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# PHYSICAL CHANGES DUE TO A DEFORMING POROUS MEDIA – A FINITE ELEMENT SOLUTION

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**Abstract:** *This paper discusses a finite element solution to investigate the physical changes due to the deforming porous media. The mathematical model was derived based on Biot's self-consistent theory, which describes a fully coupled governing equation system for a multiphase flow in a three dimensional reservoir system. It consists of the equilibrium and continuity equations for oil, gas and water-phases. An elastoplastic reservoir rock model based on the Mohr Coulomb yield criteria was used for simulating the deformation behaviour of the reservoir. The compressibility factors are calculated based on the unknowns solved in each time step. Finally, the recent values of porosity and permeability are calculated for various types of reservoir rocks.*

## INTRODUCTION

In reservoir engineering practice, the reservoir rock deformation, also known as reservoir compaction, is normally neglected except in a few cases where unconsolidated formation can cause a considerable effect on reservoir performance Although reservoir compaction and the associated subsidence are not commonly encountered their occurrence may result in many problems However, compaction due to effective compressibility occurs in most undersaturated oil reservoir and this can act as a drive mechanism with a considerable effect on the ultimate recovery Many papers in the literature have investigated the problem in various situations (Finol & Farouq 1975).

This paper deals with the effects of the reservoir compaction processes on the values of permeability and porosity For this purposes, different type of reservoir rocks saturated with three immiscible and compressible flowing fluids are considered here Initially, the reservoir and the overburden layers are in equilibrium hence the distribution of the effective stresses due to excess pore pressures are zero. As the withdrawal of fluids from the reservoir begins, the pore pressures start to decline in both systems, and therefore increase the overburden load at the top of the reservoir. If the reservoir system consists of unconsolidated sand, this load will reduce the thickness of the formation. The process is called reservoir compaction, which in some case result in a considerable amount of physical changes of reservoir rock (Lewis & Sukirman 1993) Merle *et al* (1975) have presented evidence that initially some aquifers and oil reservoirs behave as if the producing formation is relatively incompressible and becomes significantly more compressible after large pressure drops in the reservoir. At this stage, the formation rock may compact inelastically where further pressure decline can result in formation collapse, especially around the producing zones.

Lewis & Schrefler (1987) have investigated the effect of compaction on the physical properties of the reservoir formation For example the influence of difference values of Young modulus E, Poisson's ration $v$, the degree of rock cohesion $c$, the friction angle $\phi$ and rock compressibility factors $C_r$ on the degree of compaction has occurred. In many cases, numerical solutions have been successfully used to analyse the problems However, very few cases have been reported in the literature which deal with the simulation of physical changes for three-phase fluids flowing in a compacting reservoir. In this paper, the developed fully coupled model was used to investigate the changes of porosity and permeability of a compacting saturated oil reservoir

## EQUILIBRIUM EQUATION

In the case of consolidation-fluid flow problems, the fundamental relationship if to define the interaction behaviour between the fluid and the soil skeleton The well known Terzaghi 'effective stress principle' and its extension into more general three-dimensional consolidation relationships by Biot have been widely used in this class of problem The derived relationships are the simultaneous partial differential equations with unknown parameters of the displacement of the skeleton and pore fluid pressures.

For a general non linear material, the effective stress relationship is expressed in a tangential form thus allowing for plasticity, creep and other factors influencing strains to be included (Lewis & Schrefler 1987). This can be written as follows

$$d\sigma' = \mathbf{D}_T (d\varepsilon - d\varepsilon_c - d\varepsilon_p - d\varepsilon_o) \tag{1}$$

Where $\mathbf{D}_T$ is the tangential elastic stiffness matrix, $d\varepsilon$ represents the total strain of the skeleton, $d\varepsilon_c$ is the creep strain and $d\varepsilon_p$ represents the overall volumetric strain caused by uniform compression of the particles. Lewis & Schrefler (1987) defined the $\varepsilon_o$ strains as all other type of strains not directly associated with stress changes (swelling, thermal, chemical, etc) or referred as the 'autogeneous' strains.

