

# The Application of Combined Momentum – Blade Element Theory for Aerodynamics Analysis Helicopter Rotor Blade in the Forward Flight

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**Abstract:** Present work introduced the aerodynamics analysis of rotor blade helicopter in forward flight. The analysis used a combination between a Momentum Theory and The Blade Element Theory. Here the inflow ratio was assumed a uniform over the disk plane and it was predicted by using the momentum Theory. As the inflow ratio is available, then by using the Blade element theory, the aerodynamics loading along the blade span of the rotor are computed, which finally the thrust coefficient  $C_T$  can be obtained. For a given a rotor blade configuration and flight condition, the Thrust coefficient  $C_T$  is unknown, while the momentum theory required this value to be known in predicting the inflow ratio. As result an iteration process is required in implementing those two combined approaches. For the assessment purposes, four test cases had been studied. The difference between one case to other case had been selected in term: 1) twist distribution, 2) the presence of coning angle and 3) the required aerodynamics characteristics. The result showed that the combination of Momentum Theory) and The blade element theory could provide a fast solution in predicting the aerodynamics performance of rotor blade helicopter. However a comparison result with the experiment result was required in order to asses the degree of accuracy of this approach. This was suggested as future work.

## Notation:

$C_T$	Thrust Coefficient	$\alpha_{\text{eff}}$	Effective Angle Of Attack
$C_Q$	Torque Coefficient	$\beta$	Coning Angle
$C_d$	Drag Coefficient	$\lambda_{\text{ave}}$	Uniform inflow ratio
$C_l$	Lift Coefficient	$\mu$	Advance ratio
$P$	Power	$\psi$	Azimuth angle
$Q$	Torque	$\Omega$	Rotor rotational speed
$R_B$	Blade Radius	$\epsilon_{\text{ct}}$	Prescribed value
$U_T$	Velocity normal to blade leading edge line	$\theta$	Pitching angle
$U_P$	Out plane velocity		
$U_R$	Radial Velocity		
$U_\infty$	Incoming flow velocity		
$V_i$	Induced inflow		
$\alpha$	Angle Of Attack		

## I. Introduction

The aerodynamics analysis of the rotor blade helicopter software is, now, available in the market. For the preliminary design stage may one can use RaPid software. RaPiD is stand for from: Rotorcraft Analysis for Preliminary Design which developed by Dr. Omri Rand from Faculty of Aerospace Engineering, Technion – Israel Institute of

Technology, <sup>1</sup>. This software is allowing one to model and analyze general rotorcraft configurations, conventional helicopter as well as tilt rotors models. The uniform inflow model had been used as the basic concept in solving the aerodynamics problem in this code.

Another computer code for helicopter aerodynamics analysis may one use a code a called "FLIGHTLAB"<sup>2</sup>. This code developed by Advanced Rotorcraft Technology Inc, US based company. This software was designed for a flight vehicle modeling and analysis tool so that allows users to interactively produce models from a library of modeling components by arbitrarily selecting the modeling components, interconnecting them into a custom architecture, and assigning aircraft specific data to the parameters of these components. Strictly speaking FLIGHTLAB's represent simulation language, which provides an interpretive operating environment with a Matlab-like syntax that supports vector/matrix operations and interactive data and command access. In solving the aerodynamics problem, FLIGHTLAB introduces that each blade is divided into segments and the local inertial and aerodynamic loads at each blade segment is separately computed to model the load distribution along the blade. Here the FLIGHTLAB code also uses a uniform inflow model as the way how to predict the aerodynamics performance of the rotor blade helicopter.

The most comprehensive computer code for helicopter analysis might be given by CAMRAD II<sup>3, 4, 5</sup>. This code incorporates a combination of advanced technology, which including multibody dynamics, nonlinear finite elements, and rotorcraft aerodynamics. As result the code provide the capability for the design, testing, and evaluation of rotors and rotorcraft at all stages. The aerodynamics rotor blade analysis was developed by using a lifting line theory supported by a sophisticated wake analysis to calculate non uniform induced velocities. Here one has an option to use either Rigid, prescribed or free wake geometry.

