

DEVELOPMENT OF SAND PRODUCTION PREDICTION COMPUTER MODEL

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ABSTRACT

During production, sand inflow from poorly consolidated reservoir will cause problems of wear, casing collapse and surface facility deterioration. However, if the sand control completion is used to minimize sand production risk, it may lead to high skins and resulting in large productivity losses. On the other hand, completion without sand control device may lead to sand production problems with economical loss and safety risks. The best way to assess the problem is prediction of sand production, which basically come from the perforation. Therefore, a two-dimensional numerical model will be developed to simulate and monitor the stability of perforation and predict when sand will be produced during production phase. In developing the perforation cavity stability prediction model, the rock is assumed to be isotropic and homogeneous with the pores completely filled with fluid. The deformation condition is considered as a plane strain. The prediction model will employ an appropriate elastoplastic constitutive law, coupled with two-phase fluid flow model (oil and water). The prediction model will account for the perforation pattern, density, phasing angle and two-phase fluid production. The modeling geometry will be one foot of the borehole segment with perforations. The finite-element method will be used to develop the prediction model. A two-dimensional non-linear thermo-elasto-plastic consolidation program called *PLASCON* will be used as the platform to develop the prediction model. The developed prediction model can be used to predict the onset of sand production.

Key Words : Sand Production, Sand Control, Perforation Stability, Finite Element Method

Background

Mechanical failure of the formation around perforation can cause sand production that may lead to serious engineering and economic problems. If the sand control completion is used to minimize sand production risk, it may lead to high skins and resulting in large productivity losses. On the other hand, completion without sand control device may lead to sand production problems with economical loss and safety risks. The best way to assess the problems is prediction of sand production that basically come from the perforation. By using the sand production prediction model, condition lead to sand production can be estimated and prevention can be done before it occurs.

Sand production prediction refers to the forecasting of perforation stability. Considerable efforts have been invested in prediction of production conditions leading to sand release. However, the methods for prediction of perforation behaviour during production phase still lack of engineering precision. Some complex 3-D numerical models (Morita & Whitfill, 1987; Abdulazeez et al., 1997) for sand production prediction have been developed, but the simulation of the complete borehole segment with perforations still out of reach. To better predict sand production, the prediction model must be capable of modelling a complete borehole segment with perforation cavities, which will be accomplished through the development of a sand production prediction computer model.

In this paper, brief discussion on the development of computer model for predicting sand production will be presented. The main objectives of development are to simulate the behaviour of borehole with perforation cavities during production phase, predicting the onset of sand production, determine the sand production rate (or quantity of sand produced) and study the effect of perforation pattern, perforation density, phasing angle and 2-phase fluid (oil and water) production on the perforation cavity stability. The computer model is developed in terms of 2-D space and 2-phase fluid. Only one foot of borehole segment with perforation cavities will be simulated by the computer model. The finite element method will be used to develop the computer model. A 2-D nonlinear thermo-elasto-plastic consolidation program called *PLASCON* will be used as platform to develop the sand production prediction computer model.

Model Description

The perforation cavity geometry is assumed as a cylinder with an open end and the other end is semi sphere. The single perforation cavity geometry is shown in Figure 1. For the borehole segment, before modelling works take place, the borehole is opened up to become a rectangle with the open end of perforation cavity as shown in Figure 2. The simulation starts from the one slice and then proceeds to its neighbouring slices until all the slices have been simulated. After this, the simulation will start again with the first slice. The simulation will stop when the stopping criterion is met, i.e. simulation time or the maximum quantity of sand produced. The interactions between slices are considered by taking into account the third stress component that act between these slices. The 8-node isoparametric element will be used to construct the perforation cavity geometry.

Reservoir rock surrounding the perforation cavity is assumed isothermal, isotropic and homogeneous with its pores completely filled with fluid. The deformation condition is considered plane strain and nonlinear. The perforation boundary that contact with cement is assumed under no flow and displacement condition. The oil and water either in perforation cavity or in pores are considered totally immiscible.

