

DYNAMIC TOPOLOGICAL DESCRIPTION OF BRAINSTORM DURING
EPILEPTIC SEIZURE

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To my beloved father and mother,
sisters and brother,
brothers in-law and nephew.

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ABSTRACT

Electroencephalograph is one of the useful and favoured instruments in diagnosing various brain disorders especially in epilepsy due to its non-invasive characteristic and ability in providing wealthy information about brain functions. At present, a large amount of quantitative methods for extracting “hidden” information which cannot be seen by “naked” eye from an electroencephalogram has been invented by scientist around the world. Among those, Flat Electroencephalography (Flat EEG) is one of the novel methods developed by Fuzzy Research Group (FRG), UTM which has been intended to localize epileptic foci of epilepsy patients. The emergence of this invention has led to the development of several Flat EEG based research (e.g., Non Polar C_{EEG} and Fuzzy Neighborhood Clustering on Flat EEG). The verification of the method has been made via comparison with some substantial clinical results. However, in this thesis, theoretical foundation of the method is justified via the construction of a dynamic mathematical transformation called topological conjugacy whereby isomorphism between dynamics of epileptic seizure and Flat EEG is established. Firstly, these two dynamic events are composed into sets of points. Then, they are forced to be strictly linearly ordered and composed into topological spaces. Subsequently, an isomorphism is constructed between corresponding mathematical structures to show that their properties are preserved and conjugate topologically. The constructed topological conjugacy is generalized into a class of dynamical systems. Within this class of dynamical system, Flat EEG’s flow is shown to be structurally stable. Additionally, topological properties on the event of epileptic seizure and Flat EEG have also been established.

ABSTRAK

Elektroensifalograf adalah salah satu instrumen yang berguna dan digemari dalam mendiagnosis pelbagai masalah gangguan otak terutamanya epilepsi disebabkan oleh sifat semulajadinya yang tidak invasif dan keupayaannya dalam memberi maklumat yang banyak mengenai fungsi otak. Pada masa kini, sejumlah besar kaedah kuantitatif untuk mengekstrakan maklumat “tersembunyi” yang tidak dapat dilihat dengan mata kasar dari elektroensifalogram telah dicipta oleh saintis di seluruh dunia. Antaranya, Elektroensifalografi Rata (Flat EEG) merupakan salah satu kaedah baru yang berjaya dibangunkan oleh Kumpulan Penyelidikan Kabur (FRG), UTM atas tujuan menentukan lokasi fokus sawan pesakit epilepsi. Kemunculan ciptaan ini telah mendorong kepada beberapa pembangunan penyelidikan yang berasaskan Flat EEG (contohnya, C_{EEG} Tidak Berkutub dan Pengelompokan Kejiranan Kabur pada Flat EEG). Penentusahkan bagi kaedah ini sudah pun dibuat melalui perbandingan dengan keputusan-keputusan klinikal. Walau bagaimanapun, dalam tesis ini, teori asas bagi kaedah tersebut akan dijustifikasikan menerusi suatu pembinaan transformasi dinamik yang dipanggil topologikal konjugasi dimana isomorfisma diantara dinamik ketika serangan sawan dan Flat EEG akan dibina. Pertama sekali, dinamik ketika serangan sawan akan digubah kepada set. Kemudian, ia akan dijadikan set linear tegas dan digubah kepada ruang topologi. Seterusnya, suatu isomorfisma akan dibina diantara struktur matematik yang sepadan untuk menunjukkan sifat-sifat mereka dikekalkan dan konjugat dari segi topologi. Topologi konjugasi yang dibina tersebut juga diitlakan ke dalam suatu kelas system dinamik. Dalam kelas sistem dinamik ini, aliran Flat EEG telah ditunjukkan stabil dari segi strukturnya. Di samping itu, sifat topologi semasa serangan sawan dan Flat EEG juga dipaparkan.