Present work was intended to develop computer for the aerodynamics analysis of rotor blade helicopter in forward flight. The code used a combination Momentum Theory and The blade Element Theory<sup>6, 7, 8</sup> where the inflow was assumed to be uniform. The inflow ratio was calculated by using the Momentum Theory with given an initial value for the thrust coefficient  $C_{T0}$ . This inflow ratio, then, will be used as input for the blade element theory in determining the aerodynamics load along blade span. This is carried out through superimposed the inflow velocity

with the incoming free stream velocity, rotor blade rotational speed and the velocity generated by the blade coning angle  $\beta(\Psi)$ . As the resultant velocity at each blade section was obtained then the effective angle of attack at that section can be found. Hence local lift and drag at each blade section by using a look up airfoil table can be obtained. The thrust coefficient  $C_T$  and torque coefficient  $C_Q$  was, finally, easily obtained through integrating over span wise and azimuth position to represent the average value for one rotation of rotor. The obtained value of  $C_T$  would not, of course, the same as the initial value  $C_{T0}$ . Hence iteration process was required to update the of  $C_T$  used in the Momentum Theory until result between two successive iteration would not exceed a certain prescribed value  $\epsilon_{ct}$ .

For the assessment of the capability of the developed code, four test cases of two bladed rotor blade helicopters were studied. The blade was assumed a simple rectangular plan form with uniform cross section. The rotational speed of the rotor blade is fixed to 400 RPM, the blade radius is equal to 6 m and the chord length for the airfoil section is equal to 0.4 m. The difference between one case with other case in term of twist distribution, the presence of coning angle and the aerodynamics data for the airfoil section.

The result showed that the combined Momentum Theory and The Blade Element Theory could provide a fast solution in predicting the aerodynamics performance of the rotor blade helicopter. The iteration process is almost accomplished in less than 10 iterations. A comparison with experiment result might require in order to assess the degree of accuracy of this approach. This suggested for the future work.

## II. Methodology

### 2.1 Theoretical Background

As mentioned in the previous subchapter, the present work used a combined two approaches: The momentum theory for predicting the inflow ratio whiles the Blade element theory for the detailed blade loading calculation. The detailed derivation of those two approaches can be found in Ref. 6 and 7.

Strictly speaking, the momentum theory modeled the flow past through helicopter blade in forward flight as depicted in the Figure 2.1.

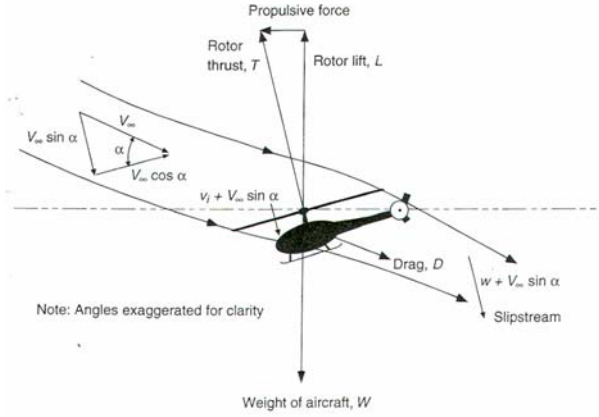


Figure 2.1: Momentum theory of a rotor in forward flight (Leishman, 2000)

For a given a thrust coefficient  $C_T$ , the incoming velocity  $U_\infty$  and the disk plane angle  $\alpha$  accordingly the Momentum theory gives the inflow ratio  $\lambda$  governed by :

$$\lambda = \mu \operatorname{tg} \alpha + \frac{C_T}{\sqrt{\mu^2 + \lambda^2}} \quad 1-1$$

If the rotor rotational speed and the rotor blade radius are denoted by  $\Omega$  and  $R_B$  respectively. Hence, the advance ratio  $\mu$  in above equations is defined as:

$$\mu = \frac{U_\infty \cos \alpha}{\Omega R_B} \quad 1-2$$

The term  $\Omega R_B$  is called as the blade tip speed  $U_{tip}$ .

