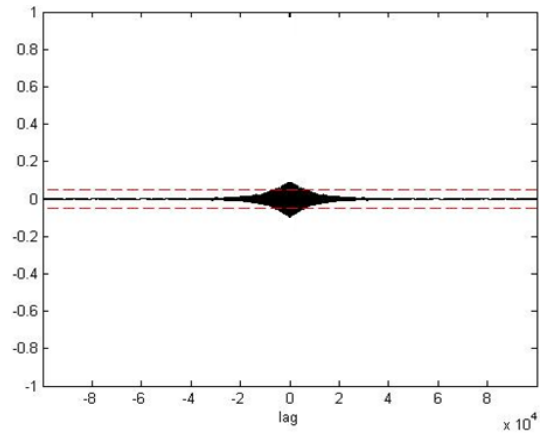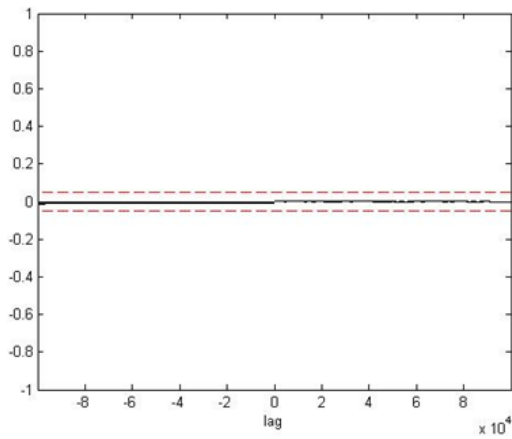
(b) Cross-correlation of input and residuals

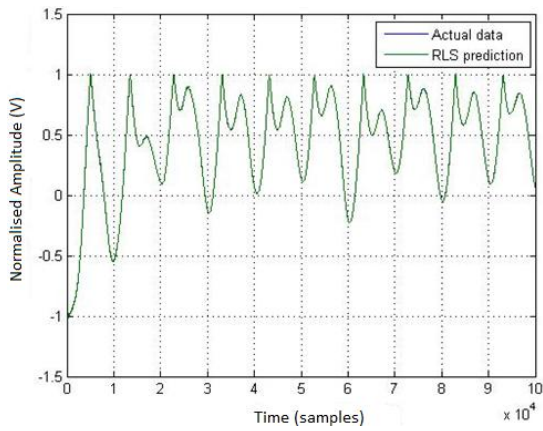(c) Cross-correlation of input square and residuals
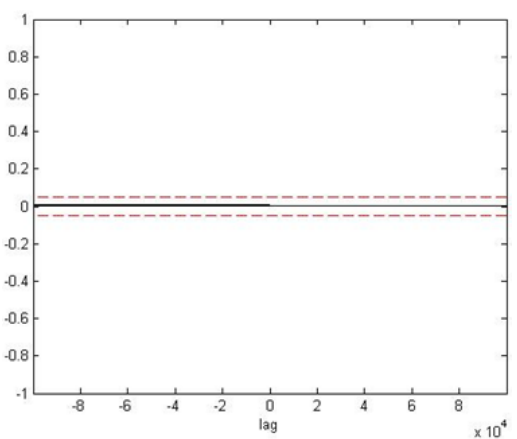
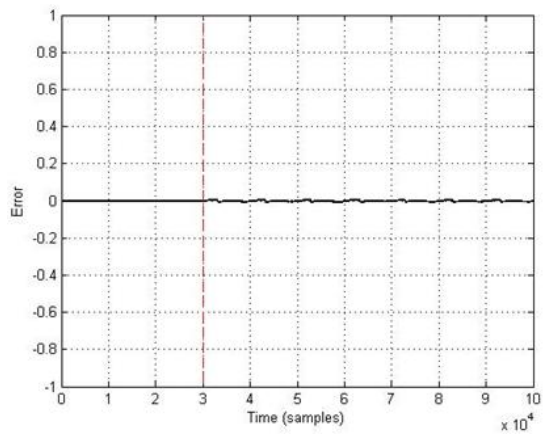(d) Cross-correlation of input square and residual square
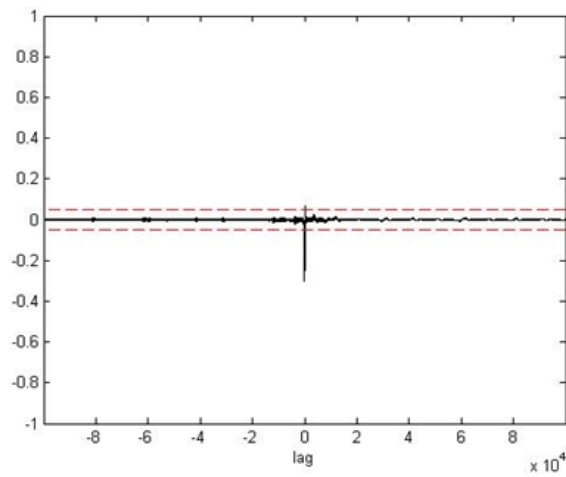
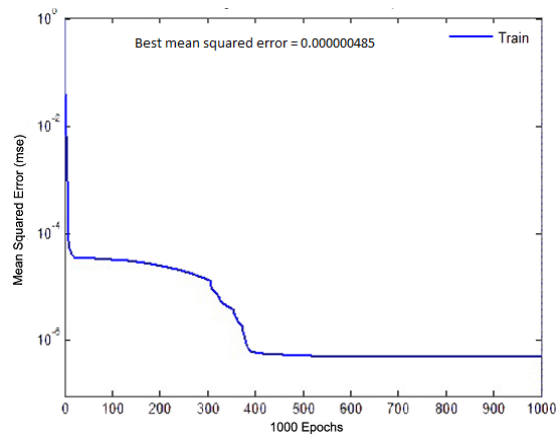(e) Cross-correlation of residuals and (input*residuals)

