A DIRECT PROBABILISTIC GLOBAL SEARCH METHOD FOR THE SOLUTION OF CONSTRAINED OPTIMAL CONTROL PROBLEMS

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To my family

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ABSTRACT

This research focuses on the development of a new direct stochastic algorithm to address the global optimization of the constrained optimal control problem where the interaction between state and control variables is governed by a system of ordinary differential equations. The objective of this method is to localize a globally optimal control curve in the feasible control space of the problem in such a way that the performance index attains its minimum value. The stochastic methodology is used on the development of the method. Thus, the resulting method is still effective when the complexity of the arising problems prohibits applying gradient-based methods. In this approach, the aforementioned control problem has first to be transformed into a nonlinear programming problem via a suitable discretization technique. The resulting problem is then solved using a stochastic method called Probabilistic Global Search Johor (PGSJ). The idea underpinning the PGSJ is to intelligently sample among potential solutions while no recombination or mutation operator is used. The sampling procedure is performed in accordance with some probability density functions (pdf) which are first initialized uniformly and then iteratively biased towards a globally optimal solution using the information obtained by evaluating the sampling points. After the PGSJ has been successfully implemented, it is found that it is able to arrive at an acceptable solution of the applied optimal control problems. The algorithm is also furnished with some theoretical supports verifying its convergence in probabilistic sense. In addition, some existing global stochastic methods which are based on using pdf are also applied on the optimal control problems where simulations reveal that the PGSJ method is superior to its competitors in terms of computation time and solution quality. These investigations lead to the extension of PGSJ into PGSJ-LS where LS indicates a line search operator added to the original method. These are then assessed and compared by applying them to a practical problem of controlling avian influenza H5N1 where it is verified that the PGSJ-LS performs slightly better than PGSJ.

ABSTRAK

Kajian ini memberi tumpuan kepada pembangunan algoritma langsung berstokastik baharu untuk menangani masalah pengoptimuman sejagat kawalan optimum berkekangan dengan interaksi di antara pembolehubah keadaan dan kawalan ditadbir oleh sistem persamaan terbitan biasa. Objektif kaedah ini adalah untuk membendung lengkung kawalan optimum sejagat dalam ruang kawalan tersaur sehinggakan suatu indeks prestasi mencapai nilai minimumnya. Metodologi stokastik digunakan pada pembangunan kaedah. Lantas kaedah terhasil masih berkesan walaupun kerumitan masalah yang timbul membataskan penggunaan kaedah berasaskan kecerunan. Dalam pendekatan ini, masalah kawalan tersebut perlu diubah terlebih dahulu menjadi masalah pengaturcaraan tak linear melalui teknik pendiskretan yang bersesuaian. Masalah terhasil kemudiannya diselesai menggunakan suatu kaedah stokastik dinamakan Kaedah Carian Sejagat Berkebarangkalian Johor (PGSJ). Idea asas kepada PGSJ ialah melakukan persampelan bijak di kalangan penyelesaian berpotensi dengan tiada operator penggabungan semula atau mutasi digunakan. Prosedur pensampelan dilakukan sejajar dengan beberapa fungsi kebarangkalian ketumpatan (pdf) yang diberi nilai awal secara seragam dan kemudiannya dicenderungkan secara lelaran ke arah penyelesaian optimum sejagat menggunakan maklumat yang diperoleh daripada penilaian titik pensampelan. PGSJ telah dilaksanakan dengan jayanya, didapati bahawa kaedah ini berkemampuan untuk menumpu kepada penyelesaian masalah kawalan optimum yang boleh diterima pakai. Algoritma ini juga dilengkapi dengan beberapa teori sokongan yang mengesahkan penumpuannya daripada aspek kebarangkalian. Di samping itu, beberapa kaedah stokastik sejagat sedia ada yang berasaskan fungsi ketumpatan kebarangkalian juga diguna pakai pada masalah kawalan optimum. Simulasi tersebut menunjukkan bahawa kaedah PGSJ adalah lebih baik berbanding pesaing-pesaing lain dari segi masa pengiraan dan kualiti penyelesaian. Kajian ini menjurus kepada perlanjutan kaedah PGSJ kepada kaedah PGSJ-LS dengan LS mewakili operator gelintaran garis yang telah ditambah kepada kaedah asal. Kaedah-kaedah ini ditaksir berbanding satu sama lain dengan mengguna kan masalah praktikal pengawalan selesema burung H5N1 di mana kajian ini mengesahkan bahawa kaedah PGSJ-LS berprestasi lebih baik daripada PGSJ.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

