

**UNSTEADY MAGNETOHYDRODYNAMICS FLOW OF A MICROPOLAR  
FLUID WITH HEAT AND MASS TRANSFER**

AURANGZAIB MANGI

UNIVERSITI TEKNOLOGI MALAYSIA

UNSTEADY MAGNETOHYDRODYNAMICS FLOW OF A MICROPOLAR  
FLUID WITH HEAT AND MASS TRANSFER

AURANGZAIB MANGI

A thesis submitted in fulfilment of the  
requirements for the award of the degree of  
Doctor of Philosophy (Mathematics)

Faculty of Science  
Universiti Teknologi Malaysia

MAY 2013

*To My Beloved Family*

## **ACKNOWLEDEMENT**

First, and foremost, “All Almighty Allah SWT is with me at each step” and I am very thankful of His Great Gratitude. I offer my humblest, sincerest and billons of Darood to Holy Prophet Hazrat Muhammad (PBUH) who exhorts his followers to seek knowledge from cradle to grave.

I would like to sincere thanks to my honorable and devoted supervisor Associate Professor Dr. Sharidan Shafie for his supervision and valuable suggestion and extraordinary efforts from beginning of my study. Without his efforts it is not possible for me to complete this difficult task.

Lastly and most importantly, I thank my family for love and encouragement throughout the whole period of my study.

## ABSTRACT

The unsteady boundary layer flow has become of great interest in the field of fluid mechanics including the area of convective double diffusion. This is due to the complexity of the problem by including extra independent time variable, especially in the study of magnetohydrodynamic flow immersed in a micropolar fluid. In this thesis, the unsteady two-dimensional laminar boundary layer and mixed convection stagnation point flow towards a stretching or shrinking sheet immersed in magnetohydrodynamic micropolar fluid are considered. Specific problems are considered with different effects such as Soret and Dufour effects, thermophoresis effect and slip effect. Along with these effects, the micropolar parameter, the magnetic parameter and the suction or injection parameter are also considered. The governing non-linear equations are transformed into a system of differential equations by using appropriate non-dimensional variables which are then solved numerically using an implicit finite difference scheme. Numerical results for the skin friction, the Nusselt number and the Sherwood number as well as the velocity, microrotation, temperature and concentration profiles for different physical parameters are presented graphically and in tabular form. The results obtained show that there is a smooth transition from small time solution to large time solution. It is also found that with an increase of Soret and Dufour numbers, the momentum boundary layer thickness increases whereas the microrotation boundary layer thickness decreases for assisting flow while a reverse trend is observed for opposing flow. The thermal and concentration boundary layer thicknesses increase in both cases. By increasing the values of the slip parameter, all the boundary layer thicknesses decrease. In addition, by increasing the values of thermophoresis, the concentration boundary layer thickness decreases.

## ABSTRAK

Aliran lapisan sempadan tak mantap telah menjadi suatu kajian yang amat menarik di dalam bidang mekanik bendalir termasuklah juga bidang resapan kembar berolak. Ini disebabkan oleh penambahan pembolehubah tak bersandar masa yang menjadikan masalah ini semakin rumit, terutamanya di dalam kajian aliran hidrodinamik magnet di dalam bendalir mikropolar. Di dalam tesis ini, aliran lapisan sempadan lamina dan olakan campuran titik genangan dua matra tak mantap ke arah kepingan meregang atau mengecut di dalam bendalir mikropolar hidrodinamik magnet dipertimbangkan. Masalah yang dipertimbangkan melibatkan pelbagai kesan, khususnya kesan Soret dan Dufour, kesan termoforesis dan kesan gelincir. Bersama dengan kesan ini, parameter mikropolar, parameter magnetik dan parameter sedutan atau suntikan juga dipertimbangkan. Persamaan menakluk tak linear diubah ke sistem persamaan pembezaan dengan menggunakan pembolehubah tak bermatra yang bersesuaian yang kemudiannya diselesaikan secara berangka menggunakan skim beza terhingga tersirat. Keputusan berangka bagi geseran kulit, nombor Nusselt dan nombor Sherwood beserta profil halaju, mikroputaran, suhu dan kepekatan bagi pelbagai parameter fizikal yang berbeza dipersembahkan secara grafik dan berjadual. Keputusan yang diperolehi menunjukkan adanya peralihan yang lancar daripada penyelesaian pada masa kecil kepada penyelesaian pada masa besar. Keputusan juga menunjukkan bahawa dengan meningkatnya nombor Soret dan Dufour, ketebalan lapisan sempadan momentum meningkat sedangkan ketebalan lapisan sempadan mikroputaran berkurangan bagi aliran berbantu manakala keadaan sebaliknya berlaku bagi aliran bertentang. Ketebalan lapisan sempadan terma dan kepekatan diperhatikan meningkat untuk kedua-dua kes. Peningkatan nilai parameter gelincir menyebabkan ketebalan kesemua lapisan sempadan berkurangan. Di samping itu, peningkatan nilai termoforesis juga mengakibatkan ketebalan lapisan sempadan kepekatan berkurangan.

