# ANALYTICAL SOLUTIONS OF DISSIPATIVE HEAT TRANSFER ON THE PERISTALTIC FLOW OF NON-NEWTONIAN FLUIDS IN ASYMMETRIC CHANNELS

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To my parents (late)

~ Thank you for everything

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# ABSTRACT

Peristalsis is a natural mechanism responsible for the propulsion and the segmentation of biofluids in living structures, and this mechanism is important due to its efficient pumping characteristics. An essential feature of peristalsis is dissipation, thus dissipative heat transfer must be considered in the propulsion of biofluids. Most biofluids exist with different non-Newtonian fluid characteristics and experimental investigations reveal that the physiological structures are non-uniform with asymmetric peristaltic waves. This research focuses on the development of mathematical models which take into account the dissipative heat transfer on the peristaltic flow of non-Newtonian fluids. The non-Newtonian fluids include Walter's B, fourth grade and Sisko fluids and the flow have been considered in the horizontal and inclined asymmetric channels. Governing equations are first modeled in the laboratory frame and then transformed into the wave frame. Resulting equations are non-dimensionalized and the nonlinearity has been reduced by adopting the long wavelength and small Reynolds number approximations. Explicit forms of the analytical solutions have been obtained using the regular perturbation method. Influences of various parameters such as velocity slip parameter, Sisko fluid parameter, Brinkman, Eckert, Deborah, Soret and Schmidt numbers on the flow quantities namely velocity, shear stress, pumping, trapping, temperature, concentration and heat transfer coefficients have been investigated. Results show that pumping, trapping and temperature are reduced for increasing velocity slip parameter. Temperature and heat transfer coefficients are increased with the increase of Brinkman, Eckert and Deborah numbers. Concentration decreases with the increase of Brinkman, Soret and Schmidt numbers. Comparative study amongst viscous, shear thinning and shear thickening fluids has also been presented.

# ABSTRAK

Peristalsis adalah mekanisme semula jadi yang bertanggungjawab bagi pendorongan dan pensegmenan biobendalir dalam struktur hidup, dan mekanisme ini adalah penting kerana ciri pengepamannya yang cekap. Suatu ciri utama dalam peristalsis adalah pelesapan, maka pemindahan haba lesapan mesti dipertimbangkan untuk pendorongan biobendalir. Kebanyakan biobendalir wujud dengan pelbagai ciri bendalir bukan Newtonan dan kajian secara eksperimen mendedahkan bahawa struktur fisiologi adalah tidak seragam dengan gelombang peristalsis tak simetri. Kajian ini memberi tumpuan kepada pembangunan model matematik dengan mengambil kira pemindahan haba lesapan pada aliran peristalsis bagi bendalir bukan Newtonan. Bendalir bukan Newtonan termasuklah bendalir Walter B, bendalir gred keempat dan bendalir Sisko, manakala aliran telah dipertimbangkan dalam saluran tak simetri mendatar dan saluran condong. Persamaan menakluk asalnya dimodelkan dalam kerangka makmal, diubah kepada kerangka gelombang. Persamaan yang terhasil adalah persamaan tanpa dimensi dan ketidaklinearan diturunkan menjadi linear dengan mengadaptasi penghampiran gelombang panjang dan nombor Reynolds yang kecil. Bentuk tak tersirat bagi penyelesaian analisis telah diperoleh dengan menggunakan kaedah usikan biasa. Pengaruh pelbagai parameter seperti parameter halaju gelinciran, parameter bendalir Sisko, nombor Brinkman, nombor Eckert, nombor Deborah, nombor Soret dan nombor Schmidt terhadap kuantiti aliran seperti halaju, tegasan ricih, keupayaan mengepam, keupayaan memerangkap, suhu, kepekatan dan pekali pemindahan haba telah dikaji. Hasil kajian menunjukkan bahawa keupayaan mengepam, keupayaan memerangkap dan suhu berkurangan dengan peningkatkan parameter halaju gelinciran**.** Suhu dan pekali pemindahan haba meningkat dengan peningkatan nombor Brinkman, nombor Eckert dan nombor Deborah. Kepekatan menurun dengan peningkatan nombor Brinkman, nombor Soret dan nombor Schmidt. Kajian perbandingan antara bendalir likat dengan bendalir penipisan ricih dan bendalir penebalan ricih juga dibentangkan.