The Numerical Model Development

The sand production prediction computer model will be written in Fortran. Generally, the computer model comprised of 2 main elements: flow continuity equation (for oil and water phase) and equilibrium equation. The flow continuity equations calculate the pore pressure distribution around perforation cavity for a given set of boundary conditions. This equation included the rate of total strain change, rate of grain volume

change due to pressure changes, rate of saturation change, rate of fluid density change and change of grain size due to effective stress changes. The equilibrium equation with the pore pressure evaluated by flow continuity equation, calculate stress state, elastic deformation and plastic deformation. The flow continuity equation for oil phase in terms of oil phase pressure, p_o and water phase pressure, p_w is:

$$-\nabla^T \left\{ \mathbf{k} \frac{k_{ro}}{\mu_o B_o} \nabla(p_o + \rho_o g h) \right\} + \lambda_{oo} \frac{\partial p_o}{\partial t} + \lambda_{wo} \frac{\partial p_w}{\partial t} + \frac{S_o}{B_o} \left(\mathbf{m}^T - \frac{\mathbf{m}^T \mathbf{D}_T}{3K_s} \right) \frac{\partial \epsilon}{\partial t} + \frac{S_o}{B_o} \frac{\mathbf{m}^T \mathbf{D}_T \mathbf{c}}{3K_s} = 0 \quad (1)$$

Where

$$\begin{aligned} \lambda_{oo} &= -\frac{\phi}{B_o} S_w' + \phi S_o \left(\frac{1}{B_o} \right)' + \frac{S_o}{B_o} C_{rm} \overline{SO} \\ \lambda_{wo} &= \frac{\phi}{B_o} S_w' + \frac{S_o}{B_o} C_{rm} \overline{SW} \\ C_{rm} &= \frac{(1-\phi)}{K_s} - \frac{1}{(3K_s)^2} \mathbf{m}^T \mathbf{D}_T \mathbf{m} \\ \overline{SO} &= S_o - p_o S_w' + p_w S_w' \\ \overline{SW} &= S_w + p_o S_w' - p_w S_w' \end{aligned}$$

and for water phase is:

$$-\nabla^T \left\{ \mathbf{k} \frac{k_{rw}}{\mu_w B_w} \nabla(p_w + \rho_w g h) \right\} + \lambda_{ow} \frac{\partial p_o}{\partial t} + \lambda_{ww} \frac{\partial p_w}{\partial t} + \frac{S_w}{B_w} \left(\mathbf{m}^T - \frac{\mathbf{m}^T \mathbf{D}_T}{3K_s} \right) \frac{\partial \epsilon}{\partial t} + \frac{S_w}{B_w} \frac{\mathbf{m}^T \mathbf{D}_T \mathbf{c}}{3K_s} = 0 \quad (2)$$

Where

$$\begin{aligned} \lambda_{ow} &= \frac{\phi}{B_w} S_w' + \frac{S_w}{B_w} C_{rm} \overline{SO} \\ \lambda_{ww} &= -\frac{\phi}{B_w} S_w' + \phi S_w \left(\frac{1}{B_w} \right)' + \frac{S_w}{B_w} C_{rm} \overline{SW} \end{aligned}$$

While the equilibrium equation with body force, \mathbf{b} and boundary traction, $\hat{\mathbf{t}}$ can be written as:

$$\begin{aligned} \int_{\Omega} \delta \epsilon^T \mathbf{D}_T \frac{\partial \epsilon}{\partial t} d\Omega + \int_{\Omega} \delta \epsilon^T \left(\frac{\mathbf{D}_T \mathbf{m}}{3K_s} - \mathbf{m} \right) \overline{SO} \frac{\partial p_o}{\partial t} d\Omega + \int_{\Omega} \delta \epsilon^T \left(\frac{\mathbf{D}_T \mathbf{m}}{3K_s} - \mathbf{m} \right) \overline{SW} \frac{\partial p_w}{\partial t} d\Omega \\ - \int_{\Omega} \delta \epsilon^T \mathbf{D}_T \mathbf{c} d\Omega - \int_{\Omega} \delta \epsilon^T \mathbf{D}_T \frac{\partial \epsilon_a}{\partial t} d\Omega - \int_{\Omega} \delta \mathbf{u}^T \frac{d\mathbf{b}}{dt} d\Omega - \int_{\Gamma} \delta \mathbf{u}^T \frac{d\hat{\mathbf{t}}}{dt} d\Gamma = 0 \end{aligned} \quad (3)$$

The \mathbf{D}^T in equations (1), (2) and (3) is replaced by \mathbf{D}^e for elastic deformation and \mathbf{D}^{ep} for plastic deformation.

To increase precision of computer model, a fully coupled solution of the 2-phase flow equation in elastoplastic porous medium is adopted. The finite element

discretization of the flow continuity equations and equilibrium equation may now be expressed in terms of the nodal point displacement, $\bar{\mathbf{u}}$ and nodal point fluid pressures, i.e. $\bar{\mathbf{p}}_o$ and $\bar{\mathbf{p}}_w$ by using the Galerkin method. The unknowns are related to their nodal point values by the following expressions:

$$\mathbf{u} = \mathbf{N}\bar{\mathbf{u}} \quad \boldsymbol{\varepsilon} = \mathbf{B}\bar{\mathbf{u}} \quad \mathbf{p}_o = \bar{\mathbf{N}}\bar{\mathbf{p}}_o \quad \mathbf{p}_w = \bar{\mathbf{N}}\bar{\mathbf{p}}_w \quad (4)$$

Substituting equation (4) into equation (1), (2) and (3), then combine them and rearrange, the following expression are obtained.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{H}_p & 0 \\ 0 & 0 & \mathbf{W}_p \end{bmatrix} \begin{Bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{p}}_o \\ \bar{\mathbf{p}}_w \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{L}_o & \mathbf{L}_w \\ \mathbf{H}_u & \mathbf{H}_o & \mathbf{H}_w \\ \mathbf{W}_u & \mathbf{W}_o & \mathbf{W}_w \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{p}}_o \\ \bar{\mathbf{p}}_w \end{Bmatrix} = \begin{Bmatrix} \frac{d\mathbf{f}}{dt} + \mathbf{C} \\ \bar{\mathbf{F}}_o \\ \bar{\mathbf{F}}_w \end{Bmatrix} \quad (5)$$

The detailed explanation of the coefficient in equations (5) are given by Lewis and Schrefler (1987).

Equations (5) represent a set of ordinary differential equations in time. Discretization in time of equations (5) is carried out based on the Kantorovich approach which have been described in detail by Lewis and Schrefler (1987). Discretization of equations (5) w.r.t. time gives the following results.

$$\begin{bmatrix} \mathbf{K} & \mathbf{L}_o & \mathbf{L}_w \\ \mathbf{H}_u & \mathbf{H}_o + \alpha\mathbf{H}_p\Delta t_k & \mathbf{H}_w \\ \mathbf{W}_u & \mathbf{W}_o & \mathbf{W}_w + \alpha\mathbf{W}_p\Delta t_k \end{bmatrix}_{k,\alpha} \begin{Bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{p}}_o \\ \bar{\mathbf{p}}_w \end{Bmatrix}_{t_k + \Delta t_k} = \begin{bmatrix} \mathbf{K} & \mathbf{L}_o & \mathbf{L}_w \\ \mathbf{H}_u & \mathbf{H}_o - (1-\alpha)\mathbf{H}_p\Delta t_k & \mathbf{H}_w \\ \mathbf{W}_u & \mathbf{W}_o & \mathbf{W}_w - (1-\alpha)\mathbf{W}_p\Delta t_k \end{bmatrix}_{k,\alpha} \begin{Bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{p}}_o \\ \bar{\mathbf{p}}_w \end{Bmatrix}_{t_k} + \begin{Bmatrix} \frac{d\mathbf{f}}{dt} + \mathbf{C} \\ \bar{\mathbf{F}}_o \\ \bar{\mathbf{F}}_w \end{Bmatrix} \Delta t_k \quad (6)$$

Where

$$\alpha = \frac{t - t_k}{\Delta t_k}$$

Equations (6) represent a fully coupled and highly nonlinear system for 2-phase flow in porous medium.