## TABLE OF CONTENTS

<b>CHAPTER</b>	<b>TITLE</b>	<b>PAGE</b>
	<b>DECLARATION</b>	ii
	<b>DEDICATION</b>	iii
	<b>ACKNOWLEDEMENT</b>	iv
	<b>ABSTRACT</b>	v
	<b>ABSTRAK</b>	vi
	<b>TABLE OF CONTENTS</b>	vii
	<b>LIST OF TABLES</b>	x
	<b>LIST OF FIGURES</b>	xii
	<b>LIST OF SYMBOLS</b>	xx
	<b>LIST OF APPENDICES</b>	xxv
<b>1</b>	<b>INTRODUCTION</b>	<b>1</b>
	1.1 Research Background	1
	1.2 Problem Statements	5
	1.3 Objectives and Scope	6
	1.4 Significance of Study	7
	1.5 Thesis Outline	8
<b>2</b>	<b>LITERATURE REVIEW</b>	<b>10</b>
	2.1 Introduction	10
	2.2 Flow over a Stretching Sheet in a Micropolar Fluid	10
	2.3 Flow over a Shrinking Sheet in a Micropolar Fluid	15
	2.4 Micropolar Fluid in a Porous Medium	20

<b>3</b>	<b>MATHEMATICAL FORMULATION</b>	<b>23</b>
	3.1 Introduction	23
	3.2 Governing Equations	23
	3.3 Boussinesq Approximation	29
	3.4 Non-dimensional Equations	31
	3.5 Boundary Layer Approximation	34
	3.6 Non-similar Transformation	36
	3.7 Physical Quantities	38
<b>4</b>	<b>THERMOPHORESIS AND SUCTION OR INJECTION EFFECTS ON THE STAGNATION POINT FLOW TOWARDS A HORIZONTAL SHEET</b>	<b>40</b>
	4.1 Introduction	40
	4.2 Governing Equations	41
	4.3 Non-dimensional Equations	43
	4.4 Non-similar Transformation	43
	4.5 Solution Procedure	45
	4.6 Results and Discussions	46
<b>5</b>	<b>THERMOPHORESIS AND SLIP EFFECTS ON THE STAGNATION POINT FLOW TOWARDS A SHRINKING SHEET</b>	<b>64</b>
	5.1 Introduction	64
	5.2 Governing Equations	65
	5.3 Non-similar Transformation	66
	5.4 Results and Discussions	67
<b>6</b>	<b>MIXED CONVECTION STAGNATION POINT FLOW TOWARDS A PERMEABLE SHRINKING SHEET WITH SLIP EFFECT</b>	<b>85</b>
	6.1 Introduction	85
	6.2 Governing Equations	86
	6.3 Non-dimensional Equations and Boundary Layer Approximation	87



6.4	Non-similar Transformation	88
6.5	Results and Discussions	90
<b>7</b>	<b>MIXED CONVECTION STAGNATION POINT FLOW TOWARDS A STRETCHING SHEET IN A POROUS MEDIUM WITH SORET AND DUFOUR EFFECTS</b>	<b>107</b>
7.1	Introduction	107
7.2	Governing Equations	108
7.3	Non-dimensional Equations and Boundary Layer Approximation	109
7.4	Non-similar Transformation	111
7.5	Results and Discussions	113
<b>8</b>	<b>CONCLUSION</b>	<b>132</b>
8.1	Summary of Research	132
8.2	Suggestion for Future Research	136
	<b>REFERENCES</b>	<b>138</b>
	Appendices A-C	150-182