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# LIST OF SYMBOLS

## Roman Letters

| | | |
|---|---|---|
| $a$ | - | Amplitude ratio at upper wall |
| $a_1$ | - | Wave amplitude at upper wall |
| $a_2$ | - | Wave amplitude at lower wall |
| $b$ | - | Amplitude ratio at lower wall |
| $\mathbf{b}$ | - | Body force per unit volume |
| $\mathbf{B}_0$ | - | Uniform applied magnetic field |
| $Br$ | - | Brinkman number |
| $c$ | - | Wave speed |
| $C$ | - | Fluid concentration |
| $C_0$ | - | Concentration at upper wall |
| $C_1$ | - | Concentration at lower wall |
| $d$ | - | Channel width ratio |
| $d_1$ | - | Upper channel width |
| $d_2$ | - | Lower channel width |
| $D$ | - | Coefficient of mass diffusivity |
| $\frac{D}{D\bar{t}}$ | - | Substantial derivative |
| $\frac{dp}{dx}$ | - | Axial pressure gradient |
| $\mathbf{e}$ | - | Rate of strain tensor |
| $Er$ | - | Eckert number |
| $F$ | - | Dimensionless time mean flow rate in wave frame |
| $Fr$ | - | Froude number |
| $\mathbf{g}$ | - | Acceleration due to gravity |

| | | |
|---|---|---|
| $\bar{H}_1$ | - | Shape of upper wall in laboratory frame |
| $\bar{H}_2$ | - | Shape of lower wall in laboratory frame |
| $\bar{h}_1$ | - | Shape of upper wall in wave frame |
| $\bar{h}_2$ | - | Shape of lower wall in wave frame |
| $h_1$ | - | Dimensionless shape of upper wall |
| $h_2$ | - | Dimensionless shape of lower wall |
| $\mathbf{I}$ | - | Identity tensor |
| $k$ | - | Thermal conductivity |
| $K$ | - | Dimensionless permeability parameter |
| $K_T$ | - | Thermal diffusion ratio |
| $k_0$ | - | Short memory coefficient |
| $M$ | - | Hartmann number |
| $\bar{P}$ | - | Pressure in laboratory frame |
| $\bar{p}$ | - | Pressure in wave frame |
| $p$ | - | Dimensionless pressure |
| $\mathrm{Pr}$ | - | Prandtl number |
| $\mathrm{Re}$ | - | Reynolds number |
| $\bar{\mathbf{S}}$ | - | Extra stress tensor |
| $Sr$ | - | Soret number |
| $Sc$ | - | Schmidt number |
| $\bar{t}$ | - | Time |
| $T$ | - | Fluid temperature |
| $\bar{\mathbf{T}}$ | - | Cauchy stress tensor |
| $T_m$ | - | Mean temperature |
| $T_0$ | - | Temperature at upper wall |
| $T_1$ | - | Temperature at lower wall |
| $\bar{U}$ | - | Axial velocity component in laboratory frame |
| $\bar{u}$ | - | Axial velocity component in wave frame |
| $u$ | - | Dimensionless axial velocity component |
| $\mathbf{V}$ | - | Fluid velocity |
| $\bar{V}$ | - | Transverse velocity component in laboratory frame |

| | | |
|---|---|---|
| $\bar{v}$ | - | Transverse velocity component in wave frame |
| $v$ | - | Dimensionless transverse velocity component |
| $Z_{h_1}$ | - | Heat transfer coefficient at upper wall |
| $Z_{h_2}$ | - | Heat transfer coefficient at lower wall |

**Greek Letters**

| | | |
|---|---|---|
| $\alpha$ | - | Channel inclination |
| $\beta$ | - | Velocity slip parameter |
| $\chi$ | - | Electrical conductivity |
| $\delta$ | - | Wave number |
| $\phi$ | - | Phase difference |
| $\varphi$ | - | Dimensionless concentration |
| $\gamma$ | - | Thermal slip parameter |
| $\eta$ | - | Dimensionless temperature |
| $\eta_0$ | - | Limiting viscosity at small shear rates |
| $\kappa$ | - | Viscoelastic parameter |
| $\lambda$ | - | Wave length |
| $F_{\lambda 1}$ | - | Frictional force at upper wall |
| $F_{\lambda 2}$ | - | Frictional force at lower wall |
| $\mu$ | - | Dynamic viscosity |
| $\theta$ | - | Dimensionless time mean flow rate in laboratory frame |
| $\rho$ | - | Fluid density |
| $\sigma$ | - | Concentration slip parameter |
| $\zeta$ | - | Magnetic field inclination |
| $\psi$ | - | Stream function |
| $\xi$ | - | Specific heat at constant volume |
| $\Gamma$ | - | Deborah number |
| $\Delta p_{\lambda}$ | - | Pressure rise per wavelength |

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# CHAPTER 1

# INTRODUCTION

## 1.1 Research Background

The preliminary details pertaining to various important phenomena involved in the present research and motivations have been presented in this section. Following subsections give general introduction by presenting the details on peristalsis, heat and mass transfer phenomena, the non-Newtonian fluids and the slip condition.