The equilibrium equation relating the total stress $\sigma$ to the body force $\mathbf{b}$ and the boundary traction $\hat{\mathbf{t}}$ specified at the boundary $\Gamma$ of the domain $\Omega$ is formulated in terms of the unknown displacement vector u. Based on the principle of virtual work, the equation can be written as follows

$$\int_\Omega \delta\varepsilon^T d\sigma d\Omega - \int_\Omega \delta\varepsilon^T db\Omega - \int_\Gamma \delta u^T d\hat{t} d\Gamma = 0 \tag{2}$$

Incorporating the effective stress relationship into equation (2), the following expression is obtained,

$$\int_\Omega \delta\varepsilon^T d\sigma' d\Omega - \int_\Omega \delta\varepsilon^T m dP b\Omega - d\hat{f} = 0 \tag{3}$$

where

$$d\hat{f} = \int_\Omega \delta u^T db d\Omega + \int_\Omega d\hat{t} d\Gamma \tag{4}$$

represents the change in external force due to boundary and body loading.

The final form of the equilibrium equation can be written as follows

$$\begin{aligned} &\int_\Omega \delta\varepsilon^T \mathbf{D}^T \frac{\partial\varepsilon}{\partial t} d\Omega \int_\Omega \delta\varepsilon^T S_w \left( \mathbf{m} - \frac{\mathbf{D}_T \mathbf{m}}{3K_s} \right) \frac{\partial P_w}{\partial t} d\Omega \\ &- \int_\Omega \delta\varepsilon^T S_o \left( \mathbf{m} - \frac{\mathbf{D}_T \mathbf{m}}{3K_s} \right) \frac{\partial P_o}{\partial t} d\Omega - \int_\Omega \delta\varepsilon^T S_g \left( \mathbf{m} - \frac{\mathbf{D}_T \mathbf{m}}{3K_s} \right) \frac{\partial P_g}{\partial t} d\Omega - \\ &- \int_\Omega \delta\varepsilon^T \mathbf{D}_T c d\Omega - \int_\Omega \delta\varepsilon^T \mathbf{D}_T \frac{\partial\varepsilon_o}{\partial t} d\Omega - \frac{\partial\hat{\mathbf{f}}}{\partial t} = 0 \end{aligned} \tag{5}$$

Note that equation (5) can be used for simulating reservoir compaction problems in various conditions.

## MULTIPHASE FLOW EQUATIONS

The multiphase flow model consists of the mathematical formulation of each flowing phase taking into account the effects of fluid and rock compressibility factors, capillary pressure, relative permeability contrasts and gas solubility in the liquid-phases. The following expression is obtained for a unit volume of fluid flowing at reservoir conditions.

$$(\text{Accumulation rate}) = \nabla\left(\frac{\mathbf{k}\rho}{\mu}(P + \rho g h)\right) \tag{6}$$

where $P$ is the fluid pressure, $\rho$ the density of fluid, $\mathbf{k}$ is the absolute permeability matrix, $\mu$ is the dynamic viscosity, and h the height above an arbitrary datum (Sukirman 1993). The factors, which contribute to the rate of fluid accumulation of each following phase, were discussed in Reference [4]

The derivations of the continuity equations for water and gas phases are essentially the same as that for the oil flow equation. The effect of the gravity term $\nabla\rho g h$ has been omitted throughout the study (Sukirman 1993). The total reservoir compressibility factor $C_r$ can be defied as follows

$$C_r = \left[S_o \frac{\partial}{\partial P_o}\left(\frac{1}{B_o}\right) + S_w \frac{\partial}{\partial P_w}\left(\frac{1}{B_w}\right) + S_g \frac{\partial}{\partial P_g} \times \left(\frac{1}{B_g}\right)\right]$$
$$+ \left(S_o C_{rm} + S_w C_{rm} + S_g C_{rm}\right) \times \left(S'_o + S'_w + S'_g\right) + \left(\mathbf{m}^T - \frac{\mathbf{m}^T \mathbf{D}_T}{3K_s}\right) \tag{10}$$

As the total fluid saturation values are equal to unity, similarly for $\left(S'_o + S'_w + S'_g\right)$, therefore, equation (10) becomes

$$C_r = C_{rf} + C_{rm} + C_{rc} \tag{11}$$

where

$$C_r f = S_o \frac{\partial}{\partial P_o}\left(\frac{1}{B_o}\right) + S_{wo}\frac{\partial}{\partial P_w}\left(\frac{1}{B_w}\right) + S_{go}\frac{\partial}{\partial P_g}$$

$$\times\left(\frac{1}{B_o}\right) = S_o\left(\frac{1}{B_o}\right) + S_w\left(\frac{1}{B_w}\right) - S_g\left(\frac{1}{B_g}\right) \qquad (12)$$