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LIST OF SYMBOLS AND ABBREVIATIONS

Symbols

Hz	-	Hertz
\mathbb{N}	-	set of natural numbers
\mathbb{Z}	-	set of integers
\mathfrak{R}	-	set of real numbers
\mathfrak{R}^n	-	n -dimensional Euclidean space
(x_1, x_2, \dots, x_n)	-	n -tuples
\min	-	minimum
\max	-	maximum
$\mu(B_z)$	-	membership value of B_z
e_p	-	electrical potential
∞	-	infinity
$\frac{dx}{dy}$	-	differentiation of x with respect to y
$f : X \rightarrow Y$	-	function f mapping from X to Y
$f^{-1} : Y \rightarrow X$	-	inverse function of f mapping from Y to X
$f(x)$	-	value of function f at x
$f \circ g$	-	composition of function f and g
I_d	-	identity mapping
\forall	-	for all
\in	-	element of
\ni	-	such that
$=$	-	equal to
\neq	-	not equal to

$<$	-	less than
$>$	-	greater than
\leq	-	less than or equal to
\geq	-	greater than or equal to
$\{\dots\}$	-	set consisting of
\subset	-	strict subset of
\subseteq	-	subset of
\cup	-	union
\cap	-	intersect
$A \times B$	-	cross product of A and B
\sim	-	related to
$\not\sim$	-	not related to
\prec	-	precede
\Rightarrow	-	implies
\Leftrightarrow	-	if and only if
τ	-	topology
$Int(A)$	-	interior of A
\bar{A}	-	closure of A
B	-	basis
S	-	subbasis
N	-	neighborhood
$Bdry(A)$	-	boundary of A
$\{X_\alpha\}_{\alpha \in A}$	-	indexed family of set X_α , $\alpha \in A$
$\coprod_{\alpha \in A} X_\alpha$	-	disjoint union of indexed family of sets X_α
C^r	-	r times differentiable continuous function
C^∞	-	infinitely differentiable continuous function
$C^r(\mathfrak{R}^m, \mathfrak{R}^m)$	-	set of r times differentiable continuous functions mapping from \mathfrak{R}^m to \mathfrak{R}^m
$Diff^r(\mathfrak{R}^n)$	-	set of r times differentiable diffeomorphisms from \mathfrak{R}^m to \mathfrak{R}^m
$Diff_{\nabla}^r(\mathfrak{R}^n)$	-	set of restricted r times differentiable

		diffeomorphisms from \mathfrak{R}^m to \mathfrak{R}^m
$\ \cdot \ $	-	norm
$\varphi_t(x)$	-	flow
$O_{\varphi_t(x_k)}$	-	an orbit of the flow $\varphi_t(x_k)$ initiated from x_k
$\overset{R}{\varphi}_t(x)$	-	flow with restricted domain and codomain
<i>dig</i>	-	digital
$D^k f(x)$	-	k -th derivative of $f(x)$

Abbreviations

ILAE	-	International League Against Epilepsy
AAN	-	American Academy of Neurology
FRG	-	Fuzzy Research Group
MC	-	Magnetic Contour Plane
BM	-	Base Magnetic Plane
FM	-	Fuzzy Magnetic Field
TM	-	Topographic Magnetic Field
MI	-	Magnetic Image Plane
BI	-	Base Image Plane
FI	-	Fuzzy Image Field
TI	-	Topographic Image Field
MC _{dig}	-	Magnetic Contour Plane digital
BM _{dig}	-	Base Magnetic Plane digital
FM _{dig}	-	Fuzzy Magnetic Field digital
TM _{dig}	-	Topographic Magnetic Field digital
FTTM	-	Fuzzy Topographic Topological Mapping
FTTM1	-	Fuzzy Topographic Topological Mapping Version 1
FTTM2	-	Fuzzy Topographic Topological Mapping Version 2
FTTM _{dig}	-	Fuzzy Topographic Topological Mapping Digital
MEG	-	Magnetoencephalography
EEG	-	Electroencephalography
QEEG	-	Quantitative Electroencephalography
Flat EEG	-	Flat Electroencephalography
C _{EEG}	-	FRG coordinate system
NPC _{EEG}	-	Non Polar C _{EEG} or Generalized Flat Electroencephalography
FCM	-	Fuzzy C-Means
SBRG	-	Seed-Based Region Growing
LOTS	-	Linearly Ordered Topological Space
ANC	-	Adaptive Noise Cancellation