The equation 1-1 represents the non linear equation in term of the inflow ratio  $\lambda$ . Hence the solution for the  $\lambda$  needs to be done iteratively. Using a Newton Raphson iterative method, the iterated value of  $\lambda$  at the nth iteration would be:

$$\lambda_n = \lambda_{n-1} - \frac{f(\lambda_{n-1})}{f'(\lambda_{n-1})} \quad 1-3$$

Where:

$$f(\lambda) = \lambda - \mu \operatorname{tg}(\alpha) - \frac{C_T}{\sqrt{\mu^2 + \lambda^2}} \quad 1-4a$$

$$f(\lambda) = 1 - \lambda \frac{C_T}{(\mu^2 + \lambda^2) \sqrt{\mu^2 + \lambda^2}} \quad 1-4b$$

The initial value of the inflow ratio  $\lambda_0$  for starting the iteration process is given by:

$$\lambda_0 = \sqrt{\frac{C_T}{2}} \quad 1-5$$

Here one can implement the criteria for finishing the iteration process by the following equation:

$$\left| \frac{\lambda_n - \lambda_{n-1}}{\lambda_n} \right| \leq \varepsilon \quad 1-6$$

The  $\varepsilon$  represents a prescribed value which can be chosen arbitrary. It could be, normally, below 0.005. If the chosen value  $\varepsilon$  is set equal to 0.005, then the iteration process would be terminated at the difference value between two successive iteration results would not exceeded more than 0.5 %.

The inflow ratio  $\lambda$  was obtained using above method, would be as input for the Blade element Theory. Basically, The Blade Element Theory is similar to the strip theory in fixed wing aerodynamics. The blade is considered as composed of a number of aerodynamically independent cross-sections, whose characteristics are the same as a blade at a proper angle of attack. In this respect, as the blade is assumed to be made of several infinitesimal strips of width  $\Delta r$ . The lift and drag are estimated at the strip using 2-D airfoil characteristics of the airfoil at that strip accordingly to the local flow velocity. It is, therefore, necessary to determine the magnitude and direction of the airflow in the immediate vicinity of the blade element under consideration.

In forward flight, the incoming flow velocity  $U_\infty$  to the disk plane can be resolved into two component velocities, namely the component velocity parallel to the disk plane  $U_{\infty //}$ , and the component velocity  $U_{\infty \perp}$  which is perpendicular to it. Suppose that the motion of the rotor blade under consideration has a variable coning angle  $\beta$ . As the blade rotates, the variation of the coning angle  $\beta$  as function of blade azimuth position  $\Psi$  can be written as:

$$\beta(\Psi) = \beta_0 + A \cos \beta + B \sin \beta \quad 1-7$$

The coefficient  $\beta_0$ , A and B above equation are specified. Each rotor blade configuration might had their own values.

In the presence of angular velocity of the blade  $\Omega$ , induced velocity  $v_i$ , and the variation of coning angle  $\beta(\Psi)$  combine all together with the incoming velocity produces a resultant velocity. This resultant velocity can be split into three component velocity. They are namely: the component velocity normal to blade leading edge line  $U_T$ , the radial velocity  $U_R$  and the out plane velocity  $U_p$ . Those three component velocities, of course, would be function of blade azimuth position  $\Psi$  and the location of blade section  $r$  to the center of blade rotation. Figure 2 show the schematic diagram of velocities work on the rotor blade.

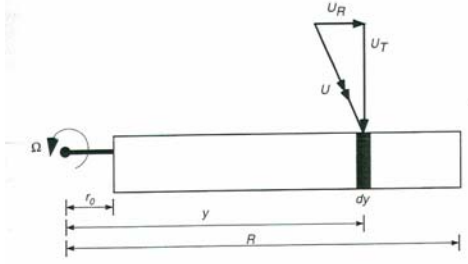


Fig.2.2 Diagram Velocities over the rotor blade helicopter (Leishman, 2000)

The three component velocities as mentioned above can be written, respectively as:

$$U_p = (U_\infty \sin \alpha_{TTP} + v_i) + r \frac{\partial \beta}{\partial t} + \mu \Omega R_B \beta(\Psi) \cos(\Psi) \quad 1-8a$$

$$U_T = \Omega r + U_\infty \cos \alpha_{TTP} \cos(\Psi) + \beta'(\Psi) + \quad 1-8b$$

$$U_R = U_\infty \cos \alpha_{TTP} \sin(\Psi) \quad 1-8c$$

Let consider a typical element or strip shown in the Figure 2.3.