represents the total fluid compressibility and

$$C_{rc} = \left(\mathbf{m}^T - \frac{\mathbf{m}^T \mathbf{D}_T}{3K_s}\right) \qquad (13)$$

is the total compressibility due to rock compaction as the result of pore pressure decrease during fluid withdrawal, whilst $C_{rm}$ is the effective rock compressibility. The above equations indicate that the two rock compressibility components, $C_{rm}$ and $C_{rc}$, depend on the type of rock i.e. these are determined by Young modulus $E$, Poisson's ration ν and solid bulk modulus $K_s$. Lewis and Schrefler (1987) and Finol *et al* (1975) have investigated the influences of these variables on the degree of reservoir compaction. In this paper the effects of the total compressibility factor Cr on the porosity and permeability changes were investigated in detail. The values of porosity and the absolute permeability were changed with time due o the occurrence of reservoir compaction. Many authors agreed that these values are related to one another and therefore must be included when predicting the performance of a compacting reservoir (Finol & Farouk 1975). Aziz & Settari (1979) have defined porosity changes as a function of a reservoir pressure drop $\Delta P$ and the rock compressibility $C_f$, as follows;

$$\phi = \phi^o\left[1 + C_f \Delta P\right] \qquad (14)$$

where

$\phi^o$ - porosity at time zero

$\Delta P = \bar{P} - P^o$

$\bar{P}$ is the average reservoir pressure whilst $P^o$ represents the initial pressure for the reservoir system. On differentiating equation (14) w r t time then

$$\frac{\partial\phi}{\partial t} = C_f \phi^o \frac{\partial\bar{P}}{\partial t} = \frac{\partial\phi}{\partial\bar{P}}\frac{\partial\bar{P}}{\partial t} \qquad (15)$$

where

$$\frac{\partial \phi}{\partial P} = C_f \phi^o \tag{16}$$

If the reservoir rock is assumed to be incompressible, i.e. $C_f \rightarrow$ then, the rate of change of porosity becomes zero or $\partial\phi/\partial t \rightarrow 0$ which means that the porosity is a constant value.

The changes of porosity with pressure can be approximated from equation (16) as follows

$$\phi^{m+1} = \phi^m \left[1 + c_r \left(P_o^{m+1} - P_o^m\right)\right] \tag{17}$$

where

$$c_r = \frac{c_m + (1-\phi)c_r}{\phi} \tag{18}$$

$c_m$ is the unixial compaction coefficient
$c_r$ is the rock matrix compressibility
$c_k$ is a coefficient of permeability reduction
$P_o$ is the pressure in the oil phase

and the superscripts ($m + 1$), ($m$) refer to new and old time step levels respectively. In practice, the unixial compaction coefficient $c_m$ can be obtained from laboratory compressibility data by

$$c_m = \frac{1}{3}\frac{1+\nu}{1-\nu}(1-\beta)c_b \tag{19}$$

where $\nu$ is poisson's ratio

$\beta$ is ration of matrix/bulk compressibility
$c_b$ is the bulk compressibility

In cases where $c_k$ is not known then, it can be assumed to be equal to the value of $c_m$.
Lewis and Schrefler (1987) defined the settlement/void ration relationship for a one-dimensional consolidation model as follows

$$\delta = d\frac{e_1 - e_2}{1 + e_1} \tag{20}$$

where $\delta$ is the average settlement of each element

$d$ is the initial depth
$e_1$ is the initial void ratio
$e_2$ is the final void ratio

In this paper, the variation of porosity with time was defied in a similar manner to the approach used by Aziz and Settari (1975). The rate change of porosity can be written as

$$\frac{\partial \phi}{\partial t} = \left( \mathbf{m}^{T} - \frac{\mathbf{m}^{T}\mathbf{D}_{T}}{3K_{s}} \right) \frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial t}\left( \frac{\mathbf{m}^{T}\mathbf{D}_{T}\mathbf{c}}{3K_{s}} \right) + \left( \frac{1-\phi}{K_{s}} - \frac{1}{3K_{s}^{2}} \mathbf{m}^{T}\mathbf{D}_{T}\mathbf{m} \right) \frac{\partial \overline{P}}{\partial t} \quad (21)$$

assuming the effect of creep strain is negligible Therefore equation (21) can be simplified as follows

$$\frac{\partial \phi}{\partial t} = C_{re} \frac{\partial \varepsilon}{\partial t} + C_{rm} \frac{\partial \overline{P}}{\partial t} \quad (22)$$

The equation (22) calculates the porosity changes caused by the effective rock compressibility and the displacement of the solid skeleton. Finol & Farouk (1975) presented the variation of permeability with time as follows

$$k^{m+1} = k^{m}\left[1 + c_{k}\left(p_{o}^{m+1} - p_{o}^{m}\right)\right] \quad (23)$$

where

$k^{m+1}$ – is the permeability at time level $(m+1)$
$k^{m}$ – is the permeability at time level $m$

In this paper, the time derivative of permeability can be defined in similar manner as for the porosity changes in equations (23) Lewis & Schrefler (1987) reported that the variable permeability scheme is of practical use only if the expected consolidation is important.