### 1.1.1 Peristalsis

The transportation of biofluids inside the living body is carried out with the help of mass movements. These mass movements are accountable for biofluid transport in the gastrointestinal tract (GIT) (Guyton (1986)) as well as in the reproductory organs (Vries *et al.* (1990) and Chalubinski *et al.* (1993)). In GIT, medical physiologists (Guyton (1986)) divide these movements into two classes voluntary (conscious) and involuntary (unconscious) movements. Further, the involuntary movements are categorized into two types; the propulsive peristaltic movements and the segmentation movements; responsible for the propulsion and the mechanical breakdown of the food, respectively. The GIT and reproductory (myometrium) tracts are composed of different interlacing muscular layers out of which two are important; the longitudinal muscle layer and the circular muscle layer (Ivy (1942)) (Figure 1.1). The propulsive peristaltic movements are initiated by the irritation of the myenteric nerve plexus and controlled by the longitudinal muscle layer while segmentation movements are initiated by the irritation of the meissner nerve plexus and controlled by the circular muscle layer (Figure 1.2). So peristalsis consists of contraction and expansion of tract performing the progressive waves which propel the contents forward along the tract (Latham (1966)). The stimulus for these waves is the distension of the tract with fluid material such as food, blood, secretions from glands, urine, embryo and others. This distension of tract at any cross section irritates the inner layer of the tract (mucosa in case of gastrointestinal tract), at the same time the nerve plexus (network of intersecting nerves) connected with the central nervous system via fibres initiates the peristaltic waves along the walls of the tract. This mechanism regulates the flow from the area of lower pressure to area of higher pressure.

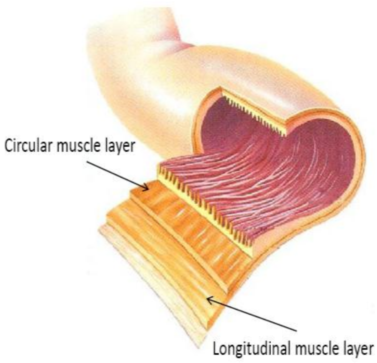


**Figure 1.1** Structure of GIT

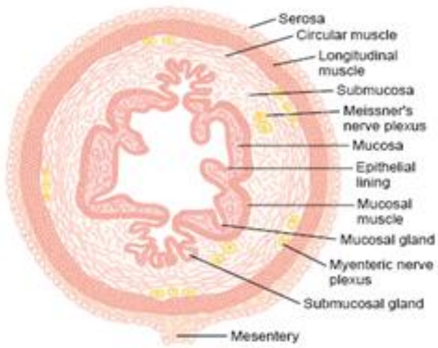


**Figure 1.2** Cross section of GIT

Peristalsis has its immense applications in medical physiology as well as in industry. In medical physiology, it is involved in the motion of food material in the GIT. For instance, in propulsion of food bolus in the oesophagus, conversion of food bolus into chyme in the stomach and movement of chyme in the intestines (Guyton (1986)). In reproductive parts, it helps in the transportation of spermatozoa in the ductus efferentes of the male reproductive tract and in the curvical canal, in the embryo implantation in the fundus site of female uterus and in the movement of ovum in the fallopin tubes of the female reproductive parts. It also helps in the vosomotion of small blood vessels as well as blood flow in arteries, transport of urine from kidney to ureter and movement of secretions in glandular ducts (bile and saliva). Some worms also moves with the help of peristalsis. Nowadays, peristalsis has exploited its significance in industry, like in sanitary fluid transport, artificial blood pumps in heart-lung machine and transport of corrosive fluids where the contact of fluid with the boundary is prohibited.