Constitutive Model

Materials of rocks are generally highly nonlinear. Hence, elastoplastic constitutive model which taking into account the material response along different stress paths is used to describe the nonlinear behaviour of the rock. Two constitutive models have been identified suitable to apply in the perforation cavity stability simulation, which are Mohr-Coulomb yield surface and Drucker-Prager yield surface (Veeken et al., 1991; Brady, 1994). The expression defining the Mohr-Coulomb yield surface is:

$$F = (\sqrt{3}\cos\theta_0 - \sin\theta_0\sin\varphi)\mathbf{q} - 3p\sin\varphi - 3c\cos\varphi = 0 \quad (7)$$

$$Q = (\sqrt{3}\cos\theta_0 - \sin\theta_0\sin\psi)\mathbf{q} - 3p\sin\psi - 3\bar{c}\cos\psi = 0 \quad (8)$$

and for Drucker-Prager yield surface is:

$$F = -3\alpha p + \frac{1}{\sqrt{3}} q - k' = 0 \quad (9)$$

$$Q = -3\beta p + \frac{1}{\sqrt{3}} q = 0 \quad (10)$$

Where

$$\alpha = \frac{2 \sin \phi}{\sqrt{3}(3 + \sin \phi)}$$

$$k' = \frac{6cc \cos \phi}{\sqrt{3}(3 + \sin \phi)}$$

$$\beta = \frac{2 \sin \psi}{\sqrt{3}(3 + \sin \psi)}$$

Using the concept of associated flow rule, the tangential elastoplastic modulus matrix, \mathbf{D}^{ep} can be derived as:

$$\mathbf{D}^{ep} = \left[\begin{array}{c} \mathbf{D}^e - \frac{\mathbf{D}^e \frac{\partial Q}{\partial \boldsymbol{\sigma}'} \left\{ \frac{\partial F}{\partial \boldsymbol{\sigma}'} \right\}^T \mathbf{D}^e}{\mathbf{A} + \left\{ \frac{\partial F}{\partial \boldsymbol{\sigma}'} \right\}^T \mathbf{D}^e \frac{\partial Q}{\partial \boldsymbol{\sigma}'}} \end{array} \right] \quad (11)$$

The term \mathbf{A} is hardening modulus, which is responsible for controlling the position and size of yield surface is defined by the hardening rule. In this context, hardening is considered only depend on the plastic strain. Hence, it can be expressed as:

$$\mathbf{A} = - \left\{ \frac{\partial F}{\partial \boldsymbol{\epsilon}^p} \right\}^T \frac{\partial Q}{\partial \boldsymbol{\sigma}'} \quad (12)$$

Model Validation

Validation of the computer model must be done before it can be fully utilized. The developed computer model will be validated with laboratory experiment results which have been conducted by Surej (1997) and Chuong (2000). Parameters involved in the validation of the developed computer model are stress-strain behaviour of the rock with perforation during fluid production (or dynamic test), the onset of sand production and sand production rate or quantity of sand produced (at failure). From the validation result, the most suitable elastoplastic model will be selected (either Mohr-Coulomb or Drucker-Prager yield surface) and it will be used for further studies.

Conclusion

The developed sand production prediction computer model is able to simulate the behaviour of perforated borehole during production period. The onset and amount of the sand produced is also expected to be predicted by the developed computer model.