## NUMERICAL SOLUTIONS

In this paper, the finite element method, which is Galerkin-based, was used to discretize the developed governing equation which describes three-phase fluid flow coupled with the equilibrium equations in a three dimensional model An eight noded brick element was used in the present three-dimensional model. Applying the finite element discretization method to the governing equations will result as follows

For the equilibrium equation

$$\mathbf{K}\frac{d\bar{\mathbf{u}}}{dt}+\mathbf{L}_w\frac{d\bar{\mathbf{P}}_w}{dt}+\mathbf{L}_o\frac{d\bar{\mathbf{P}}_o}{dt}+\mathbf{L}_g\frac{d\bar{\mathbf{P}}_g}{dt}-\mathbf{C}-\frac{d\mathbf{f}}{dt}=0 \tag{24}$$

and for the oil-phase, equation (8) will take the form

$$\mathbf{H}_p\bar{P}_o+\mathbf{H}_w\frac{d\bar{\mathbf{P}}_w}{dt}+\mathbf{H}_o\frac{d\bar{\mathbf{P}}_o}{dt}+\mathbf{H}_g\frac{d\bar{\mathbf{P}}_g}{dt}+\mathbf{H}_u\frac{d\bar{u}}{dt}+\bar{\mathbf{F}}_o=0 \tag{25}$$

The finite element discretization for water and gas-phase are essentially the same as that for oil flow equation These equations represent a set of ordinary differential equation in time. In this paper, the time discretization method used is based on a Kantorovich (Lewis & Schrefler 1987) scheme, which may be regarded as a one-dimensional finite element scheme. The time integration takes the same form as used for the spatial integration, which gives the following form

$$\begin{bmatrix} \mathbf{K} & \mathbf{L}_w & \mathbf{L}_o & \mathbf{L}_g \\ \mathbf{W}_u & \mathbf{W}_w & \mathbf{W}_o & \mathbf{W}_g \\ \mathbf{H}_u & \mathbf{H}_w & \mathbf{H}_o & \mathbf{H}_g \\ \mathbf{G}_u & \mathbf{G}_w & \mathbf{G}_o & \mathbf{G}_g \end{bmatrix}_{k,a} \begin{Bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{P}}_w \\ \bar{\mathbf{P}}_o \\ \bar{\mathbf{P}}_g \end{Bmatrix}_{n+1} = \begin{bmatrix} \mathbf{K} & \mathbf{L}_w & \mathbf{L}_o & \mathbf{L}_g \\ \mathbf{W}_u & \mathbf{W}_w & \mathbf{W}_o & \mathbf{W}_g \\ \mathbf{H}_u & \mathbf{H}_w & \mathbf{H}_o & \mathbf{H}_g \\ \mathbf{G}_u & \mathbf{G}_w & \mathbf{G}_o & \mathbf{G}_g \end{bmatrix}_{k,a} \begin{Bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{P}}_w \\ \bar{\mathbf{P}}_o \\ \bar{\mathbf{P}}_g \end{Bmatrix}_{n} + \begin{Bmatrix} \bar{\mathbf{F}}_u \\ \bar{\mathbf{F}}_w \\ \bar{\mathbf{F}}_o \\ \bar{\mathbf{F}}_g \end{Bmatrix}\Delta \tag{27}$$

Equations (27) are applied at all nodes within the domain and those on the boundary where the unknowns are not prescribed These equations represent a fully coupled and highly non-linear system, for three-phase flow in a deforming porous media Since all the coefficients are dependent on the unknowns, iterative procedures are performed within each time step to obtain the final solution

A material balance error check per unit time was performed by calculating the algebraic sum of the residuals of each phase in every gridblock. In this paper, an implicit pressure-explicit saturation scheme (IMPES) has been applied to solve the unknowns

## NUMERICAL EXAMPLES

The developed finite element model was employed to analyse the physical changes due to reservoir compaction. In this paper, for a hypothetical saturated reservoir model was used (Sukirman 1993) The simulations results for the reservoir compaction and the consequent changes in porosity and permeability values with production time are shown in Figures 1 to 4. It can be seen that the changes in porosity and permeability values are minimum for higher value E and vice versa. This is an expected results because a higher value of E implies a more rigid formation rock and therefore tending to incompressible behaviour. The results obtained show the reduction of porosity and permeability values with production time.