### 1.1.2 Heat and Mass Transfer

Heat transfer analysis is prevalent in the study of peristaltic flows due to its large number of applications in processes like hemodialysis (method used for removing waste products from blood in the case of renal failure of kidney) and oxygenation. Bioheat is currently considered as heat transfer in the human body. In view of thermotherapy (application of heat to the body for treatment, examples pain relief, increase of blood flow and others) and human thermoregulation system (ability of living body to maintain body temperature with in certain limits in case of surrounding temperature variations) as mentioned by Srinivas and Kothandapani (2008), bioheat transfer has attracted many biomedical experts. Heat transfer analysis is important especially in case of non-Newtonian peristaltic rheology as there involves many intricate processes like heat conduction in tissues, heat transfer during perfusion (process of delivery of blood to capillary bed) of arterial-venous blood, metabolic heat generation and heat transfer due to some external interactions like mobile phones and radioactive treatments. It is also helpful in the treatment of diseases like removal of undesirable tissues in cancer.

Dissipative heat transfer is the most important and essential feature of peristaltic flows as suggested by Shapiro *et al.* (1969). In peristaltic flows when the fluid is forced to flow due to the sinusoidal displacements of the tract boundaries, the fluid gains some velocity as well as kinetic energy. The viscosity of the fluid takes that kinetic energy and converts it into internal or thermal energy of the fluid. Consequently, the fluid is heated up and heat transfer occurs. This phenomenon is modelled by the energy equation with dissipation effects. For two dimensional flows the energy equation reduces to a second order partial differential equation that is parabolic in nature. The mode of dissipative heat transfer is studied in the present research.

Moreover, due to the intricate nature of the bio-fluid dynamics, both heat and mass transfer occur simultaneously giving complex relations between fluxes and driving potentials as debated by Srinivas and Kothandapani (2009) and Eckert and Drake (1972). The mass flux caused by the temperature gradient called Soret effect or thermal-diffusion discussed by Alam *et al.* (2006) is often negligible in heat and mass transfer processes due to its small order of magnitude. However, for the non-Newtonian fluids with light or medium molecular weight, it is not appropriate to neglect Soret effects as studied by Dursunkaya and Worek (1992) and Postelnicu (2004). Therefore, in the present study, due attention has been given to the combined effects of heat and mass transfer with Soret effect. Further, in the present analysis Dufour effect (energy flux caused by the composition gradients) has been neglected, which is one of the limitation of the present research.

### 1.1.3 Non-Newtonian fluids

In non-Newtonian fluids, the shear stress may depend upon the shear rate. Both shear stress and shear rate may be time dependent and the fluid may have viscous as well as elastic characteristics (Sajid (2006), Khan (2008)). Because of the different rheological properties of non-Newtonian fluids, there exists no single universal constitutive relationship between stress and rate of strain by which all the non-Newtonian fluids can be examined. Therefore, several models of non-Newtonian fluids have been suggested and considered. The complexity in non-Newtonian fluids starts due to the non-linear terms appearing in their constitutive relationships. Several researchers considered various models under different approximations and geometries by assuming the fluid content as a Newtonian fluid which is suitable in some particular cases like urine transport. However, most of the biological and industrial fluids are constituted of Newtonian and non-Newtonian fluids behaving collectively as a non-Newtonian mixture (Joseph (1980)). The examples of non-Newtonian fluids includes semi-solid food called bolus in oesophagus (Guyton (1986)), semi-liquid food (chyme) in stomach and intestines, blood in arteries or veins, cervical mucus in bones and semen and ovum in reproductory tracts. Whereas in case of industrial fluids, waste inside the sanitary ducts, toxic materials, metal alloys, oil and grease in automobiles or machines, nuclear slurries inside the nuclear reactors and many others.

To investigate the non-Newtonian characteristics of the physiological fluids, different non-Newtonian fluids namely Walter's B, fourth grade and Sisko fluids have been considered in the present research. Walter's B fluid (Beard and Walters (1964)) is a viscoelastic fluid model defines both viscous as well as elastic characteristics. Physically it describes the elastic nature of the physiological fluids. Walter's B fluid model has been widely studied by various researchers through different configurations. The details can be seen in the studies of Ariel (1992), Baris (2002a), Baris (2002b), Joneidi *et al.* (2010), Mohiddin *et al.* (2010) and Nandeppanavar *et al.* (2010). Some recent investigations on the peristaltic flow of Walter's B fluid in inclined tube and endoscope have been conducted by Nadeem and Akbar (2010a) and Nadeem *et al.* (2011a).