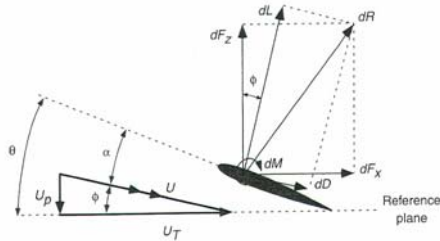


Figure 2.3: Velocity Diagram on the blade section (Leishman, 2000)

This element has a pitch angle equal to  $\theta$ . That is, the angle between the plane of rotation and the line of zero lift. Many rotor blades are twisted, so the pitch angle  $\theta$  varies with  $r$  and it should be noted as  $\theta(r)$ . The blade sees an in-plane velocity  $U_T$ , which represent the velocity in the direction of tangential to the plane of rotation. If there were no an out plane velocity  $U_{\infty\perp}$ , and induced inflow  $v_i$ , this angle would be the section angle of attack. Those two components of velocity  $U_{\infty\perp}$ , and  $v_i$  change the flow direction by amounts  $\Phi$ , as shown in the figure above, namely,

$$\Phi = \text{arctg} \left( \frac{U_p}{U_T} \right) \quad 1-9$$

Considering velocity components equation 1-8a and 1-8b for  $U_p$  and  $U_T$ . The first equation described that the  $U_p$  is uniform in radial as well as in the azimuth direction, which is opposite with the in plane component velocity  $U_T$ . As result that the inflow angle  $\Phi$  need to be presented as  $\Phi(r, \Psi)$ .

The effective angle of attack  $\alpha_{\text{eff}}$ , then, can be defined as :

$$\alpha_{\text{eff}}(r, \Psi) = \theta(r) - \Phi(r, \Psi) \quad 1-10$$

The airfoil lift and drag coefficients  $C_l(\alpha_{\text{eff}})$  and  $C_d(\alpha_{\text{eff}})$  at this effective angle of attack  $\alpha_{\text{eff}}$  may be looked up from a table of airfoil characteristics. The lift and drag forces will be perpendicular to, and along the apparent stream direction. This effective velocity works upon the differential blade element  $\Delta r$ , creates the differential lift  $\Delta L(r, \Psi)$  and the differential drag  $\Delta D(r, \Psi)$  are given by :

$$\Delta L(r, \Psi) = \frac{1}{2} \rho (U_p^2 + U_T^2)^2 c(r) C_{+}(\alpha_{\text{eff}}) \Delta r \quad 1-11a$$

$$\Delta D(r, \Psi) = \frac{1}{2} \rho (U_p^2 + U_T^2)^2 c(r) C_{d}(\alpha_{\text{eff}}) \Delta r \quad 1-11b$$

Those two differential forces must be rotated in directions normal to, and tangential to the rotor disk, respectively, and producing the differential thrust  $\Delta T(r, \Psi)$  and the differential axial force  $\Delta F_x(r, \Psi)$  as given below :

$$\Delta T(r, \Psi) = (\Delta L \cos(\Phi) - \Delta D \sin(\Phi)) = \frac{1}{2} \rho (U_T^2 + U_p^2) c (C_l \cos(\Phi) - C_d \sin(\Phi)) \Delta r \quad 1-12a$$

$$\Delta F_x(r, \Psi) = (\Delta D \cos(\Phi) + \Delta L \sin(\Phi)) = \frac{1}{2} \rho (U_T^2 + U_p^2) c (C_d \cos(\Phi) + C_l \sin(\Phi)) \Delta r \quad 1-12b$$

The differential torque  $\Delta Q(r, \Psi)$  and differential power  $\Delta P(r, \Psi)$  can be obtained, respectively, as :

$$\Delta Q(r, \Psi) = r \Delta F_x(r, \Psi) \quad 1-13a$$

$$\Delta P(r, \Psi) = \Omega r \Delta F_x(r, \Psi) = \Omega \Delta Q(r, \Psi) \quad 1-13b$$

