

IMPLEMENTATION OF MIXED INTEGER LINEAR PROGRAMMING FOR  
HYDRO-THERMAL GENERATION SCHEDULING WITH RIVER AND  
RESERVOIR CONSTRAINTS

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RESERVOIR CONSTRAINTS

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*To my lovely father, mother, and my country Iran*

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## ABSTRACT

A Short-term Hydro-thermal Scheduling (HTS) model based on Mixed Integer Linear Programming (MILP) is developed and presented in this thesis. For countries such as Malaysia that are close to the equator, high precipitation throughout the year replenishes existing water resources. The efficient scheduling of hydro and thermal units considering a large amount of water resources and river systems can significantly affect the total operation costs of the system. The HTS is a highly complex problem involving a large number of continuous and integer variables with nonlinearity and nonconvexity/nonconcavity characteristics in its objective function and constraints. A comprehensive MILP hydraulic model for unit-wise, and cascaded multi-chain reservoir system considering head variation effects has been developed. Incorporation of the detailed reservoir and river modelling with variable head makes the HTS problem even more complex with an additional number of integer/continuous variables as well as the constraints. A piecewise linear approximation is used to transform all nonlinearities into an equivalent linear model. Multi-thread computing is utilised to expedite the solution process of MILP Branch and Bound and Cut (BB & C) method using a certain number of concurrent threads. Obtained results show the successful implementation of the multi-chain river system modelling on several test cases including 69-unit, 132-unit and 287-unit. The proposed MILP-HTS algorithm is compared with a Lagrangian Relaxation (LR) algorithm that is currently employed by a real-world utility. Based on the similar input data, the MILP-HTS algorithm offers more optimal hydro-thermal generation strategy, taking into account a detailed hydraulic modelling. Based on the simulation results, the proposed MILP algorithm outperforms several other deterministic and heuristic techniques in terms of objective cost and execution time. Comparison with other equivalent MILP models over the same test conditions demonstrated that the proposed MILP model with the formulation presented in this thesis creates tighter relaxation (better cuts) in the BB & C solution process. This results in a cheaper objective value with a lesser computation time. Implementation of multi-thread computing improves the execution time performance for all case studies as compared with the serial computation time. Simulation results also suggest that the multi-threading can allow taking tighter optimality gap resulting in a more accurate solution (near-optimal) for large-scale problems in a moderate time, even with more detailed hydraulic modelling.

## ABSTRAK

Satu model penjadualan jangka pendek Hidro-Termal (HTS) berasaskan pengaturcaraan campuran Integer Linear (MILP) telah dibangunkan dan dibentangkan di dalam tesis ini. Negara seperti Malaysia yang terletak berhampiran dengan khatulistiwa, kadar penurunan hujan yang tinggi sepanjang tahun telah meningkatkan sumber air yang ada. Penjadualan unit hidro dan termal yang cekap perlu mengambil kira jumlah sumber air yang besar dan juga sistem pengaliran sungai, jumlah keseluruhan kos operasi bagi sistem tersebut. HTS boleh menjejaskan dengan ketara yang melibatkan bilangan integer berterusan dan pelbagai yang banyak dengan ciri-ciri ketidaklelurus dan tidak boleh dikira dalam fungsi objektif dan kekangannya. Sebuah model hidraulik yang dilengkapi MILP untuk unit-cerdik dan sistem takungan berbilang-rangkaian lara dengan mengambil kira pelbagai turus perubahan telah dihasilkan. Penubuhan takungan secara terperinci dan pembentukan sungai dengan kesan perubahan turus menyebabkan masalah HTS bertambah rumit dengan bertambahnya bilangan integer/pembolehubah berterusan sekaligus kekangannya. Satu kaedah penghampiran titik-titik linear digunakan untuk mengubah ketidaklelurus kepada sebuah model lurus yang seragam. Pengkomputeran Berbilang-Uliran digunakan untuk mempercepatkan proses penyelesaian kaedah MILP Cabang dan Had dan Potong (BB & C) dengan menggunakan jumlah serempak uliran yang tertentu. Keputusan yang diperolehi menunjukkan kejayaan dalam pelaksanaan model multi-rangkaian sistem sungai terhadap beberapa kes-kes ujian termasuk 69-unit, 132-unit dan 287-unit. Cadangan algoritma MILP-HTS dibandingkan dengan algoritma Santaian Lagrangean (LR), dimana merupakan kaedah yang digunakan oleh utiliti dunia sebenar. Berdasarkan data input yang sama, algoritma MILP-HTS menawarkan strategi untuk janakuasa hidro-termal yang lebih optimum, dengan mengambil kira model hidraulik yang terperinci. Berdasarkan keputusan simulasi, algoritma MILP yang dicadangkan melebihi prestasi beberapa kaedah berketentuan dan heuristik dari segi kos objektif dan masa pelaksanaan. Perbandingan dengan model MILP lain yang setaraf ke atas ujian yang sama menunjukkan bahawa model MILP yang dicadangkan berserta rumus yang dihuraikan di dalam tesis ini menghasilkan santaian yang lebih ketat (potongan yang lebih baik) bagi BB & C proses penyelesaian. Keputusan menunjukkan nilai objektif yang lebih murah dengan masa pengiraan yang lebih singkat. Pelaksanaan berbilang-uliran meningkatkan prestasi pelaksanaan masa untuk semua kajian kes berbanding dengan masa pengiraan bersiri. Keputusan simulasi juga mencadangkan bahawa berbilang-uliran boleh membenarkan pengambilan sela optimaliti yang lebih rapat, yang menyebabkan penyelesaian yang lebih tepat (menghampiri optimum) untuk masalah berskala besar dalam masa yang sederhana, walaupun dengan model hidraulik yang lebih terperinci.