ACO	-	Ant Colony Optimization
ACK	-	Ackleys Problem
ARS	-	Adaptive Random Search
BB	-	Branch and Bound
BC	-	Bee Colony
BCP	-	Bernstein based control parameterization
CRS	-	Controlled Random Search
CM	-	Cosine Mixture Problem
EA	-	Evolutionary Algorithms
GA	-	Genetic Algorithm
GARS	-	Generalized Adaptive Random Search
GW	-	Griewank Problem
HAS	-	Hesitant Adaptive Searches
IDP	-	Iterative Dynamic Programming
IVP	-	Initial Value Problem
LHS	-	Latin Hypercube Sampling
LM1	-	Levy and Montalvo 1 Problem
LM2	-	Levy and Montalvo 2 Problem
LMM	-	Linear Multistep Methods
MS	-	Monkey Search
NLP	-	Nonlinear Programming Problem
NP	-	Nested Partitions
NLP	-	Nonlinear Programming Problem
OCP	-	Optimal Control Problem
pdf	-	probability density function
pdfs	-	probability density functions
PGSJ	-	Probabilistic Global Search Johor
PGSL	-	Probabilistic Global Search Lausanne

PRS	-	Pure Random Searches
PSO	-	Particle Swarm Optimization
SA	-	Simulated Annealing
SIN	-	Sinusoidal Problem
SIVP	-	Stiff Initial Value Problem
TS	-	Taylor Series

LIST OF SYMBOLS

A	-	The acceptable probability density
b	-	The number of bisecting procedure
d	-	Dimension of the problem
D	-	The box of feasible controls
f	-	The objective function,
H	-	Hamiltonian function
I_n^i	-	The ith interval in the nth iteration
I_n^{ij}	-	The jth subinterval of the ith interval in the nth iteration
M	-	Maximum number of iterations
N	-	The number of partitions on each interval
P	-	Probability of sampling from complementary search space
S	-	The number of samples in each iteration
t_0	-	Initial time
t_f	-	Final time
u	-	The control curve
u^*	-	The optimal control curve
x	-	The state variable
x^*	-	The optimal state variable
$\dot{x}$	-	The first derivative of the state variable
x_0	-	Initial value of the state variable
Ω	-	The box of feasible region
σ	-	The scale factor
ϵ	-	The accuracy required
ξ	-	Increment in probability updating procedure
κ	-	The maximum number of updating iterations
λ	-	Adjoint variable
μ	-	Multiplier variable
λ^*	-	The optimal adjoint variable

μ^*	-	The optimal multiplier variable
χ	-	The characteristic function
Ω^c	-	The Complementary search space
π	-	Projection function

LIST OF APPENDICES

LIST OF ALGORITHMS

CHAPTER 1

INTRODUCTION

1.1 Introduction

A set of interrelated objects when considered as a unit is usually recognized to be a system where the objects obey certain rule and regulations. Especially when a system is about artificial process, these regulations can often be mathematically modeled into an ordinary differential equation involving input and output variables. The input variables are usually collectively shown by u and also called control variables. These are actually the systems parameters that facilitate influencing the system to achieve desirable outputs. The output variables, in turn, are collectively represented by x and also called state variables. These variables are the system respond to a selected set of the inputs parameters.

In this case, a performance index in the form of a real valued function can be designed to measure the quality of control variables regarding to a standard desired for the behavior of the system. Therefore, the performance index is used to measure how the output of a system is close to the required standard when a control profile is considered. When the need for the best possible control variables to minimize or maximize a performance index arises, an Optimal Control Problem (OCP) occurs.

Possibly, this problem for the first time occurred in ancient civilization when human started to practice farming. As long as human perceived the cultivation system can be influenced, perhaps the first question was what strategy can optimize the food production process? Then, even in the early food producing era this problem might have been arisen and it was probably answered through trial and error. However, in the industrial applications the scale of the systems is often quite large and it is not affordable to search for the best control strategy through trial and error.

The first time this problem appeared in the open literature was when the Italian astronomer and mathematician Galileo Galilei (1564–1642) posed the brachistochrone problem in 1638 (Sargent, 2000). This problem is about a smooth wire and a bead that could slide along the wire under gravity assuming no friction. The objective of this problem is to attain the best possible shape of the wire such that the bead could traverse from one end-point to another in minimum time where the end-points are not located in a vertical line.

The solution of this problem which is a segment of a cycloid was unknown until Isaac Newton (1642–1727) and his contemporaries Leibniz, L'Hospital, Johann Bernoulli and his brother were challenged on this problem, and their efforts led to solution of this problem. When this solution was published, it stimulates the interest of researchers on this kind of problems leading to the initiation of the calculus of variations approach.