Differential type fluids (Rajagopal (1982), Rajagopal *et al.* (1986) and Dunn and Rajagopal (1995)) are considered to be the best fluid models that capture most of the non-Newtonian characteristics. Fourth grade fluid is the most general subclass of the differential type fluids. This model is capable of describing normal stress effects, shear thinning effects and shear thickening effects. For the flow of fourth grade fluid, the reader may refer to the studies of Hayat *et al.* (2002a), Wang and Hayat (2004), Hayat and Wang (2003), Hayat *et al.* (2005), Kaloni and Siddiqui (1987) and Erdogan (1981). Studies on the peristaltic flow of fourth grade fluid include the work of Haroun (2007b), Hayat *et al.* (2007b), Hayat and Noreen (2010) and Hayat *et al.* (2010).

Sisko fluid model (Sisko (1958)) is important because for different values of material parameter power law index, it describes three different kinds of fluids namely: shear thinning, viscous and shear thickening fluids. Physically it describes the shear thinning and shear thickening characteristics of the physiological fluids. On the flow of Sisko fluid, the studies includes the works of Khan *et al.* (2010a), Khan *et al.* (2010b), Khan *et al.* (2011), Molati *et al.* (2009), Sajid and Hayat (2008), and Akyildiz *et al.* (2009) and for peristaltic flows, recent investigations includes the studies of Wang *et al.* (2008), Nadeem and Akbar (2010b) and Nadeem *et al.* (2011b).

### 1.1.4 Slip Condition

On the interaction of a fluid with the solid surface, the conditions where the molecules of the fluid near to the surface stick with the surface having the same velocity, is called no-slip condition. While in the case of many polymeric liquids with high molecular weight, the molecules near to the surface show slip or stick-slip on the surface. To tackle this problem, Navier (1823) suggested the general slip boundary condition defining that the difference of fluid velocity and the velocity of the surface is proportional to the shear stress at that surface. The coefficient of proportionality is the slip parameter having the dimension of length. The slip condition is of great importance especially when fluids with non-Newtonian or elastic characters are considered. In such cases, the slippage may occur under a large tangential traction. Both no slip and slip boundary conditions have been considered in the present research.

Motivated by the facts discussed above, the present research considers mode of dissipative heat transfer on the peristaltic flow subject to both no-slip and slip boundary conditions with mass transfer. Different types of fluids considered are viscous, Walter's B, fourth grade and Sisko fluids. Flow is considered through asymmetric and inclined asymmetric channels. Series solutions are obtained by employing the regular perturbation method. Long wavelength and small Reynolds number approximations are taken into account for the linearization of the governing equations. Present research has been divided into four main problems presented in Chapters 3 to 6. These problems are novel, have not yet been considered as noticed in detailed literature review presented in Chapter 2. Further, the problem statement and the research objectives explicitly describe the contributions of the present research. The following Section 1.2 presents the problem statement. Further, Section 1.3 describes the objectives of the research and Section 1.4 presents the scope of the research. Moreover, significance of the research is given in Section 1.5 and research methodology is explained in Section 1.6. Finally, Section 1.7 provides details the on dimensionless parameters and Section 1.8 describes the outlines of the thesis.

## 1.2 Problem Statement

In their classical study, Shapiro *et al.* (1969) suggested that dissipation is an essential feature of peristalsis. In peristaltic flows, the kinetic energy induced by the fluid motion is transformed to the thermal energy by the viscosity of the fluid. Consequently, heat transfer occurs through the mode of dissipation. This important mode of dissipative heat transfer in the peristaltic flows leads us to the following research questions:

What are the effects of dissipative heat transfer on the magnetohydrodynamic peristaltic flow of viscous fluid through a porous asymmetric channel in presence of slip condition and mass transfer? What are the effects of dissipative heat transfer on the peristaltic flow of Walter's B fluid in an asymmetric channel with mass transfer? What is the influence of dissipative heat transfer on magnetohydrodynamic peristaltic flow of fourth grade fluid in an inclined asymmetric channel subject to slip conditions? What are the simultaneous effects of slip and dissipative heat transfer on the peristaltic flow of Sisko fluid in an asymmetric channel?

## 1.3 Objectives of the Research

This research aims to investigate the dissipative heat transfer analysis in the peristaltic flow. It constitutes the development of mathematical models, solution of the governing equations and analysis of the influences of various pertinent parameters on the considered flow problems. The main objectives of this study are:

1. To investigate the slip effect on dissipative heat and mass transfer of magnetohydrodynamic peristaltic flow of viscous fluid in a porous asymmetric channel.

2. To investigate the dissipative heat and mass transfer on the peristaltic flow of a Walter's B fluid through an asymmetric channel.

3. To investigate the influence of slip and dissipative heat transfer on the magnetohydrodynamic peristaltic flow of a fourth grade fluid in an inclined asymmetric channel.