BSS	-	Blind Source Separation
MRI	-	Magnetic Resonance Imaging
fMRI	-	Functional Magnetic Resonance Imaging
PET	-	Positron Emission Tomography
SPECT	-	Single-Photon Emission Tomography
CIT	-	Cortical Imaging Technique
LORETA	-	Low-Resolution Electromagnetic Tomography
ADTF	-	Adaptive Directed Transfer Function
VARETA	-	Variable Resolution Electromagnetic Tomography
DNA	-	Deoxyribonucleic Acid
FDG	-	2-[¹⁸ F] Fluoro-2-Deoxy-D-Glucose
ICU	-	Intensive Care Unit
RNN	-	Recurrent Neural Networks
EoES	-	Event of epileptic seizure
EoFE	-	Event of Flat EEG

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Disease is an anomalous condition whereby the ordinary functions of any parts of the body of an organism are interrupted. Usually, they are associated with symptoms and signs. One of the various formal ways of defining the term disease would be “any impairment that interferes or modifies the performance of normal functions, including responses to environmental factors such as nutrition, toxicants, and climate; infectious agents; inherent or congenital defects; or combinations of these factors” as defined by Wobeser (1997). Seizure is a type of condition that arises from disease. It is the physiology alteration which usually occurs unexpectedly due to the malfunction or synchronous abnormal discharges of electrical activity inside the brain. Such condition can happen to anyone at any age regardless of gender, but are more likely to strike on elderly. Statistically, it affects approximately 4% of the world populations of age 80 or lesser (Susan, 2004).

Seizure was defined by Perkin *et al.* (2007) as the sudden disturbance of electrical function inside the brain associated with changes of neurologic function. Generally, seizures can be classified into two major groups depending upon how they begin. At present, two of the most widely accepted and universally employed seizure classifications are the 1981 and 1989 International Classifications of Epilepsies, Epileptic Syndromes and Related Seizures Disorders, proposed by International League Against Epilepsy (ILAE) (Jerome, 2006). ILAE is a physician’s association

which seeks to create a better life to people with seizures and is the most preeminent in world. Through this classification, epileptologists are able to communicate between each other using a standard reference. Basically, the two major groups of seizure classified by 1981 and 1989 ILAE are partial (also called local or focal) seizures and generalized seizures (Shorvon, 2010). Partial seizure involved only specific and small part of the brain cortex, usually in one hemisphere, whereas generalized seizure involves large area of cortex in both hemispheres of the brain.

Causes of seizure can be of drug overdose, imbalance of chemical substances in the body, withdrawal of alcohol or drug, high fever, kidney or liver failure, infection in the brain, brain tumor, or abrupt reduction of blood or oxygen flow to the brain (Bricker *et al.*, 1994; Peacock, 2000; Williams and Wilkins, 2007; Atamon, 2008). On the other hand, condition that may be experienced during seizure includes sudden unintentional or uncontrolled muscle movements, sensory disturbances, loss or alteration in consciousness (one of the typical condition for generalized seizure), short-term anomalous sensation, visual disturbances and etc. (Pitkanen *et al.*, 2006). Some seizures are accompanied by symptom (also called as aura) in which may serve as an initial warning for sufferer to take precaution or safety measure. Examples of symptoms are irregular smell, sounds or taste, strange feelings, headache, feeling dizzy or numb (Sadock *et al.*, 2007). However, not every seizure comes with such clue. Consequently, it would be life threatening if the person is driving, swimming alone, crossing a busy road and etc.

Most of the time seizure last for only 3 to 5 minutes (American Academy of Orthopedic Surgeons, 2010). It rarely last longer than 15 minutes. Nonetheless, a seizure can be recurrent i.e., occur more than once. If a seizure is recurrent and unprovoked, it is potentially due to epilepsy (Engel *et al.*, 2008). This implies that epilepsy is a type of seizure and that not all seizures are due to epilepsy (Appleton and Marson, 2009). The general term for people with epileptic seizure is epilepsy. Similar as seizure, in epilepsy there is also a miniature brainstorm of certain groups of brain cells. The source or origin of the current sources, that is, the location which generate the corresponding tiny electric current, is known as epileptic foci.