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## LIST OF ABBREVIATIONS

BB	Branch and Bound
BB & C	Branch and Bound and Cut technique
DP	Dynamic Programming
ED	Economic Dispatch
EP	Evolutionary Programming
GA	Genetic Algorithm
GZBDF	Generalized Z-bus based Distribution Factor
LR	Lagrangian Relaxation
MILP	Mixed Integer Linear Programming
MVS	Microsoft <sup>®</sup> Visual Studio
OPF	Optimal Power Flow
PL	Priority List
SCUC	Security Constraint Unit Commitment
STHTS	Short-term Hydro-thermal Scheduling
UC	Unit Commitment

## LIST OF SYMBOLS

$c_j^p(t)$	-	production cost of unit j in period t
$c_j^u(t)$	-	startup cost of unit j in period t
$c_j^d(t)$	-	shutdown cost of unit j in period t
$f_h$	-	the water inflow rate of the h-th hydro unit
$G$	-	the total number of thermal units with gas fuel
$H$	-	the total number of hydro units
$J$	-	total number of generator units
$NL$	-	total number of line
$P_j^{min}$	-	minimum capacity of generation in unit j
$P_j^{max}$	-	minimum capacity of generation in unit j
$p_j^R(t)$	-	maximum available power output of unit j in period t
$P_m(t)$	-	the active power flow in line m in period t
$P_m^{max}$	-	the maximum active power capacity of line m
$p_h(t)$	-	active power generation output at hydro unit h
$q_h(t)$	-	the rate of water flow from hydro unit h in interval t
$qtot_h$	-	the prespecified volume of water available for the h-th hydro unit

$R(t)$	-	spinning reserve requirement in period $t$
$RD_j$	-	rampdown limit of unit $j$
$RU_j$	-	ramp-up limit of unit $j$
$Si_h$	-	the initial volume of water of the reservoir of the $h$ -th hydro unit
$Sf_h$	-	the final volume of water of the reservoir of the $h$ -th hydro unit
$T$	-	total number of time interval
$t$	-	time interval
$t^{insR}$	-	the instant reserve requirement time
$T_j^{off}$	-	minimum down time of unit $j$
$T_j^{on}$	-	minimum up time of unit $j$
$V_g^P(t)$	-	the gas volume of unit $g$ in period $t$
$v_j(t)$	-	binary variable that specifies the status of units
$V_{total}$	-	the total available volume of gas.
$X_j^{on}(t)$	-	ON time of unit $j$ at time $t$
$X_j^{off}(t)$	-	OFF time of unit $j$ at time $t$

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## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Overview**

Economic growth in many countries increased the demand for electricity. Electrical energy is the main component to drive economic growth and enhanced social welfare in today's modernized world. Providing an adequate generation to feed the varying load has long been an obligation for utilities [1]. In this sense, sufficient generation for future demand needs to be planned together with strategies that one takes in order to manage current generating plants to supply the demand. The basic requirement for utilities, however, is to match the demand with enough generation in an economic, reliable and secure manner.