4. To investigate the effect of dissipative heat transfer on peristaltic flow of a Sisko fluid in an asymmetric channel subject to slip conditions.

## 1.4 Scope of the Research

This study provides the dissipative heat transfer analysis in the peristaltic transport. Slip effects and chemical reaction are also taken into account. The fluid models chosen for the study are viscous fluid, Walter's B fluid, fourth grade fluid and Sisko fluid models. The flow is assumed to be unsteady, two dimensional and isochoric. The configurations that have been considered in this research are asymmetric channel and inclined asymmetric channel. Series solutions of the resulting equations are obtained by the regular perturbation method. The wavelength is taken long and the Reynolds number is assumed to be small.

## 1.5 Significance of the Research

This study significantly provides the profound understanding of dissipative heat transfer analysis on peristaltic flow of different types of fluid in various configurations in the living body. Particularly, asymmetric channel represents the sagittal cross section of the non-pregnant uterus where the flow is induced by the symmetric or asymmetric myometrial peristaltic contractions as studied by Eytan and Elad (1999) and Eytan *et al.* (1999). Whereas, inclined asymmetric channel represents the inclined geometry of the stomach. The viscoelastic nature of chyme or intrauterine fluid with embryo is characterized by the Walter's B fluid (Usha and Rao (1995) and Nadeem and Akbar (2010a)). Shear thinning or thickening effects of the biological fluid are modeled by the fourth grade fluid (Haroun (2007b)) and Sisko fluid whereas fourth grade fluid also exhibits the normal stress effects. Heat transfer analysis is important because of the heat generation during the metabolic processes and the mechanical breakdown of the food in the stomach and intestines. Mass transfer is taken into account because of its importance in the chemical breakdown of the food and chemical reactions due to the amalgamation of gastric juices with the food. Due to the non-Newtonian nature of biological fluids the slip effects become essential. Regular perturbation method is the best method which provides the convergent analytical solutions of highly nonlinear problems. So, the solutions obtained in this research can be used for comparison as well as for validation of numerical simulations in future work. This study will be helpful in understanding the biological fluid transport and contribute to the advancement of medical science.

## 1.6 Research Methodology

This section describes the methodology adopted for the solutions of the considered research problems. The step wise procedures along with the details on the regular perturbation method have been explained in the following subsections.

### 1.6.1 Problem Formulation in Laboratory Frame

For the considered flow problems, the mathematical formulation is carried out in the laboratory frame of reference (fixed frame). Since the laboratory frame is fixed, the flow there is unsteady. The mathematical model constitutes of partial differential equations along with the corresponding boundary conditions and wall geometries.

### 1.6.2 Laboratory Frame into Wave Frame Transformations

A wave frame is introduced which is moving forward with the constant wave speed *c* in the direction of wave propagation. Coordinates along with the flow quantities are related by defining the laboratory frame to wave frame transformations. Using the transformations, the governing equations, boundary conditions and wall geometries are transformed from laboratory frame into the wave frame. The boundaries in the moving wave frame appear to be stationary and the flow becomes steady. Consequently in the wave frame, the governing equations, boundary conditions and the wall geometries becomes time independent.

### 1.6.3 Non-dimensionalization

In order to carry out the non-dimensional analysis, in each problem, dimensionless variables have been introduced. In the wave frame, the resulting time independent partial differential equations, boundary conditions and wall geometries are derived into dimensionless forms using these dimensionless relations.

### 1.6.4 Stream Function

We introduce the stream function related with the velocity components in the present analysis. Invoking these relations in our mathematical formulation, we write the governing equations and the boundary conditions in terms of stream function. Further, eliminating pressure from the axial and transverse components of momentum equation we obtain the vorticity transport equation. The solution of vorticity transport equation gives the expressions for stream function in the explicit forms. Using the explicit solutions of stream function, we obtain the solutions for other flow quantities.

### 1.6.5 Linearization

Governing equations in terms of stream functions are highly nonlinear and coupled. The closed from solutions for these equations seem impossible to obtain. Evidently, long wave length approximation is appropriate and applicable in the peristaltic flows as mentioned by Barton and Raynor (1968), Radhakrishnamacharya (1982), Zien and Ostrach (1970) and Jaffrin and Shapiro (1971). Peristaltic waves propagate with long wavelengths along the boundaries of tracts having small diameter or widths (Shapiro *et al.* (1969)). In the assumption of long wavelength, the ratio of channel width to wavelength becomes very small of negligible order. Physically, the transverse flow quantities become small and thus negligible as compared to the flow quantities in longitudinal directions. Further, peristalsis acts as a pump providing pressure rise in the flow direction. In such a case, the inertial effects are smaller as compared to the viscous effects (Shapiro *et al.* (1969)). This assumption results in the small Reynolds number. These assumptions simplify the nonlinearity of the governing equations and boundary conditions. Consequently, the highly nonlinear governing equations along with boundary conditions are partially linearized under the long wavelength and small Reynolds number approximations.