## CONCLUSIONS

A fully coupled finite element model has been applied for simulating the physical changes due to the deformation of a saturated reservoir. The derivation of the governing equations considered the equilibrium equation and the continuity of the fluid flow The model has been applied to study the effects of rock deformation on the values of permeability and porosity

The results indicate that the changes in the permeability and porosity values are more critical for the case of unconsolidated reservoirs. It is felt that the developed model can also be used for predicting the drive mechanism due to the compacting porous media.

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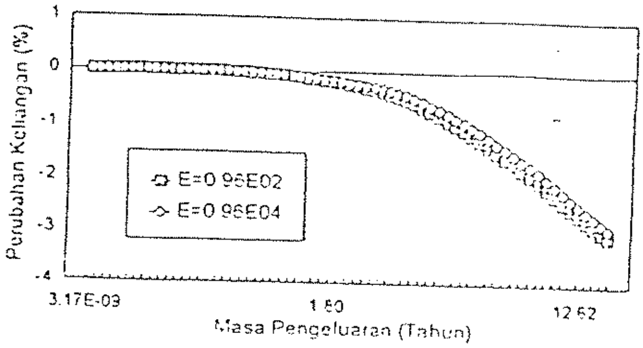


Figure 1-Porosity vs Time

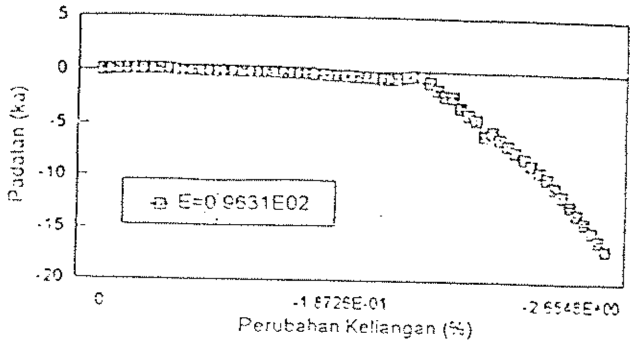


Figure 2-Deformation vs Volume Change

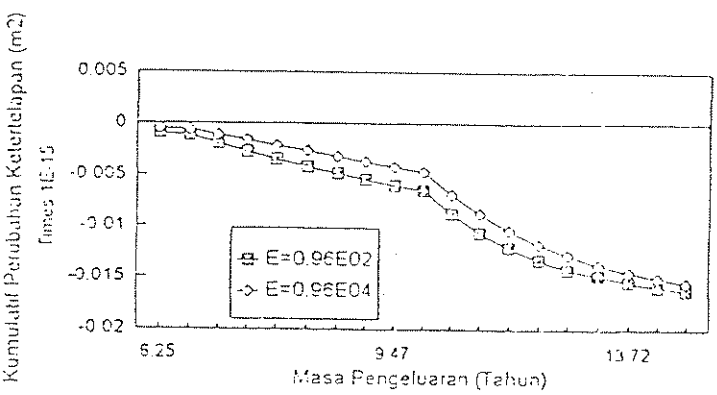


Figure 3-K, Permeability vs Time

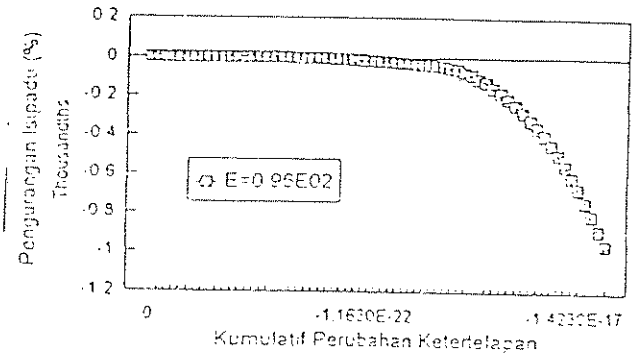


Figure 4 – Reservoir volume vs permeability