Finally, the thrust  $T(\Psi)$ , torque  $Q(\Psi)$  and power  $P(\Psi)$  may be found by integrating  $\Delta T(r, \Psi)$ ,  $\Delta Q(r, \Psi)$  and  $\Delta P(r, \Psi)$  above from root to tip ( $r=0$  to  $r=R_B$ ), and multiplying the results by the total number of blades  $N_b$  for a given certain blade azimuth position  $\Psi$ . They are namely:

$$T(\Psi) = \int_{r=R_0}^{r=R_B} \frac{1}{2} \rho (U_T^2 + U_p^2) c (C_l \cos(\Phi) - C_d \sin(\Phi)) dr \quad 1-14a$$

$$Q(\Psi) = \int_{r=R_0}^{r=R_B} \Delta F_x(r, \Psi) r = \int_{r=0}^{r=R_B} \frac{1}{2} \rho (U_T^2 + U_p^2) c (C_d \cos(\Phi) + C_l \sin(\Phi)) r dr \quad 1-14b$$

$$P(\Psi) = \int_{r=R_0}^{r=R_B} \Delta F_x(r, \Psi) \Omega r = \int_{r=0}^{r=R_B} \frac{1}{2} \rho (U_T^2 + U_p^2) c (C_d \cos(\Phi) + C_l \sin(\Phi)) \Omega r dr \quad 1-14c$$

The average thrust  $T_{\text{ave}}$ , torque  $Q_{\text{ave}}$  and power  $P_{\text{ave}}$  can be found by integrating  $T(\Psi)$ ,  $Q(\Psi)$  and  $P(\Psi)$  from  $\Psi = 0^0$  to  $\Psi = 360^0$ . as :

$$T_{\text{ave}}(\Psi) = \frac{1}{2\pi} \int_0^{2\pi} \left( \int_{r=R_0}^{r=R_B} \frac{1}{2} \rho (U_T^2 + U_p^2) c (C_l \cos(\Phi) - C_d \sin(\Phi)) dr \right) d\Psi \quad 1-15a$$

$$Q_{\text{ave}}(\Psi) = \frac{1}{2\pi} \int_0^{2\pi} \left( \int_{r=0}^{r=R_B} \frac{1}{2} \rho (U_T^2 + U_p^2) c (C_d \cos(\Phi) + C_l \sin(\Phi)) r dr \right) d\Psi \quad 1-15b$$

and

$$P_{\text{ave}}(\Psi) = \frac{1}{2\pi} \int_0^{2\pi} \left( \int_{r=0}^{r=R_B} \frac{1}{2} \rho (U_T^2 + U_p^2) c (C_d \cos(\Phi) + C_l \sin(\Phi)) \Omega r dr \right) d\Psi \quad 1-15c$$

The above integration can, in general, be only numerically done since the chord  $c$ , the sectional lift and drag coefficients may vary along the span wise as well as in the azimuth direction. The inflow velocity  $v_i$  depends on  $T$ . Thus, an iterative process will be needed to find the quantity  $v_i$ . The iteration process would be accomplished, if the difference value between two successive iterations for the average thrust coefficient  $C_T^{i+1}$  and  $C_T^i$  defined as:

$$\left| \frac{C_T^{i+1} - C_T^i}{C_T^{i+1}} \right| \leq \varepsilon \quad 1-16$$

In the right hand side above equation  $\varepsilon$  represents an arbitrary prescribed value which here may one can chose to be equal 0.005.

## 2.2 Numerical Procedure.

The implementation of the combined Momentum theory and the blade Element theory for the aerodynamics rotor blade helicopter analysis can be described as shown by the flow diagram as depicted in the Figure 2.4.

Given rotor blade parameter geometry which involve: chord distribution  $c(r)$ , twist distribution  $\theta(r)$ , aerodynamic airfoil data for the blade section ( $c_{+}(\alpha)$  and  $c_{d}(\alpha)$ ), Coning angle as function of blade azimuth position  $\beta(\Psi)$ , blade radius and blade number. The input for the flight condition involve: the incoming flow velocity  $U_{\infty}$ , the angle of attack with respect to the disk plane  $\alpha_{TPP}$ , and the rotational speed rotor blade  $N_{RPM}$ .