Figure 7: Correlation tests of RLS

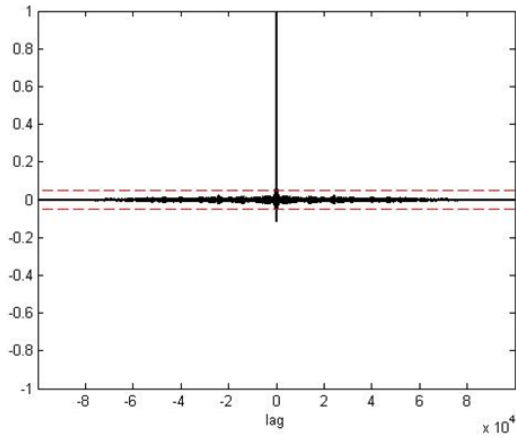(a) Actual and predicted output
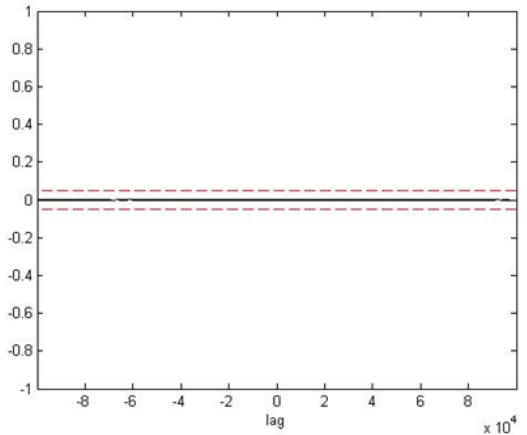

(b) Error between actual and predicted output

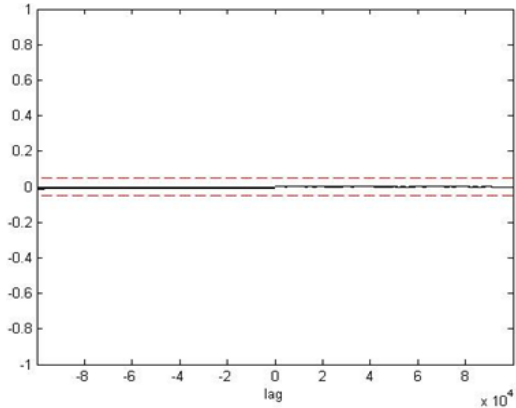(c) Mean-squared error vs. number of training passes
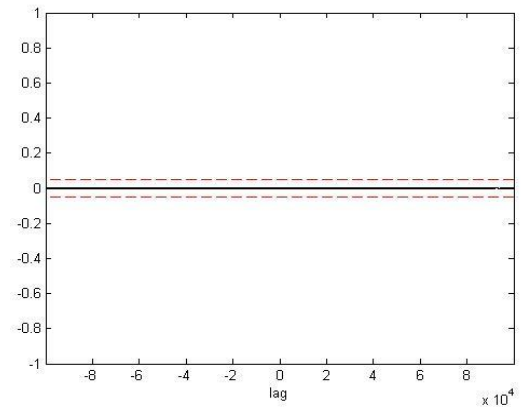
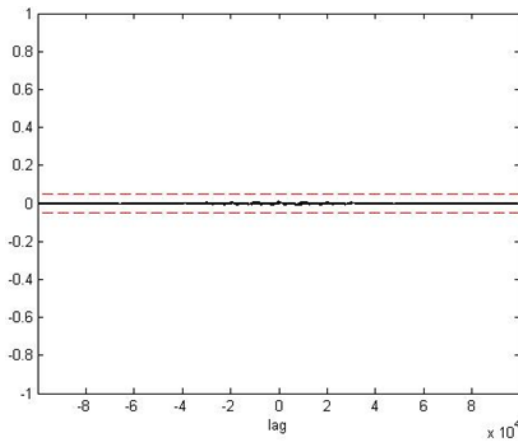Figure 8: MLP-NN prediction

(a) Auto-correlation of residual

(b) Cross-correlation of input and residuals

(c) Cross-correlation of input square and residuals

(d) Cross-correlation of input square and residual square

(e) Cross-correlation of residuals and (input*residuals)

Figure 9: Correlation tests of MLP-NN

5.0   CONCLUSIONS

In this dynamic modelling, horizontal plane equations have been developed by other researchers which resemble a similarity of a Twin Rotor MIMO System of a helicopter. System identification is a tool to model non-standard aircraft configurations, which then impose into the TRMS system. Time domain analyses have been analysed to investigate and develop confidence in the obtained model. The extracted model has predicted the system behaviour well. A Recursive Least Square and MLP-NNs approaches have been used in this study. It has been assessed with one set of data. The predicted model has achieved the best mean-squared error using the MLP-NN approach as compared to the RLS method. Every approach has its own advantages and disadvantages. Amongst the disadvantages of both techniques is the delay in processing time in order to obtain the best result.

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