### 1.6.6 Regular Perturbation Method

In the past, several methods have been employed for the investigation of peristaltic flows under various assumptions and approximations. For the viscous fluid, the analytical solutions for the flow problems have been obtained by Barton and Raynor (1968), Fung and Yih (1968), Jaffrin (1973), Mishra and Rao (2003) and Shapiro *et al.* (1969). Also, several researchers (Brown and Hung (1977), Takabatake and Ayukawa (1982), Takabatake *et al.* (1988), Pozridikis (1987), Ratishkumar and

Naidu (1995) and Xiao and Damodaran (2002)) have numerically investigated the peristaltic flow of viscous fluid through different geometries. Regular perturbation method about a small amplitude ratio was employed by Yin and Fung (1969) in their investigation of peristaltic flow of viscous fluid in a cylindrical tube. Further, Li (1970) and Mekheimer *et al.* (1998) have presented perturbation solutions about small wave number in their studies.

Later, regular perturbation method has been widely used in various studies for the viscous fluid conducted by Elnaby *et al.* (2003), Ali *et al.* (2008), Elnaby and Haroun (2008), Muthuraj and Srinivas (2010a), Radhakrishnamacharya and Srinivasulu (2007), Srinivas *et al.* (2011), Srinivas and Muthuraj (2011), Srinivas *et al.* (2012a), Srinivas *et al.* (2012b) and Vajravelu *et al.* (2007). In the peristaltic flow of non-Newtonian fluids, the constitutive relationships bring nonlinearity in the governing equations. Regular perturbation method is advised appropriate and efficient for such nonlinear systems (Dyke (1975) and Bush (1992)). In the present era, this method is widely used in the peristaltic flow of non-Newtonian fluids. For details, the reader may refer to the references (Siddiqui and Schwarz (1993), Siddiqui and Schwarz (1994), Siddiqui *et al.* (1991), Elshehawey and Mekheimer (1994), Elmaboud and Mekheimer (2011), Hayat *et al.* (2002b), Haroun (2007a), Elshahed and Haroun (2005) and Vajravelu *et al.* (2011)).

In the resent research, close form solutions of the highly nonlinear systems have been obtained by using the regular perturbation method. For perturbations solutions, we express the flow quantities in terms of small perturbation parameter as

$$\text{Flow quantity} = \sum_{i=0}^{\infty}\left(\text{Flow quantity}\right)_i \cdot \left(\text{Small parameter}\right)^i, \qquad (1.1)$$

and invoke these expansions into the governing equations and boundary conditions. The terms of indefinitely higher order become smaller and smaller as involving the ascending powers of small parameter (Dyke (1975)) giving the convergent regular perturbation solutions for flow quantities as debated by Hinch (1991), Holmes (1995) and Bush (1992). The leading term is roughly correct and further terms are corrections of decreasing size (Dyke (1975)). On comparing the coefficients of like powers of small parameter, the corresponding systems of zeroth and first order are obtained. First the zeroth order system is solved and the solutions are obtained for the flow quantities. Using these zeroth order solutions into the first order system, the solutions are obtained for the flow quantities up to the first order of small perturbation parameter. For the calculation of flow quantities namely pressure rise and frictional forces, numerical integration has also been performed.

## 1.7 Dimensionless Parameters

The dimensionless parameters appearing in the present research are amplitude ratio, channel width ratio, phase difference, Hartmann number, permeability parameter, velocity slip parameter, thermal slip parameter, concentration slip parameter, Reynolds number, wave number, viscoelastic parameter, Deborah number, power law index, Sisko fluid parameter, Brinkman number, Prandtl number, Eckert number, Schmidt number and Soret number. Particularly, the amplitude ratio defines the ratio of wave amplitude to the upper channel width, channel width ratio defines the ratio of lower channel width to upper channel width, phase difference gives the phase angle between waves at upper and lower walls, Hartmann number defines the ratio of magnetic forces to viscous forces, permeability parameter gives the permeability of porous medium. Further, velocity, thermal and concentration slip parameters defines the ratio of slip length to upper channel width, respectively. Reynolds number gives ratio of inertial forces to viscous forces. Wave number gives ratio of upper channel width to the wavelength.