After a long period of evolution and investigation, this field of study evolved into the optimal control theory. Among greatest contributions on this area, in 1953 the American mathematician Richard Bellman (1920–1984) invented the Dynamic Programming method. Later in 1962 the Soviet mathematician Lev Pontryagin (1908–1988) laid the theoretical foundation of the minimum principal for this problem. These discoveries have been perceived as major breakthroughs in the last century.

Thereafter, many great mathematicians were interested to work on this problem leading to the establishment of this field into an active research area that attracted the interest of many researchers from many disciplines. However, the interest on this problem really flourished when the efficient and affordable computes become everywhere accessible, and the industrial dynamic optimization problems become more complex.

1.2 Motivations

As the technology being used in the industry is developing, the safety standards are strictly regulated and the demands for high quality products are continuously growing. These procedures are affecting the applied control problems to be more and more complicated. Therefore, in today's industrial applications sophisticated techniques have to be developed to synthesis the optimal control for arising problems.

Moreover, as no method can be applied for every problem, there always is the need for developing more efficient methods to accommodate arising more complex problems.

In addition, the investigation into OCP has been of importance not only for traditional application in aerospace (Krotov and Kurzhanski, 2005) and chemical process engineering (Ilse *et al.*, 2002), but also this class of problems has been recognized in diverse areas ranging from agriculture (Straten, 1999) to food production process (Banga *et al.*, 2003; Ouseguia *et al.*, 2012), leading to numerous significant studies focusing on different aspects of these problems either in computational or theoretical areas.

1.3 Background of the Problem

As the explicit use of analytical results is often prohibitive due to complexity and scale of applied problems, many numerical techniques have been proposed. These methods in attempt to find solution use techniques as diverse as classical variational methods to inspire heuristic approaches. The classical computation methods mostly apply either the necessary condition of optimality or direct techniques. The former approach, leads to a boundary problem, and the solution of this problem helps to obtain gradient information. The classical direct techniques (Polak, 1973) are also gradient based methods, and consequently these methods may converge to a local optimum.

On the other hand, the difficulty to achieve the optimal control for multimodal control problems is to localize a globally optimum control among many locally optimum controls. This class of problems have already been recognized as challenging problems in control system engineering (Grune and Junge, 2008), control of chemical process (Choi *et al.*, 1999; Ferrari *et al.*, 2010), control of electrical power systems (Cao *et al.*, 1998; Robandia *et al.*, 2001; Yan *et al.*, 2010; Amjady and Nasiri-Rad, 2010), food production process (Garcia *et al.*, 2006), water management problems (Moles *et al.*, 2003; Faria and Bagajewicz, 2011), agricultural management problems (Cruz *et al.*, 2003b), and other nonlinear and nonconvex control problems in science and engineering where the control space includes several or even just two locally optimum controls.

Unfortunately, even the most advanced recent gradient-based methods (Loxton *et al.*, 2009; Marzban and Razzaghi, 2010; Cimen, 2010) are not appropriate enough to contribute a reasonable solution to challenging control problems cited above. Thus, global optimization techniques should be studied and developed to surmount the nonconvex multimodal control problems.

1.4 Optimal Control Problem

This research focuses on Mayer problem of optimal control (Shapiro, 1966; Cesari, 1983; Lewis and Syrmos, 1995). We consider a general class of these problems where the system of dynamics is governed by an ordinary differential equation, and the objective is to find the best control curve to minimize a real valued performance index. The problem also involved inequality constraints, while the control variables are constrained by a hypercube subset of a finite Euclidean space. The problem is stated by,

$$\begin{aligned} \text{min} \quad & \varphi(x(t_f)) \\ \text{subject to} \quad & \dot{x}(t) = f(x(t), u(t), t) \\ & g(x(t), u(t), t) \leq 0 \\ & x(t_0) = x_0 \\ & x \in \mathbb{R}^n, \quad u \in D \subset \mathbb{R}^m \quad t_0 \leq t \leq t_f \end{aligned} \tag{1.1}$$

where φ is a real valued function on $\mathbb{R}^n$. The time interval is $[t_0, t_f]$. As mentioned earlier x and u indicate respectively state and control variables which are actually curves from time interval to respectively $\mathbb{R}^n$ and $D \subset \mathbb{R}^m$ where D is a box. $\dot{x}$ is the first derivative of the function x. f is a function from $\mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}$ to $\mathbb{R}^n$. The function g is from $\mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}$ to $\mathbb{R}^r$ and the inequality can be understood componentwise. Finally, $x_0 \in \mathbb{R}^n$ is a given initial state.