Optimal operation of the electric power system provides an efficient use of generation resources while all system constraints are honored. In this sense, reducing the production cost is the ultimate goal for the utilities. To the consumer, a reasonable electricity bill will be the consequence of such economic operation by the utility.

## 1.2 Optimal Power System Operation

Economical operation means the optimization of total production cost, incurred as a result of the depreciation of installed plants, maintenance costs, fuel costs and labor costs. Thus, the task of optimizing these costs involves long-term and short-term studies.

A long-term planning study considers, among many other facts, future power demand and location, load forecasting, maintenance scheduling, availability of fuel supplies, and replacement of ageing power stations. Short-term scheduling, on the other hand, deals with the commitment of enough units to meet a short-term load demand of a day or a week. The solution of long-term studies is normally coordinated with the short-term scheduling. For example, the long-term plan can provide the long-term fuel consumption of the system, or the component's maintenance schedule, which are accommodated into the short-term studies. Using results of the long-term planning, one can effectively manipulate the upper bound of the fuel limitation, or the unit's initial states, whether it is on maintenance or in service.

Unit commitment and scheduling, sometimes called pre-dispatch, is the operation that bridges the gap between load forecasting, maintenance scheduling, fuel planning and real time load dispatch. In scheduling process, the most economical combination of generating units, from those available for one week/day in advance, is selected to meet the forecasted demand [2].

The duration of the period of scheduling problems means that costs due to capital investment, interest charges, salaries, and maintenance cost can be considered to be fixed. The operational cost that is to be minimized in the generation scheduling is mainly due to fuel cost, losses in the transmission system and weekly or daily maintenance costs [3]. It is because these costs are changeable; therefore, they can be minimized through the optimization process (finding the best solution).

Hydro-plants are known for their minimal production costs and thus may become candidates for base generation operation. However, if the amount of water in

the upstream reservoir of a hydro plant is insufficient for full operation throughout the unit commitment period, it must be used as a “cycling unit” or load-following unit, which has a fast ramp rate. On the other hand, these units will be scheduled to startup/shutdown whenever appropriate, which is mostly occurred during the peak load where the operation cost of the system may be high [4].

For fossil fuel, older thermal plant units are characterized by low maximum output and high running cost due to mechanical ageing over the years [5]. In contrast, modern units can be characterized by high maximum output and high start-up cost. The variations in load demands mean that it will not be possible to operate all units at maximum output throughout the commitment schedule. This has a significant effect on the unit commitment problem since fossil fuel thermal plants, once shut down, can only be re-synchronized after a specified minimum period of shut down or up time have elapsed.

### **1.3 Generation Scheduling Problem**

Compared to other real-time operation problems such as Optimal Power Flow (OPF) or Economic Dispatch (ED), which are designed to adjust the output of the on-line (available) units during the operation, the UC problem is a more complex optimization problem [6]. It is because it involved two distinct set of variables; Integer and continuous variables [7]. Integer (binary) variables are used to address the status of a unit in certain time-instance, whereas the continuous variables point the output level of the unit if it is committed (the status is ON). However, this can introduce a huge number of variables, which is difficult to be solved by conventional techniques used to solve ED, or OPF problems.

Introduction of binary (0/1) variables into the UC model creates a nonconvex optimization problem [8]. Besides, the UC is inherently a nonlinear problem because the objective cost model is built based on nonlinear input-output functions of a unit. More, it is a high dimensional problem because it deals with a large number of units over an optimization period of daily (24-interval) or weekly (168-interval)

considering all the unit-wise constraints (ramping up/down, minimum ON/OFF) and system-wise constraints (coupling operational correlation between units) constraints.

Generation scheduling can be a very difficult problem to solve [1] due to the huge number of variable and constraints in the model. It involves  $M$  load patterns (24 for daily or 168 for weekly) to be modeled. For a practical system, it deals with a large number of units ( $N$ ) with all their operational limits. In this sense, a combination of the units has to match the load at each time-instance. The maximum number of combinations for each time instance is  $(2^N - 1)$ . Therefore, the total number of possible combination over a planning horizon of  $M$ , will be  $(2^N - 1)^M$ . For a system with 69 units over a 24-hour interval, this number can be as formidable as  $(2^{69} - 1)^{24} = 31.6 \times 10^{497}$ .

