MODELING OF BLOOD FLOW AS EXTENDED KORTEWEG-DE VRIES AND EXTENDED KORTEWEG-DE VRIES BURGERS EQUATIONS

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A dissertation submitted as a partial fulfillment of the requirement for the award of the Degree of Master of Science (Engineering Mathematics)

Faculty of Science

Universiti Teknologi Malaysia

JANUARI 2013

To my beloved father and mother…

En. Saad bin Ismail & Pn. Sabariah bt Hj.Aziz

My supportive family…

Abang Det & Kak Not, Abang Cik, Kak Da & Abang Amil ,Kak Jiah & Abang Wan and my three little angels: Ammar, Amni and Dayyan.

Ands

To my naqibah & usrahmates, akhawats and all my friends.Thanks a lots and may Allah bless all of us.Ameen.

ACKNOWLEDGEMENTS

Alhamdulillah, all praise to Allah for giving me the opportunity, health, strength and patience to complete this dissertation.

I would like to express my deepest gratitude to my supervisor **Assoc.Prof. Dr Mukheta Isa** for his guidance and invaluable advice, comment and persistent encouragement throughout the course of this project.

My appreciation is also extended to all my lectures at Department of Mathematics for sharing their knowledge. And special thanks to my parents, family, classmates (MSJ), roomate and also to all my usrahmates for their concerns, helpful comments and encouragement that made the completion of this dissertation possible. May Allah bless all of us.

ABSTRACT

Blood is made of four components which are plasma, red blood cells, white blood cells and platelets. Many researchers have considered blood as an ideal fluid and the artery as an elastic tube. Due to complexity of blood deformation, a mathematical description of blood itself has not yet been completely formulated. In this research, we shall consider blood flow in an elastic artery. Our main focus is to model blood flow as KdV and KdVB type equations by using the governing equations and asymptotic series method. Specifically these equations are called the extended KdV (eKdV) and the extended KdVB (eKdVB). The hyperbolic tangent method is used in order to obtain the progressive wave solutions. It is found that for extended KdV equation, when there is no tapering $(a = 0)$, the blood flow follows the KdV model.

ABSTRAK

Darah terdiri daripada komponen plasma, sel darah merah, sel darah putih dan platelet. Kebanyakan penyelidik beranggapan bahawa darah ialah cecair yang ideal dan arteri sebagai tiub yang kenyal. Disebabkan oleh sifat darah yang kompleks, penghuraian dari segi matematik sendiri masih tidak dapat dilakukan secara tepat. Dalam kajian ini, kita tumpukan kepada aliran darah dalam tiub yang kenyal. Fokus utama ialah untuk memodelkan aliran darah sebagai persamaan Korteweg-de Vries (KdV) and KdV-Burgers (KdVB) dengan menggunakan persamaan yang ditetapkan dan kaedah siri asimptot. Secara spesifiknya, persamaan ini dikenali sebagai lanjutan KdV (eKdV) dan lanjutan KdVB (eKdVB). Kaedah garis sentuh hiperbolik digunakan bagi mendapatkan penyelesaian kepada gelombang progresif. Didapati bahawa apabila persamaan lanjutan KdV tidak mempunyai kesan sempitan terhadap tiub arteri $(a = 0)$, aliran darah akan berdasarkan model KdV.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Blood is made of four components which is plasma, red blood cells, white blood cells and platelets. Each of components plays an important role in our body. Blood flow is known to be an incompressible viscous fluid flowing in a distensible elastic membrane tube. Besides, the unique feature of arterial blood flow is its pulsatile character. The intermittent ejection of blood from the left ventricle produces pressure and flow pulse in the arterial tree. Experimental studies reveal that the flow velocity in blood vessel largely depends on the elastics properties of the vessel wall, and that they propagate towards the periphery with a characteristic pattern. Theoretical investigation for the blood waves have been developed by many researchers through the use of weakly nonlinear theories. In this research, our aim is to model blood flows as a Korteweg-de Vries (KdV) and KdV-Burgers (KdVB) equations. Specifically as the extended KdV (eKdV), the extended KdVB (eKdVB) and the extended Pertubed-KdV equations.