An illustrative example of this problem arises when studying tunnel-diode oscillator that is depicted in Figure 1.1. Considering this electric circuit where L denotes the inductility, C capacity, R resistance, and D indicates diode, and assuming the state variable $x(t)$ represents the electrical current related to this circuit at time t, and the control variable $u(t)$ denotes a suitable transformation of the voltage $V0(t)$ which is designed as a control function, the following Rayleigh equation can be derived (Maurer and Augustin, 2001),

$$\ddot{x}(t) = -x(t) + \dot{x}(t)(1.4 - p\dot{x}(t)^2) + 4u(t). \tag{1.2}$$

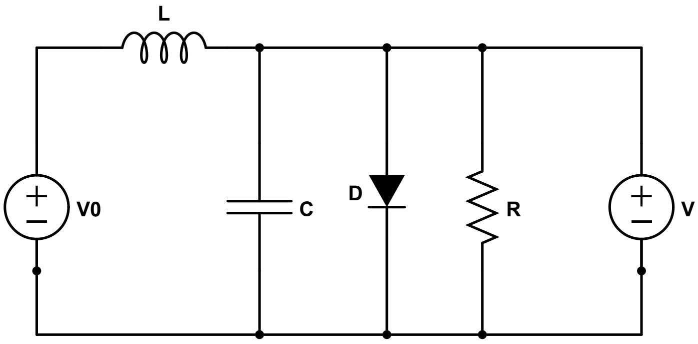

Figure 1.1: An illustration of a tunnel-diode oscillator

Additionally, assigning the value 0.14 to the parameter p in the Equation (1.2), considering the time interval to be $[0, 4.5]$, and defining the state variables $x_1(t) = x(t)$ and $x_2(t) = \dot{x}(t)$, $x_1(0) = -5$, $x_2(0) = -5$, then Equation (1.2) is turned into the following system,

$$\begin{cases} \dot{x}_1\ (t) = x_2(t) \\ \dot{x}_2\ (t) = -x_1(t) + x_2(t)(1.4 + 0.14x_2^2(t)) + 4u(t) \end{cases}$$

The above system of equations facilitates acquiring the value of electrical current functions corresponding to a given control function. In this case, the following performance index is designed to measure the quality of applied control inputs,

$$J = \int_0^{4.5} u(t)^2 + x_1^2(t)dt.$$

Introducing one more state variable, the above equation can be written as $J = x_3(4.5)$ where $x_3(t) = u(t)^2 + x_1^2(t)dt$ and $x_3(0) = 0$. In addition, the aforementioned variables have to be satisfied in,

$$u(t) + \frac{1}{6}x_1(t) \leq 0, \text{ for } t \in [0, 4.5].$$

Therefore, the optimal control problem of finding the best voltage $V0(t)$ which regulate the electrical current of the tunnel-diode oscillator circuit in such a way that it minimize the value of J can be stated as follows,

$$\begin{aligned} &\min x_3(4.5) \\ &\qquad \dot{x}_1\ (t) = x_2(t) \\ &\qquad \dot{x}_2\ (t) = -x_1(t) + x_2(t)(1.4 + 0.14x_2^2(t)) + 4u(t) \\ &\qquad \dot{x}_3\ (t) = u(t) + x_1^2(t) \\ &\qquad u(t) + \frac{1}{6}x_1(t) \leq 0 \end{aligned}$$

where $x_1(0) = -5$, $x_2(0) = -5$, $x_3(0) = 0$, and $0 \leq t \leq 4.5$. This optimal control problem is known as Rayleigh's problem (Loxton *et al.*, 2009).

1.5 Statement of the Problem

Although the form of control problem described in Problem (1.1) has long been under investigation, authors mostly contented themselves with weak forms of problem where some specific smoothness and convexity assumption satisfied. However, as discussed above, ill-behavior control problems, exhibiting attributes such as multimodality and non-smoothness are frequently arising in control system engineering. Consequently, investigation into these OCPs has been an absorbing topic in few last decades. Therefore, in this study we are aiming at expanding literature on this area by introducing a new probabilistic method for the solution of the problem mentioned above. Hence, this research addresses the development of a new stochastic search technique to be employed in achieving global optimal control.

1.6 Objectives of the Research

In order to obtain this goal, several objectives have to be pursued. These are as follow:

- To develop a new probabilistic global search technique based on probability density functions.
- To analyze and prove the convergence of the new algorithm in probabilistic sense.
- To propose a new discretization method based on control parametrization framework to convert OCP into Nonlinear Programming Problem (NLP).
- To carry out simulations to evaluate the effectiveness of the new algorithm on the solution of OCPs.
- To compare the efficiency of the new algorithm against some other existing global stochastic approaches which are based on probability density functions.