UC initially determines the start-up and shutdown schedules of all units in order to supply forecasted loads at the minimum costs, subject to satisfying all prevailing system constraints (unit-wise and system-wise) plus spinning reserve requirements. However, it is a critical process in power system operation as it is the first step towards meeting the forecasted demand with sufficient reserves [9]. In this sense, the variation in demand as well as system credible contingencies can be taken into account. Afterwards, all plant's operators need to know the schedule in advance in order to prepare the generators and manage their operation.

The generation schedules are calculated in a way to find the minimum possible operational cost in the system in order to meet hourly load demand while preserving all the system constraints. It is vital to all stakeholders, including the Grid System Operator (GSO), and consumers who end up paying the total cost. However, generation scheduling is a substantial scheduling process with tremendous money savings, if appropriately conducted.

Likewise, secure least-cost operation can be provided by adding line security constraints into the optimization problem. Important factors, such as production limits, ramping limits, and minimum up and down times, spinning reserve requirements, transmission losses that affect generation scheduling need to be considered in the UC.



Obtaining the minimum cost UC that meets all constraints within a minimum computational time has always been, and remains, an engineering challenge to date. Traditional methods were initially based on old-fashioned priority lists, which were computed from average marginal cost data. In the industry, the most widely used techniques are the Lagrangian Relaxation (LR) [10], Dynamic Programming (DP) [11] and Mixed-Integer Programming (MIP) [12, 13] methods, which have recently come into the practical use after significant algorithmic and executional improvements were introduced to MIP solvers [14].

The Unit Commitment problem is essentially a mixed integer problem because it involves shutdown (turn a unit OFF) and start-up (turn a unit ON) modes which are, in practice, represented by binary (0/1) variables. This, however, poses a challenge to solve the problem using conventional linear and nonlinear optimization techniques [15]. The first effort to solve this problem is naturally using the BB technique [12, 16] which is a mathematically accurate method. However, it still suffered from large computation time as the number of generating unit increases.

As an alternative to the BB technique, Dynamic programming (DP) was introduced to solve the UC problem. DP method decomposes problems into stages (time intervals) and traces the optimum solution by finding combination of the stages that solve the UC [17]. It also suffers from the curse of dimensionality, which leads the solution time to infinity if large scale system with a number of generating units was applied. Nonetheless, strategies have been devised to minimize the number of combinations at each stage so that DP can provide an accurate, solution of the problem that is faster than the BB technique.

Lagrangian relaxation (LR) is investigated in the quest to find a fast method that provides an accurate answer. In other words, LR is introduced as an alternative to BB and Dynamic programming technique. The LR is optimized by varying Lagrange multipliers of the objective function. There will be a gap between the primal and dual solution which cannot be closed because of the integer variables. While the LR does not suffer from dimensionality problem, unnecessary commitment of generating units may happen that is due to the enforcement of a number of heuristic manipulations required to meet duality gap (stopping parameter

in LR technique). In other words, Such approximations cause the unnecessary unit commitment and thus resulting in higher production costs for the system [18]. Furthermore, due to a number of approximations made in the LR, the method is fast but less accurate or suboptimal [19].

The MIP, LR and DP approaches essentially try to determine various commitment decisions, which are binary in nature, meaning that, whether a unit is to be on (status equals to 1) or is to be off (status equals to 0). The determination of the integer variable is subject to constraints such as minimum shutdown time or minimum uptime. For each of the combinations, the minimum cost of the combination needs to be determined so that the total generation satisfies the load and losses as well as unit and system constraints. The cost optimization of every integer variable combination is a continuous problem and in reality, is actually an economic dispatch problem, which can use conventional linear and nonlinear technique to solve the resultant problem.

Among the earliest mathematical method attempts to solve the UC is the Mixed Integer Programming (MIP) method, using Branch and Bound (BB) technique. Mathematically, it is proven that BB can find the global minimum cost. This is because it searches for all possible solutions. It is efficient because of the bounding procedure that truncates the search path, exceeding the bound [15, 20]. It means that the search space is minimized by rigorous mathematical reasons, unlike the truncated DP which truncates the search using heuristic reasoning [21].

The practical problems of using BB for unit commitment are primarily due to the curse of dimensionality, spending excessive time and memory to search over all the possible nodes in the branches of BB technique. Therefore, the programming strategy in BB is very challenging. DP also suffered from dimensionality problem but that is due to the large number of states as well the time-dependent and inter-temporal unit constraints. With arrival of complicating constraints, the DP may fail converging to the solution. In the BB method the solution can be eventually found but in a more time-consuming manner. One vital development in BB is the cutting plane technique, which reduces the search space (via rigorous mathematics) and hence increases the BB execution efficiency.