## LIST OF TABLES

TABLE NO.	TITLE	PAGE
4.1	Comparison of values of the skin friction of final steady-state flow ( $\xi = 1$ ) for various values of $f_0$ when $K = M = n = 0$	48
4.2	Results of the skin friction for various values of $K = 1, M = 1, f_0 = 1, n = 0.5, \xi = 0.6$ with different step sizes.	49
4.3	The skin friction, the Nusselt number and the Sherwood number for various values of $f_0$ and $\xi$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, n = 0.5$	49
5.1	Comparison of the values of $f''(0)$ for stretching sheet in the absence of concentration $\phi$ , thermophoresis $\tau$ and slip parameter $\delta$	70
5.2	Comparison of the values of $f''(0)$ for shrinking sheet when $K = 0$ in the absence of concentration $\phi$ , thermophoresis $\tau$ and slip parameter $\delta$ .	71
5.3	The reduced skin friction, the reduced Nusselt number and the reduced Sherwood number for various values of $\varepsilon$ and $\xi$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, n = 0, \delta = 0.2$	71
5.4	The reduced skin friction, the reduced Nusselt number and the reduced Sherwood number for various values of $\delta$ and $\xi$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, n = 0, \varepsilon = -0.75$	72
6.1	Comparison of the values of $f''(0)$ for stretching sheet in the absence of concentration $\phi$ , magnetic parameter $M$ and slip parameter $\delta$	93
6.2	Comparison of the values of $f''(0)$ for shrinking sheet when $K = 0$ in the absence of concentration $\phi$ , magnetic parameter	

	$M$ and slip parameter $\delta$ .	94
6.3	The reduced skin friction, the reduced Nusselt number and the reduced Sherwood number for various values of $f_0$ , $\lambda$ and $\varepsilon$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \delta = 0.2, \Omega = 1, \xi = 0.6$	94
6.4	The reduced skin friction, the reduced Nusselt number and the the reduced Sherwood number for various values of $f_0$ , $\lambda$ and $\delta$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \varepsilon = -0.75, \Omega = 1, \xi = 0.6$	95
7.1	Comparison of the values of $C_{fx} Re_x^{1/2}$ and $Nu_x / Re_x^{1/2}$ for different values of $Pr$ in absence of concentration $\phi$ , Soret $S_r$ and Dufour $D_f$ effects when $\chi = 1$ and $\lambda = 1$	117
7.2	Comparison of the values of $C_{fx} Re_x^{1/2}$ and $Nu_x / Re_x^{1/2}$ for different values of $Pr$ in absence of concentration $\phi$ , Soret $S_r$ and Dufour effects $D_f$ when $\chi = 1$ and $\lambda = -1$	117
7.3	Values of $C_{fx} Re_x^{1/2}$ , $Nu_x / Re_x^{1/2}$ and $Sh_x / Re_x^{1/2}$ for different values of $\lambda$ and $MD$ when $K = 1, Pr = 0.71, Sc = 0.94, n = 0, \Omega = 1, S_r = 0.2, D_f = 0.2, \chi = 1, \xi = 0.6$ for assisting ( $\lambda > 0$ ) and opposing ( $\lambda < 0$ ) flow	118
7.4	Values of $C_{fx} Re_x^{1/2}$ , $Nu_x / Re_x^{1/2}$ and $Sh_x / Re_x^{1/2}$ for different values of $\lambda$ and $S_r$ when $K = 1, MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, D_f = 0.2, \chi = 1, \xi = 0.6$ for assisting ( $\lambda > 0$ ) and opposing ( $\lambda < 0$ ) flow	119
7.5	Values of $C_{fx} Re_x^{1/2}$ , $Nu_x / Re_x^{1/2}$ and $Sh_x / Re_x^{1/2}$ for different values of $\lambda$ and $D_f$ when $K = 1, MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, S_r = 0.2, \chi = 1, \xi = 0.6$ for assisting ( $\lambda > 0$ ) and opposing ( $\lambda < 0$ ) flow	120

## LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
4.1	Physical model and coordinate system	41
4.2	The velocity profiles for various values of $\xi$ when $K = 1, M = 1, n = 0.5$ (a) suction ( $f_0 = 1$ ) and (b) injection ( $f_0 = -1$ )	50
4.3	The microrotation profiles for various values of $\xi$ when $K = 1, M = 1, n = 0.5$ (a) suction ( $f_0 = 1$ ) and (b) injection ( $f_0 = -1$ )	51
4.4	The temperature profiles for various values of $\xi$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0.5$ (a) suction ( $f_0 = 1$ ) and (b) injection ( $f_0 = -1$ )	52
4.5	The concentration profiles for various values of $\xi$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, n = 0.5$ (a) suction ( $f_0 = 1$ ) and (b) injection ( $f_0 = -1$ )	53
4.6	The velocity profiles for various values of $\xi$ when $K = 1, M = 1, n = 0$ (a) suction ( $f_0 = 1$ ) and (b) injection ( $f_0 = -1$ )	54
4.7	The microrotation profiles for various values of $\xi$ when $K = 1, M = 1, n = 0$ (a) suction ( $f_0 = 1$ ) and (b) injection ( $f_0 = -1$ )	55
4.8	The temperature profiles for various values of $\xi$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0$ (a) suction ( $f_0 = 1$ ) and (b) injection ( $f_0 = -1$ )	56

4.9	The concentration profiles for various values of $\xi$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, n = 0$ (a) suction ( $f_0 = 1$ ) and (b) injection ( $f_0 = -1$ )	57
4.10	The velocity profiles for various values of $f_0$ when $K = 1, M = 1, n = 0, \xi = 0.4$	58
4.11	The microrotation profiles for various values of $f_0$ when $K = 1, M = 1, n = 0, \xi = 0.4$	58
4.12	The temperature profiles for various values of $f_0$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \xi = 0.4$	59
4.13	The concentration profiles for various values of $f_0$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, n = 0, \xi = 0.4$	59
4.14	The velocity profiles for various values of $K$ for suction or injection when $M = 1, n = 0, \xi = 0.6$	60
4.15	The microrotation profiles for various values of $K$ for suction or injection when $M = 1, n = 0, \xi = 0.6$	60
4.16	The concentration profiles for various values of $\tau$ for suction or injection when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \xi = 0.6$	61
4.17	The velocity profiles for various values of $M$ for suction or injection when $K = 1, n = 0, \xi = 0.6$	61
4.18	The microrotation profiles for various values of $M$ for suction or injection when $K = 1, n = 0, \xi = 0.6$	62
4.19	Variation of the skin friction with $\xi$ for different values of suction or injection when $K = 1, M = 1, n = 0.5$	62
4.20	Variation of the Nusselt number with $\xi$ for different values of suction or injection when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0.5$	63
4.21	Variation of the Sherwood number with $\xi$ for different values of suction or injection when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, n = 0.5$	63
5.1	Physical model and coordinate system	65

- 5.2 The velocity profiles for various values of  $\varepsilon$  when  $K = 1, M = 1, n = 0, \delta = 0.2, \xi = 0.8$  73
- 5.3 The microrotation profiles for various values of  $\varepsilon$  when  $K = 1, M = 1, n = 0, \delta = 0.2, \xi = 0.8$  73
- 5.4 The temperature profiles for various values of  $\varepsilon$  when  $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \delta = 0.2, \xi = 0.8$  74
- 5.5 The concentration profiles for various values of  $\varepsilon$  when  $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, n = 0, \delta = 0.2, \xi = 0.8$  74
- 5.6 The velocity profiles for various values of  $\delta$  when  $K = 1, M = 1, n = 0, \varepsilon = -0.75, \xi = 0.8$  75
- 5.7 The microrotation profiles for various values of  $\delta$  when  $K = 1, M = 1, n = 0, \varepsilon = -0.75, \xi = 0.8$  75
- 5.8 The temperature profiles for various values of  $\delta$  when  $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \varepsilon = -0.75, \xi = 0.8$  76
- 5.9 The concentration profiles for various values of  $\delta$  when  $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, n = 0, \varepsilon = -0.75, \xi = 0.8$  76
- 5.10 The velocity profiles for various values of  $\xi$  when  $K = 1, M = 1, \varepsilon = -0.75, \delta = 0.2$  for (a)  $n = 0.5$  and (b)  $n = 0$  77
- 5.11 The microrotation profiles for various values of  $\xi$  when  $K = 1, M = 1, \varepsilon = -0.75, \delta = 0.2$  for (a)  $n = 0.5$  and (b)  $n = 0$  78
- 5.12 The temperature profiles for various values of  $\xi$  when  $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \varepsilon = -0.75, \delta = 0.2$  for (a)  $n = 0.5$  and (b)  $n = 0$  79
- 5.13 The concentration profiles for various values of  $\xi$  when  $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, \varepsilon = -0.75, \delta = 0.2$  for (a)  $n = 0.5$  and (b)  $n = 0$  80
- 5.14 The concentration profiles for various values of  $\tau$  when  $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \varepsilon = -0.75, \delta = 0.2, \xi = 0.8$  81
- 5.15 Variation of the reduced Sherwood number with  $\xi$  for different values of  $\tau$  when  $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \varepsilon = -0.75, \delta = 0.2$  81