1.7 Scope of the Research

This research concentrates on the stochastic global optimization methods that use probability density functions to sample new potential solutions. The study also focuses on reducing the complexity and improving the reliability of these techniques especially when they are applied on the OCPs of Mayer type. Therefore, the research is focused on efforts towards investigations that result in enhancing the quality of optimal solution and enriching reliability of these stochastic techniques. Additionally, the deterministic global approaches are beyond the scope of this research.

1.8 Significance of the Study

The existing stochastic search techniques often need long time to achieve an enough precision and reliable solution for practical problems. On the other hand, a reasonable running time for control problems is usually of great importance. Therefore, if the control problem is nonconvex multimodal and the assumptions of other approaches fail to satisfy, where the stochastic techniques are the only alternative, the research on these techniques is highly significant.

1.9 Main Contributions

The significance of this research can be explained from the following aspects:

- A probabilistic algorithm is developed based on the exploitation of the probability density functions to efficiently direct the search efforts towards global optima. This algorithm called Probabilistic Global search Johor (PGSJ) as it shares some features with another algorithm called Probabilistic Global Search Lausanne (PGSL). These algorithms consist in four nested loops, where each loop invokes a special operator. However, the operators used in the PGSJ algorithm are different. Chapter 3 includes a detailed description of this algorithm.
- One important difference between PGSJ and PGSL is distinguished when it comes to guarantee the global convergence of these algorithms. The convergence of the PGSL has not been proved, while the convergence of the PGSJ algorithm

is theoretically proved in the probabilistic sense. The details of this theory is available in Chapter 3.

- The PGSJ algorithm can be applied on NLPs. In order to use this algorithm on the solution of OCP, a new efficient method for converting OCP into NLP is proposed. In this technique the Bernstein bases function is used to discretize a control problem in the control parameterization framework. Chapter 3 includes a description of this Bernstein based control parameterization (BCP).

- The PGSJ algorithm along with BCP method implemented using C++ programming language as well as MATLAB environments. Then, some case studies were simulated to study the effectiveness and efficiency of the PGSJ method on the solution of the OCPs. In addition, complexity analysis of this algorithm including time and memory complexity was carried out and it was revealed that this algorithm is an efficient optimization method with linear time and memory complexity.

- This is the first attempt that the continuous ant colony optimization methods are applied on the OCPs. The reason behind this study is to provide a methodology to evaluate the PGSJ algorithm against the resent popular continuous ant colony optimization methods that are based on probability density functions and claimed to be very efficient.

- The new stochastic method was compared against the algorithms that use a general probability density functions for means of sampling. This class of algorithms includes some popular continuous ant colony algorithms and the PGSL algorithm. All these algorithm coded in MATLAB, and then using the same environment and same set of benchmark problems, the algorithms compared based on the number of function evaluations they need to arrive at a globally optimum solution. The results of these procedures are available in Chapters 4 and 5.

- The investigations on the advantages and disadvantages of different stochastic methods evaluated in this research directed us to consider the possibility of improving the PGSJ algorithm by adding one mere local search operator to the original algorithm. At this aim, three line search operators were selected, and the behavior of the improved PGSJ which is called PGSJ-LS were compared against the original PGSJ algorithm while they are applied on some benchmark OCPs. The results of this comparison are discussed in Chapter 5 where these methods also applied on a practical OCP arises from combating avian influenza H5N1.

1.10 Thesis Overview

In previous sections a brief introduction on the OCP is given to motivate introducing the research objectives and the scope of this study. Subsequently, it is explained how significant it is to study the objectives outlined earlier. Following that an overview on the existing methodologies documented in the literature for the solution of an OCP is presented. This is followed by a description on the methodology used for the purpose of delivering the objectives of this research. These objectives are addressed in the subsequent chapters of this thesis.

As the focus of this study is on the direct optimization methods, in Chapter 3 a new discretization method is presented to competently convert an OCP into a NLP. In addition, an efficient probabilistic global optimization method is developed to address the resulting problem. Subsequently, the theoretical convergence of the newly developed algorithm is also provided to support the method.

In the following chapter some continuous ant colony optimizations are also applied on the control problem considered in this study. Then, in Chapter 5 these methods are compared against the new method. The result of comparisons, simulations as well as theoretical and numerical studies on this problem is finally summarized in Chapter 6.

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