Another development is that the complexity in programming has been tackled commercially by a company (now owned by IBM) that developed a BB solver under the CPLEX<sup>®</sup> software framework. CPLEX<sup>®</sup> also has a solver for economic dispatch using Barrier method. A generalized solver developed by experts in the field relieves the burden of a power system analyst that solves a unit commitment problem, allowing them to concentrate and focus mainly on practical requirements and the correct modeling of the UC.

Research and development on BB using CPLEX<sup>®</sup> as a solver for the UC started in middle of 2000. Interestingly enough, another company called GUROBI<sup>®</sup> has produced a rival multi-thread BB code. CPLEX<sup>®</sup> has now evolved to be a multi-thread solver with dynamic search, which has made it computationally competitive and in fact comparable to LR. In fact, many large-scale UC problems, which cannot be solved by serial computation with a reasonable accuracy, can now be solved by using multi-thread computing.

The concept of the multi-thread computing implies that, one can achieve a computationally better (faster) execution if more computational resources can be involved with the algorithm. In this sense, based on the number of the computing cores existed in a computer, the algorithm can be concurrently run in a parallel fashion. As a result, the algorithm is handled using the maximum computational capacity of the machine. Nevertheless, each computing core has two physical threads, which indeed can further enhance the previous concurrent run-time, considering the waiting time between the parallel threads.

#### **1.4 Hydro-thermal Generation Scheduling**

The operation planning of hydro-thermal systems, usually called hydro-thermal scheduling (HTS). This problem is a more complex optimization model compared to the UC. In this sense, the HTS problem requires solving for thermal unit commitments and generation dispatch as well as the hydro schedules [22]. This coordination is necessary,

not only because of system constraints such as satisfaction of demand and reserve, but also because of plant operation characteristics, such as cascaded hydro plants. The operational interdependency of the reservoirs in the downstream to the released water from upstream reservoirs over a catchment (river scheme) introduces a big challenge in the HTS problem.

HTS differs from what is referred to as hydrothermal coordination (HTC). At HTC problem, the hydro generation, at the peak hours, is coordinated with thermal power to minimize the cost. In other words, adequate thermal units were committed and then hydro-generations are added to satisfy the upward load increase during the peak hours. Therefore, the HTC is the process of coordinating maximum hydro-power as a fixed amount of generation subtracted from the load for a certain time duration (peak hours). In contrast, the HTS problem is a complete unit commitment process for both hydro and thermal units, following the load demand at each interval. However, results for both models may become the same but the HTS problem is more efficient (the most profitable use of water resources) and better sounds the practical scheduling issues in this context.

In short, the specific features of cascaded hydro plants include i) spatial-temporal coupling among reservoirs and ii) for every plant, the nonlinear dependence between power output, water discharged, and head of the associated reservoir are precisely accounted for through a (0/1) mixed-integer linear formulation. Additionally, in order to solve the short-term HTS problem, the most accurate models can be implemented. The model will feature the hydro generation characteristic by well-described relationship between head of associated reservoir, water discharged, and the power generated. This is a nonlinear and nonconcave three-dimensional (3-D) relationship. To explain this, one can find that a concave function has a curve in which the slope of the curve decreases as the horizontal elements grow up. With this definition, if slopes of the curve are not steadily decreasing the curve is said to be nonconcave.

One of the major problems in solving MILP based HTS is the nonconcavity of each unit performance curve that made it difficult to be solved in Branch and Bound and Cutting plane technique (BB & C), resulting in a numerically unstable

solution. The proposed model represents the nonconcavities of each curve through additional binary variables. In this sense, the model ought to maintain the accuracy within an acceptable range while keeping the computational burden low. Finally, the (0/1) mixed-integer linear programming problem can be solved efficiently by the available BB & C solvers.

## 1.5 Security Constrained Generation Scheduling

The inclusion of transmission security in the UC problem can affect the solution process [23, 24]. The N-1 contingency analysis needs to be performed in order to add the transmission constraints to the UC problem. Transmission system constraints, however, pose a big challenge to the UC. In other words, the dimensionality of the constraint matrix can be highly increased, and the computational effort to solve the resultant problem becomes a challenging process. Nevertheless, most of the UC algorithms unable to address N-1 security constraints [25].