Introduce the initial value of thrust coefficient for starting calculation the induced velocity by solving the induced velocity equation derived from the Momentum Theory as given by Eq. 1.1. This equation represents the non linear equation in term of the unknown induced velocity, so an iteration process was required. It can be done by using Newton Raphson's iteration Method. As the induced velocity available the effective velocity at each blade section  $r$  and the blade azimuth position can be calculated. The effective angle angle of attack, then, can be obtained and so the differential lift and drag by using look up table airfoil data can be found. Finally the total thrust, torque and power at each blade azimuth through numerical integration along blade span can be obtained. If the blade azimuth position over one revolution divided into  $N_R$  number of blade azimuth position, then carried out the calculation rotor blade aerodynamics performance over those  $N_R$  number of blade position. Sum up to those obtained Thrust coefficient and then the result divided by  $N_R$  would give the average thrust coefficient. In a similar way for obtaining the average torque and power coefficient.

## III. Discussion and Result

It had been identified that there are various factors influence the aerodynamics performance of the rotor blade helicopter. Among of those: the number of blade, blade airfoil section, twist distribution, chord distribution and the coning angle  $\beta$  as the blade rotates from one blade azimuth position to other. Another factors are, of course, the incoming velocities with respect to the rotor disk plane and the rotational speed of the rotor blade.

For the purpose of assessment of the capability of the present method the following data which had been fixed, namely:

1. Blade number  $N_B : 2$
2. Blade radius  $R_B : 6$  meters
3. Uniform blade chord  $c(r) : 0.4$  meter
4. Rotational speed rotor blade : 400 Rpm
5. Angle of attack with respect to the rotor disk plane  $\alpha_{TPP} = 8^{\circ}$
6. Number of blade element  $N_{BE} : 40$
7. Number of division of blade azimuth position  $N_R : 60$
8. Inner blade radius for starting the Blade Element theory applied  $R_o = 0.1$  meter

The others required data to allow the present method to accomplish aerodynamics performance analysis can be considered as the test cases under investigation in this study.

First test case:

The blade was assumed to have a uniform pitch angle  $\theta(r) = 8^{\circ}$ . The aerodynamics characteristic of the airfoil section defined to follow as given in Ref. 6 as :

$$\begin{aligned} c_{+}(\alpha) &= 2\pi \alpha \\ c_{d}(\alpha) &= 0.1 + 0.025 \alpha + 0.65 \alpha^2 \end{aligned}$$

1-17

$\alpha$  in radian.

The coning angle as function of blade azimuth position  $\beta(\Psi)$  is set equal to zero.

The incoming velocity of forward flight had been selected to be 20 m/sec, 40 m/sec and 50 m/sec respectively. With the rotor rotational speed at 400 RPM and the tip to tip plane angle  $\alpha_{TPP} = 8^{\circ}$ , would correspond to the advance ratio at  $\mu = 0.111$ ,  $\mu = 0.111$  and  $\mu = 0.111$ . All calculations used the initial value for the thrust coefficient,  $C_{T0} = 0.002$ . The criteria of convergence between two successive iteration was defined by equation 1-16. The aerodynamic performance for above rotor blade for

three different values of velocity in term of thrust coefficient, torque coefficient and the required number of iteration were summarized as shown in the Table 1.

Table 1: Comparison result of thrust coefficient and torque coefficient at different velocity of the forward flight

Figure 3.1 shows the comparison result of thrust coefficient as function of blade azimuth position for three different value of forward speed. While in term of torque coefficient as shown in the Fig. 3.2.

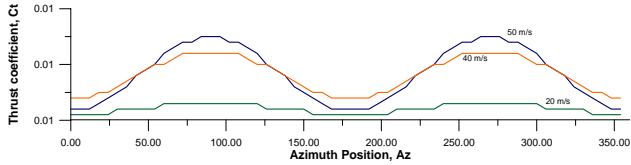


Figure 3.1

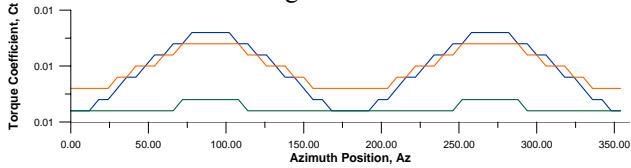


Figure 2

Second test case :