Further, viscoelastic parameter characterizes the non-Newtonian viscoelastic effects defining the ratio of elastic forces to viscous forces. Deborah number represents the non-Newtonian fourth grade effects defining the ratio of characteristics time of fluid to the characteristics time of the flow. Different values of power law index gives different fluids namely: shear thinning, viscous and shear thickening fluids. Sisko fluid parameter represents the non-Newtonian characteristics of Sisko fluid. Brinkman number defines the ratio of viscous dissipation to heat transfer rate. Prandtl number gives the ratio of momentum diffusivity to the thermal diffusivity. Eckert number characterizes the dissipation and represents the ratio of kinetic energy to the enthalpy (where enthalpy is defined as the sum of internal energy and product of pressure and volume). Schmidt number consists of the ratio of momentum diffusivity to mass diffusivity. Mass flux caused by the temperature gradients is defined as the Soret effects and these effects are characterized by the Soret number.

## 1.8 Thesis Outline

The present thesis consists of seven chapters counting from this introductory chapter which contains the general introduction, problem statement, objectives of the research, scope of the research, significance of the research, research methodology and details on the dimensionless parameters. Further, Chapter 2 presents a detailed literature review on the problems discussed in the objectives of the research. In this research, problems of dissipative heat transfer on the peristaltic flow of Newtonian and various non-Newtonian fluids in asymmetric physical configurations with slip condition and chemical reactions have been investigated. These problems are investigated and presented in four chapters (Chapters 3 to 6).

In Chapter 3, the effects of slip on dissipative heat and mass transfer in the peristaltic transport are studied. The magnetohydrodynamic (MHD) flow of viscous fluid in a porous asymmetric channel is considered. Velocity, thermal, and concentration slip conditions are taken into account. Exact solutions for the stream function, axial pressure gradient, axial velocity, shear stress, temperature and concentration fields have been obtained by adopting long wavelength and small Reynolds number approximations. Pumping and trapping phenomena have been studied for different waveforms. Flow quantities have been plotted for various increasing parameters and the results are discussed in details. Comparisons with published results are found to be in good agreement.

In Chapter 4, the effects of dissipative heat and mass transfer on peristaltic transport of Walter's B fluid in an asymmetric channel are studied. The governing equations are solved using the regular perturbation method by taking the wave number as a small parameter. Perturbation solutions for the stream function, temperature, heat transfer coefficient and mass concentration are presented in explicit forms. Solutions are graphically plotted for different values of arising parameters such as viscoelastic parameter, Prandtl, Eckert, Soret, Schmidt and Reynolds numbers. Comparison with published results for viscous fluid is also presented and a close agreement is observed.

In Chapter 5, the dissipative heat transfer and slip effects on the peristaltic transport of a magnetohydrodynamic fourth grade fluid in an inclined asymmetric channel are studied. The governing equations are firstly modelled in the laboratory frame and then transformed into the wave frame. Under the long wavelength approximation, the resulting equations are solved using the regular perturbation method. A non-Newtonian parameter, namely the Deborah number serves as small perturbation parameter. Explicit expressions of solutions for the stream function, axial velocity, axial pressure gradient, temperature, and heat transfer coefficient are presented. Pumping and trapping phenomena are analysed for increasing velocity slip parameter whereas temperature profile and heat transfer coefficient are presented for various arising parameters. It has been found that these parameters considerably affect the considered flow characteristics. Comparisons with published results are found to be in good agreement.

Chapter 6 presents the effects of dissipative heat transfer on the peristaltic flow of a Sisko fluid in an asymmetric channel in the presence of slip conditions. Employing the long wave length approximation, the analytic solutions have been obtained by taking Sisko fluid parameter as the perturbation parameter. Explicit expressions of solutions for the stream function, axial pressure gradient, axial velocity, temperature, and heat transfer coefficient are presented. The variations of various interesting parameters are graphically plotted and discussed. Comparisons of the temperature profiles and heat transfer coefficient between Newtonian, shear thinning and shear thickening fluids are also shown. Comparison with an existing study is presented to validate the results obtained. Further, Chapter 7 gives the summary of research and some recommendations for the future research. Finally, a list of appendices provides the details of the derivations of governing equations, values of the coefficients appearing in the solutions and a list of publications.

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