Bender's decomposition algorithm [26-28] is widely used to solve the security constraint UC (SCUC). It solves the original UC by relaxing the model into a master and the sub-problems. Afterwards, using Benders cuts [28], the security violation can be detected, added and eventually removed from the solution.

Using Bender's decomposition allows one to achieve the objective value in less execution time. At this point, no security violation is persisted in the solution by adding the required number of Bender's cuts whenever a violation is detected. In this sense, the imposition of security constraints can make the optimal solution even impossible to reach, as the constraint's matrix dimension is now larger.

To provide the transmission security margin through the generation scheduling process, one may alternatively outsource it from external software such as PowerWorld<sup>®</sup>. In this sense, the contingency analysis in PowerWorld<sup>®</sup> can be

integrated to the developed source files via relevant interface. Such interaction between PowerWorld<sup>®</sup> and the source files enables one to add the contingency based transmission security constraints to the model.

## 1.6 Thesis Objectives

The main aim of the thesis is to develop an efficient MILP based algorithm for HTS problem. Detailed objectives of the thesis can be stated as:

1. To develop a detailed hydraulic river (hydraulically coupled units) and reservoir systems modeling considering fixed (two-dimensional) and variable head variation (three-dimensional), minimum discharge, riparian constraint, and hydro unit-wise scheduling, non-concave performance curve linearization, head variation effects.
2. To model the actual utility data such as practical input-output curves, river system data modeling to be used in realistic hydrothermal scheduling.
3. To analyse the MILP solution in order to find an optimal point for BB & C method, which can results in cheaper solution while computational efforts maintained in a reasonable range. Improvements in computing time are studied using multi-threading technology.
4. To validate the algorithm performance based on the practical utility data as well as the large-scale test systems.

## 1.7 Scope of the work

1. The proposed algorithms have been coded using C/C++ programming with the INTEL<sup>®</sup> C++ compiler in Microsoft<sup>®</sup> Visual Studio IDE.
2. To solve large-scale linear programming problems, ILOG CPLEX<sup>®</sup> Barrier Optimizer, which is available as a callable C/C++ library is used.

3. To perform the security constraints, contingency analysis package available in PowerWorld<sup>®</sup> commercial software is integrated to the developed C/C++ source files.
4. BB & C technique is employed to perform the optimization process efficiently in the proposed MILP based HTS problem.
5. A realistic power system is chosen to validate the model in practice.

## 1.8 Thesis Contribution

The contributions of this thesis are listed as follows:

- Detailed modeling of river and reservoir systems considering head effects, minimum discharge limit, hydro unit-wise rather than plant-wise.
- Validation using practical hydrothermal system with comprehensive hydro model in every aspect such as reservoir, river network, nonlinear hydro unit performance curve, head variation effect, flow characteristics, riparian constraint, and limited water volume.
- Direct modeling of coupling constraints using the actual utility data. It is done without using any reasoning or heuristic technique which is an issue in the LR and DP methods.
- Data modeling of the practical utility for an actual hydrothermal test.
- Multi-thread parallel computing to enhance the computational performance of the MIP algorithm. Using multi-threading computations allows BB & C technique obtaining the solution in less computational efforts.
- Determining the optimal point for the BB & C method through the intersection point between two graphs of time-gap and cost-gap.

## 1.9 Thesis Outline

This thesis is organized into six chapters. The outline of the chapters is as follows.

Chapter 2 investigates the existing widely discussed algorithms in the field of UC, HTS. In this chapter, a compare-contrast study will run among the works done to date. The important findings from the previous works will be used as a guideline in this research.

Chapter 3 deals with the mathematical formulation of the developed *MILP*-HTS algorithm. The linearization and discretization strategies that were taken to facilitate the MILP based cascaded HTS solutions are highlighted in this chapter.

Chapter 4 discusses the proposed *MILP*-HTS methodology and implementation process to overcome the lacks of previous algorithms faced by utilities using a more sophisticated model. Moreover, it also covers the detailed explanation of the river system modeling in the HTS problem under the MILP algorithm.

Chapter 5 presents the results for the proposed *MILP*-HTS problem. A variety of test cases is used to verify the algorithm in different circumstances and constraints.

In Chapter 6, conclusions of the proposed *MILP*-HTS and recommendation for future works are pointed out.



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