The following test case used a similar configuration of rotor blade in the previous test case. The difference with the previous case is in term of twist angle. If in the previous case there is no twist, now, the problem in hand the blade has twist angle. Here two types of twist angle are presented, they are namely: 1) ideal twist and 2) linear twist, are given respectively as:

$$\text{Ideal twist : } \theta(r) = \frac{\theta_{\text{tip}}}{r} \quad 1-18a$$

And

$$\text{Linear twist: } \theta(r) = \theta_0 + \theta_{\text{tw}} r \quad 1-18b$$

For the ideal twist model, three values of pitch at the tip  $\theta_{\text{Tip}}$  had been selected, namely:  $\theta_{\text{Tip}} = 8^\circ$ ,  $4^\circ$  and  $\theta_{\text{Tip}} = 2^\circ$ . The problem in hand had been run at the forward speed 50 m/sec. The result in

$U_\infty$ (m/sec)	Advance ratio $\mu$	Thrust Coef. $C_T$	Torque Coef. $C_Q$	Iteration number
20	0.079	0.00621	0.00511	12
40	0.158	0.00682	0.00548	7
50	0.197	0.00684	0.00546	6

term thrust coefficient, torque coefficient and the required number of iterations as shown in the Table 2 bellows:

$\theta_{\text{Tip}}$ Angle (deg)	Advance ratio $\mu$	Thrust Coef. $C_T$	Torque Coef. $C_Q$	Iteration number
8	0.197	0.01231	0.00780	6
4	0.197	0.00477	0.00295	6
2	0.197	0.00103	0.00057	6

Figure 3.3 showed the comparison result for the thrust coefficient as function of blade azimuth position between an ideal twist and linear twist at the forward speed 50 m/sec. Both ideal and linear twist model have pitch angle at the tip  $\theta_{\text{Tip}} = 2.0$ . The linear twist used Eq. 1-18b as the pitch distribution along span wise with  $\theta_0 = 9^\circ$  and  $\theta_{\text{tw}} = -7^\circ$ . The comparison result of torque coefficients  $C_T$  plotted as function of blade azimuth position between ideal and linear twist as shown in the Figure 3.4

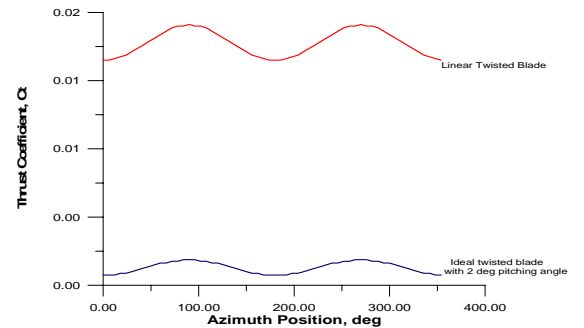


Figure 3.3

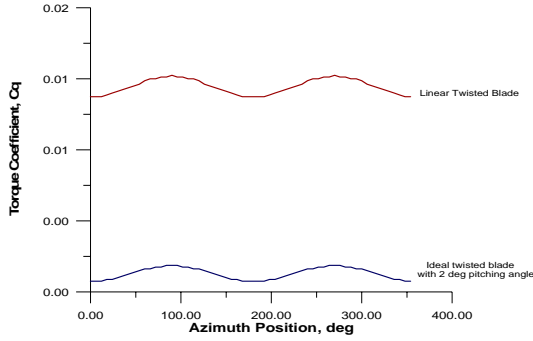


Figure 3.4

Third test case:

Here the blade had coning angle  $\beta$  which varying with respect to the blade azimuth position as adopted from Ref. 6.

$$\beta(\Psi) = 6^\circ - 4^\circ \cos(\Psi) - 4^\circ \sin(\Psi)$$

The blade was assumed to have a linear twist as it was discussed in the second test case.

The problem in hand was run under a similar condition as carried out to the first and second test case. Here the blade was assumed a linear twist as described in the second test case. Figure 3.5 showed a comparison result of thrust coefficient as function of blade position between the rotor blade with and with out coning angle  $\beta$ . While in term of torque coefficient as depicted in the Figure 3.6. The average thrust coefficient for no coning angle was obtained  $C_{Tave} = 0.01424$ , while introducing a coning angle as given by Eq. 1-19 produced  $C_{Tave} = 0.01422$ . This problem was run at forward speed 50 m/sec.