Electroencephalography (EEG) is the recording of the electrical activity originating from the brain. It is non-invasive in nature and thus harmless and painless, as recordings are done on the surface of scalp where multiple of electrodes are placed. One of the major advantages of EEG is that abnormal electrical activity inside the brain can be recorded and portrayed on an electroencephalogram for further analysis. EEG is used extensively to diagnose epilepsies, classify the type and locate the source of electrical activity (Sanei and Chambers, 2007). This device is according to Popp and Deshaies (2007), Yudofsky and Hales (2008) and Gilhus *et al.* (2011) to be one of the most important laboratory tests in identifying epilepsies. Perhaps the best reason for its wide acceptance is that EEG allows neurologists to analyze and locate damaged brain tissue and also to make planning prior to surgery to avoid or lessen the risk of injury on important parts of the brain. Recently, obtaining the graphic electrical activity inside the brain has in general become a necessary part of surgical (Miller and Cole, 2011).

Hans Berger, a German psychiatrist, was the principal inventor of electroencephalography and the first recording of human brain electrical activity was conducted by him in the year of 1924 (Ramon, 2010). Thereafter, it was discovered by him the existence of rhythmic alpha brain waves in the year of 1929 (Tong and Thakor, 2009). Since then, Hans Berger became popular and managed to achieve international recognition and fame. This powerful invention which is capable of explaining how the brain works in terms of electrical activity, has gained him the name father of human electroencephalography. Other stuffs that Hans Berger has also research on in the early years, was measuring electrical waves in the cortices of dogs and also measuring temperature oscillations using mercurial thermometer (Verplaetse, 2009).

1.2 Research Background

Lately, numerous research using various concepts and techniques to identify epileptic foci has been established in the interest of creating better life for epileptic

patients. For instances, via multimodality approach (Desco *et al.*, 2001), by using large-area magnetometer and functional brain anatomy (Tiihonen *et al.*, 2004), examining correlations among electrodes captured by linear, nonlinear and multi linear data analysis technique (Evim *et al.*, 2006), 3-D source localization of epileptic foci by integrating EEG and MRI data (Natasa *et al.*, 2003) and even approaches that are based on statistical tools such as Bayesian method (Toni *et al.*, 2005) and maximum likelihood estimation approach by Jan *et al.* (2004). Each of the methods has their own advantages and weaknesses.

Liau (2001) under Fuzzy Research Group (FRG) in UTM has also developed a novel mathematical model to solve neuromagnetic inverse problem. This model is termed as Fuzzy Topographic Topological Mapping (FTTM) and is a topological structured-based model. The main advantage of FTTM model is it requires only instantaneous data. Thus, the computing time is lower compared to statistical-based models. Generally, FTTM enables recorded signals (on flat surface) be portrayed 3-dimensionally. Since the introduction of FTTM, majority of the research by FRG has been on visualizing and extracting “hidden” information from EEG signals. All these studies were conducted to gain deeper understanding on how brain works from mathematical viewpoint.

Flat EEG signal (Flat EEG, in short) is a way of viewing EEG signals on the first component of FTTM. Hence, theoretically, EEG signals can be portrayed in 3-dimension space by FTTM model. Since the introduction of Flat EEG, most FRG research has been on extracting quantitative information within EEG via Flat EEG. Constructions of Flat EEG embark from the modeling of epileptic seizure as a dynamical system with potential difference as the feature space. Then by exploiting the dynamic temporal ordering properties on the state space trajectory of seizure, it was showed that a whole Flat EEG data can be analyze piece by piece (Fauziah, 2008). This signifies that dynamics is embedded within Flat EEG. Hence, Flat EEG is a dynamical system.