Therefore, the developed computer model can be used to predict the condition that cause perforation failure and sand production. With this simulation result, justification on the requirement of installing sand control device or using other well completion methods can be obtained. This may help in optimising the production operation of the well, which in turn can improve the well productivity.

Nomenclature

$\left(\frac{1}{B_o}\right)$ - slope of $\left(\frac{1}{B_o}\right)$ versus p_o curve

$\left(\frac{1}{B_w}\right)$ - slope of $\left(\frac{1}{B_w}\right)$ versus p_w curve

- A** - hardening modulus
- B** - formation volume factor
- B** - linear operator
- b** - body force
- c** - apparent cohesion
- c** - creep function
- D^e** - tangential elastic modulus matrix
- D^{ep}** - tangential elastoplastic modulus matrix
- D_T** - tangent matrix
- g** - gravity
- h** - the head above some arbitrary datum
- k** - absolute permeability matrix of the medium
- k_r** - relative permeability function
- K_s** - bulk modulus of solid phase
- m** - for normal stress components is unity and for shear stress components is zero
- N** - shape function
- \bar{N}** - shape function based on Galerkin method
- p** - fluid pressure
- \bar{p}** - mean stress invariant
- \underline{p}** - nodal point fluid pressure
- q** - deviator stress invariant
- q** - outflow rate per unit area of the boundary surface
- S** - fluid saturation
- t_k** - kth time step
- S_w'** - slope of the capillary curve
- \hat{t}** - boundary traction
- \bar{u}** - nodal point displacement
- δu** - virtual displacement
- ε** - total strain of rock skeleton
- ε_o** - autogeneous strain
- ε^p** - plastic strain
- φ** - angle of internal friction
- φ** - porosity
- μ** - dynamic viscosity
- θ₀** - angular stress invariant

ρ	- density
σ'	- effective stress
ψ	- dilatancy angle
Γ	- boundary
Ω	- domain
Δt_k	- the length of the k^{th} time step

Subscript

o	- oil phase
w	- water phase

Superscript

T	- matrix transpose
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Reference

1. Abdulazeez, A., Mohammad, R.A. and Salah, F.T. (1997). "A New Numerical Simulation Model and Upscaling Technique for Scaled Model Sand Production." SPE 37765.
2. Brady, B.H. (1994). "Some Recent Advances in Computational Geomechanics for Energy Resource Recovery." Computer Methods and Advances in Geomechanics, Siriwardane and Zaman (eds), Balkema Rotterdam. 21-31.
3. Chuong, P.V. (2000). "The Relationship Between Reservoir Rock Compaction and Sand Production." Master Thesis. Universiti Teknologi Malaysia.
4. Lewis, R.W. and Schrefler, B.A. (1987). "The Finite Element Method in the Deformation and Consolidation of Porous Media." Chichester: John Wiley and Sons.
5. Morita, N. and Whitfill, D.L. (1987). "Realistic Sand Production Prediction: Numerical Approach." SPE 16989.
6. Surej, K.S. (1997). "Wellbore Instability Studies by Physical Modelling." Master Thesis. Universiti Teknologi Malaysia.
7. Veeken, C.A.M., Davies, D.R., Kenter, C.J. and Kooijman, A.P. (1991). "Sand Production Prediction Review: Developing an Integrated Approach." SPE 22792.