The nonlinear KdV equation is a famous model of water waves in dispersive media while burgers equations in dissipative media, Karpman (1975). The balance between the nonlinearity can cause the steepening and dissipation that make the attenuation of waves, and nonlinearity and dispersion which causes the broadening of waves. The balance between nonlinearity and dispersion can lead to the occurrence of stable nonlinear structures called solitary waves. The existence of balance between the nonlinearity, dispersion and dissipation resulting the Korteweg-de Vries –Burgers (KdVB) equation which represents the combination of KdV and Burgers equations. In this research we assumed that arteries as a tapered, thin walled, long and circularly prestressed elastic tube and blood as an incompressible viscous fluid.

1.2 Background of the Problem

Blood is made of four components which are plasma, red blood cell, white blood cell and platelets. Each of components plays an important role in blood. Blood carries the oxygen that the body needs around the body for use then returns the oxygen poor blood back to the lungs to replace the missing oxygen. The process known as blood flow or blood circulation. Due to complexity of blood rheology, a mathematical description of blood itself has not yet been completely formulated. In the systematic circulation, the large vessels are approximated by tubes with thin, elastic walls, while the blood filling the vessels is considered as a continuum, incompressible fluids.

The Korteweg–de Vries equation (KdV) is a mathematical model of waves on shallow water surfaces. The equation is named for Diederik Korteweg and Gustav de Vries who studied it in (Korteweg & de Vries 1895), though the equation first appears in (Boussinesq 1877, p. 360). The shallow water equations are the simplest form of the equations of motion that can be used to describe the horizontal structure of an atmosphere. They describe the evolution of an incompressible fluid in response to gravitational and rotational accelerations. The KdV equation is also the best model of the dispersive waves and under certain simplifying conditions covers cases for surface waves of long wavelength in liquids, plasma waves, lattice waves and weakly nonlinear magnetohydrodynamic waves.

The existence of balance between the nonlinearity, dispersion and dissipation resulting the Korteweg-de Vries –Burgers (KdVB) equation which represents the combination of KdV and Burgers equations. The Burgers equation is the simplest model of diffusive waves and under certain simplifying assumptions, it is cover cases of turbulence, sound waves in viscous media, waves in fluid-filled visco elastic tubes and magnetohydrodynamic waves in media having finite electrical conductivity. The KdVB equation is a well-known model equation in the study of shock waves in fluids and plasma. Therefore, in this research our aim is to model blood flow as the extended KdV (eKdV), extended KdV-Burgers (eKdVB) and extended Pertubed-KdV equations by considering blood flow as an incompressible viscous fluid and treating the arteries as a tapered, thin walled, long and circularly conical prestressed elastic tube.

1.3 Statement of the Problem

Nonlinear wave propagation in blood flow is a subject that has been much studied over the past three decades. In this research, we assumed the arteries as a tapered, thin walled, long and circularly conical prestressed elastic tube. By considering the blood as an incompressible viscous fluid depending on viscosity, we will model blood flows as eKdV, eKdVB and extended Pertubed-KdV equations. Throughout this research, the effect of tapering will be discovered.

1.4 Objective of the Research

The objectives of this dissertation are

i. To model a blood flow as extended KdV, extended KdV-Burgers and extended Pertubed-KdV equations.

ii. To study the effect of tapering on blood flow based on the extended KdV equation.

1.5 Scope of the Project

In this research, the main focus will be on modeling of blood flow as e KdV, eKdVB and extended Pertubed-KdV equations. We assume the arteries as tapered, thin walled, long and circularly conical prestressed elastic tube. Tapered means becoming narrower towards one end and it is refered to the artery tubes . By considering the blood as an incompressible viscous fluid depending on viscosity, we will model blood flow as eKdV, eKdVB and extended Pertubed-KdV equations.

1.6 Methodology of the Research

This research will begin with the study on the past research papers for literature review. The derivation of eKdV, eKdVB and extended Pertubed-KdV equations will be obtained in order to model blood flow. The hyperbolic tangent method will be used to obtain the progressive wave solution.

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