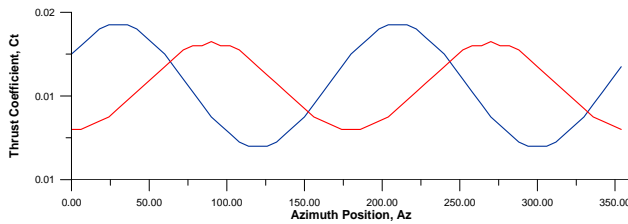


Figure 3.5

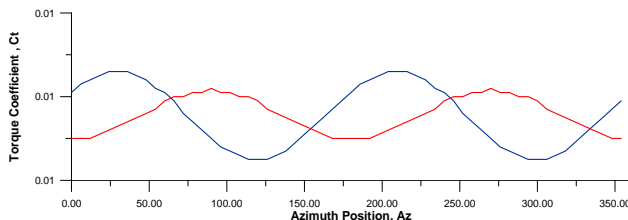


Figure 3.6

The fourth Test case

If the first to the third test case was assumed that the aerodynamics characteristic of the airfoil section was given by Eq. 1.17. In term of lift coefficient, Eq. 1.17 represents the aerodynamics characteristic of an ideal airfoil. The fourth test case was introduced showed the influence of airfoil data to the solution. Here the blade was considered to use airfoil section NACA 23015, which follow Ref. 9, the aerodynamics characteristic for this airfoil are given by :

$$\begin{aligned} c_+(\alpha) &= 1.5 - 0.0188 (\alpha - 14.0)^2 \quad \text{for } |\alpha| > 15^\circ \\ c_+(\alpha) &= 1.5 - 0.0188 (14.0 - \alpha)^2 \quad \text{for } 10^\circ \leq |\alpha| \leq 15^\circ \\ c_+(\alpha) &= 0.10 + 0.11 \alpha \quad \text{for } -10^\circ \leq |\alpha| \leq 10^\circ \end{aligned} \quad 1-20a$$

and

$$c_d(\alpha) = 0.007 + 0.0055 (|c_+(\alpha)| - 0.2)^2 \quad \text{for } -10^\circ \leq |\alpha| \leq 10^\circ$$

$$c_d(\alpha) = 0.0125 + 0.16 (|c_+(\alpha)| - 1.1)^2 \quad 1-20b$$

The comparison result in term of thrust coefficient  $C_T$  with respect to azimuth position for the case linear twist with the airfoil section characteristics Eq. 1-17 and Eq. 1-20 as depicted in the Figure 3.7, while in term of torque coefficient  $C_Q$  as shown in The Figure 3.8. The coning angle  $\beta$  as given by Eq. 1-19 was imposed those analysis. Table 3 showed the calculation result for the fourth test case for different value of forward speed.

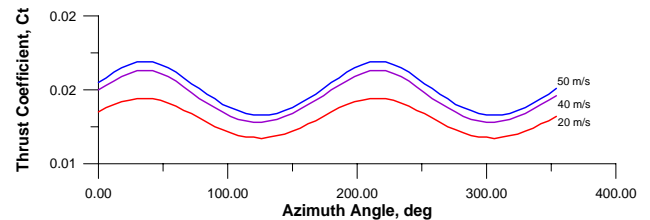


Figure 3.7



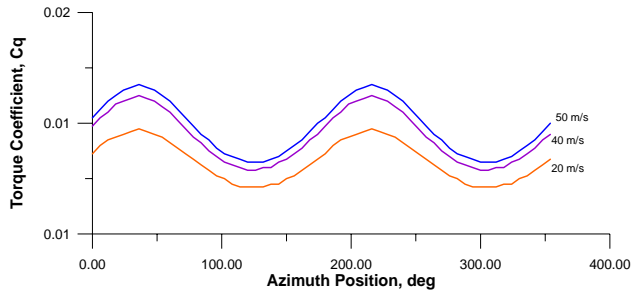


Figure 3.8

$U_\infty$ (m/sec)	Advance ratio $\mu$	Thrust Coef. $C_T$	Torque Coef. $C_Q$	Iteration number
20	0.079	0.01304	0.01066	7
40	0.158	0.01447	0.01155	6
50	0.197	0.01501	0.01189	5

#### IV. Conclusion and Future work

#### V. Reference

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