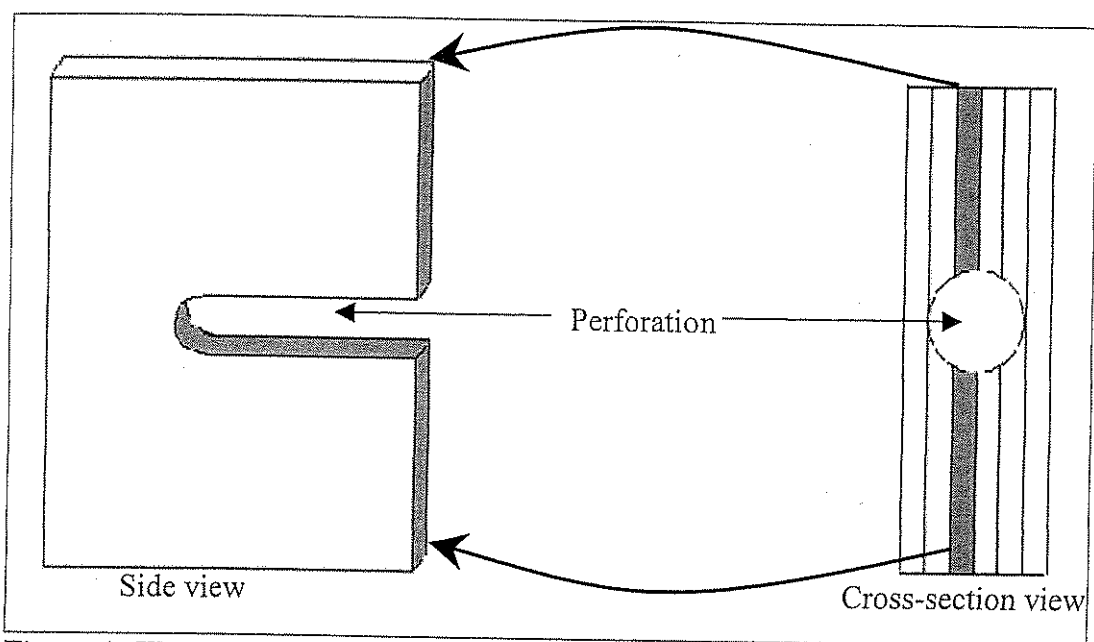


Figure 1: The geometry of single perforation.

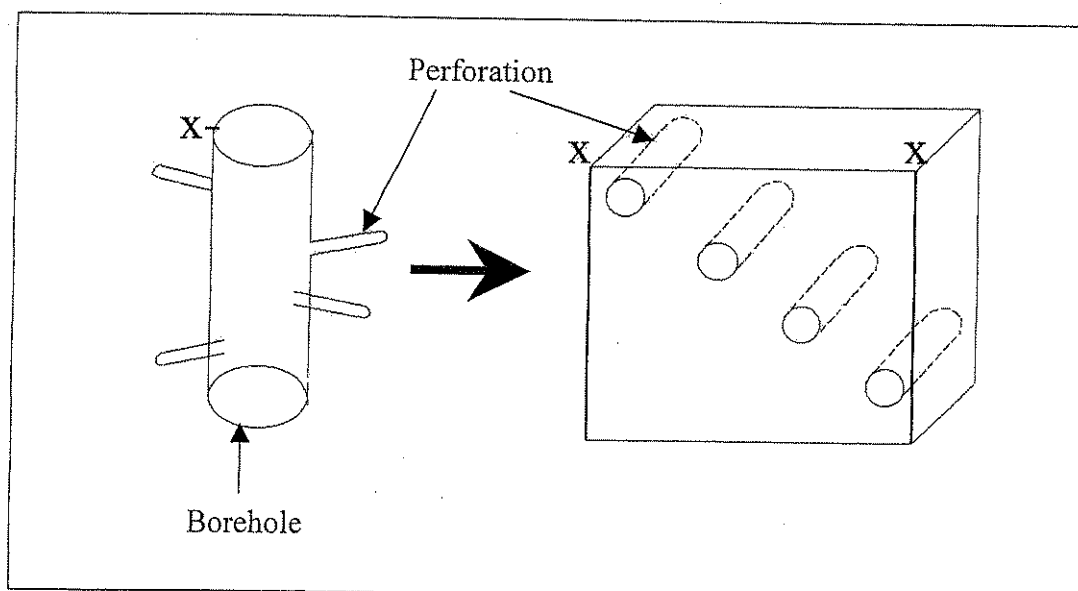


Figure 2: Modelling geometry of 4 spiral pattern perforations (4 SPF).