A large amount of advancement and outstanding achievement has been gained by FRG since the introduction of Flat EEG. For instance, Amidora (2012)

has developed a clustering method using Non-Polar C_{EEG} , which is an extension and improvement of Flat EEG in terms of portraying cluster centers of electrical activity. The results obtained from this method have been compared and validated (with significant agreement) with the results obtained via functional magnetic resonance imaging (fMRI) in one of the leading brain institute in Japan, Riken (Amidora, 2012). Furthermore, Faisal (2011) had also successfully proved that Flat EEG at any time can be written as matrix form and further be decomposed uniquely into simple groups analogous to how every integer has unique prime factorization. His discovery has received good compliments from some experts (Faisal, 2011).

1.3 Problem Statement

At present, several Flat EEG based research has been introduced and conducted with some still in progress. Most of this research intends to improve and enhance Flat EEG in terms of portraying the origin of electrical activity inside the brain. Although those developed method is reliable, it still lack of a comprehensive mathematical justification. Primarily, none mathematical formulation has been offered for transformation of dynamicity of epileptic seizure to Flat EEG. Owing to the fact that Flat EEG rely greatly upon the concept of dynamical system, this “gap” must therefore be “patched” in order to obtained verification on Flat EEG and also findings which stems from Flat EEG. Besides, transformation of EEG to Flat EEG which preserves the magnitudes renders Flat EEG to contain unwanted signals captured during recording from the surroundings. Consequently, its accuracy in representing actual electrical activity inside the brain is often affected. Hence, issue pertaining to persistence of Flat EEG to perturbations is imperative. Apart from that, there has been lack of mathematical interpretation on the event of epileptic seizure. Thusly, establishing topological properties on this event would be appealing.

1.4 Research Objectives

The objectives of this research are:

1. to construct a mathematical model which can describes the dynamicity of Flat EEG in relation to epileptic seizure;
2. to generalized the topological conjugacy between the dynamical system of epileptic seizure and dynamical system of Flat EEG into a class of dynamical systems;
3. to investigate the persistence of the dynamical system of Flat EEG to perturbations;
4. to describe the event of epileptic seizure and Flat EEG topologically.

1.5 Scope of Research

In this research, the dynamic justification of Flat EEG, Flat EEG's reliability in the presence of artifacts and the mathematical description on the event of epileptic seizure and Flat EEG will be carry out using notion of topology.

1.6 Significance of Findings

Contributions of the findings in this study are:

1. a mathematical model which can describes the dynamicity of Flat EEG in relation to epileptic seizure;
2. the development of a topological conjugacy which serves as an equivalence relation in a class of dynamical systems;
3. the development of a neighborhood of perturbations where the dynamical system of Flat EEG is structurally stable.
4. the development of topological properties on the event of epileptic seizure.

1.7 Thesis Outline

This thesis contains nine chapters. Its framework is depicted in Figure 1.1. Chapter 1 provides the general information of the research which includes the research background, problem statement, research objectives, scope of research and the significance of the findings. It enables readers to grasp the whole idea of the thesis.

Chapter 2 presents the literature reviews of relevant research. Basically, origin of electrical currents inside the brain and instrument (EEG) used to measure the electrical currents is explained. Subsequently, available methods used to locate the source of electric currents developed by FRG i.e., Flat EEG and Non-Polar C_{EEG} are presented.

Mathematical concepts will be presented in Chapter 3. In Chapter 4, the notion of modelling will be discussed and assumptions imposed in this work along with their justifications will be presented.

Chapter 5 presents the dynamic model construction for Flat EEG. Basically, a geometrical representation for Flat EEG is introduced in prior to modeling Flat EEG as dynamical system. Besides, epileptic seizure was also re-modeled as dynamical system using the notion of flow.

In Chapter 6, various mathematical structures will be established on the trajectories of dynamical systems of epileptic seizure and Flat EEG. Then a topological conjugacy will be constructed from epileptic seizure to Flat EEG. Additionally, the topological conjugacy is shown to form an equivalence relation in a class of dynamical systems.

In Chapter 7, the reliability of Flat EEG in the presence of artifacts will be investigated by means of structural stability.

Chapter 8 describes the event of epileptic seizure and Flat EEG mathematically. Particularly, notion of topology will be used to describe the events.

Finally, Chapter 9 concludes this thesis by giving the summary of every chapter, highlighting the significance of the research and providing some suggestions for future research.

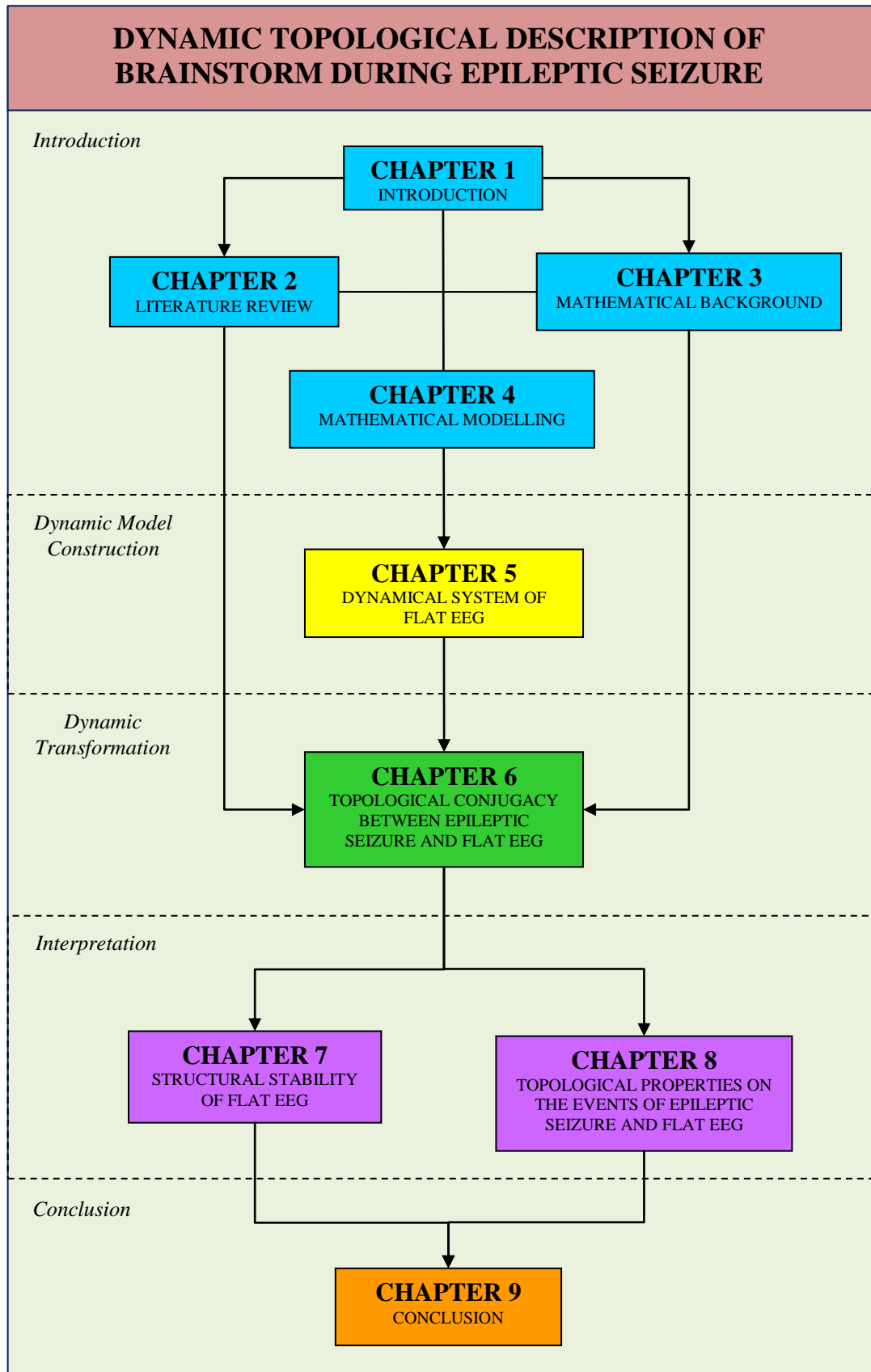


Figure 1.1: Research framework

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