5.16	Variation of the reduced skin friction with $\xi$ for different values of $\varepsilon$ when $K = 1, M = 1, n = 0, \delta = 0.2$	82
5.17	Variation of the reduced Nusselt number with $\xi$ for different values of $\varepsilon$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, n = 0, \delta = 0.2$	82
5.18	Variation of the reduced Sherwood number with $\xi$ for different values of $\varepsilon$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \delta = 0.2$	83
5.19	Variation of the reduced skin friction with $\xi$ for different values of $\delta$ when $K = 1, M = 1, n = 0, \varepsilon = -0.75$	83
5.20	Variation of the reduced Nusselt number with $\xi$ for different values of $\delta$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \varepsilon = -0.75$	84
5.21	Variation of the reduced Sherwood number with $\xi$ for different values of $\delta$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, \tau = 0.2, n = 0, \varepsilon = -0.75$	84
6.1	Physical model and coordinate system	87
6.2	The velocity profiles for various values of $\lambda$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	96
6.3	The microrotation profiles for various values of $\lambda$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	96
6.4	The temperature profiles for various values of $\lambda$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	97
6.5	The concentration profiles for various values of $\lambda$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	97
6.6	The velocity profiles for various values of $\varepsilon$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \delta = 0.2, \Omega = 1, \xi = 0.8$	98
6.7	The microrotation profiles for various values of $\varepsilon$ when $K = 1,$	

	$M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \delta = 0.2, \Omega = 1, \xi = 0.8$	98
6.8	The temperature profiles for various values of $\varepsilon$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \delta = 0.2, \Omega = 1, \xi = 0.8$	99
6.9	The concentration profiles for various values of $\varepsilon$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \delta = 0.2, \Omega = 1, \xi = 0.8$	99
6.10	The velocity profiles for various values of $M$ when $K = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	100
6.11	The microrotation profiles for various values of $M$ when $K = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	100
6.12	The temperature profiles for various values of $M$ when $K = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	101
6.13	The concentration profiles for various values of $M$ when $K = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	101
6.14	The velocity profiles for various values of $K$ when $M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	102
6.15	The microrotation profiles for various values of $K$ when $M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	102
6.16	The temperature profiles for various values of $K$ when $M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	103
6.17	The concentration profiles for various values of $K$ when $M = 1, Pr = 0.71, Sc = 0.94, n = 0, f_0 = 1, \varepsilon = -0.75, \delta = 0.2, \Omega = 1, \xi = 0.8$	103
6.18	Variation of the reduced skin friction with $\lambda$ for different values of $\varepsilon$ and $f_0$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \delta = 0.2, \Omega = 1, \xi = 0.6$	104



6.19	Variation of the reduced Nusselt number with $\lambda$ for different values of $\varepsilon$ and $f_0$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \delta = 0.2, \Omega = 1, \xi = 0.6$	104
6.20	Variation of the reduced Sherwood number with $\lambda$ for different values of $\varepsilon$ and $f_0$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \delta = 0.2, \Omega = 1, \xi = 0.6$	105
6.21	Variation of the reduced skin friction with $\lambda$ for different values of $\delta$ and $f_0$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \varepsilon = -0.75, \Omega = 1, \xi = 0.6$	105
6.22	Variation of the reduced Nusselt number with $\lambda$ for different values of $\delta$ and $f_0$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \varepsilon = -0.75, \Omega = 1, \xi = 0.6$	106
6.23	Variation of the reduced Sherwood number with $\lambda$ for different values of $\delta$ and $f_0$ when $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0, \varepsilon = -0.75, \Omega = 1, \xi = 0.6$	106
7.1	Physical model and coordinate system	108
7.2	The velocity profiles for various values of $\lambda$ when $K = 1, MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, S_r = 0.2, D_f = 0.2, \chi = 1, \xi = 0.2$	121
7.3	The microrotation profiles for various values of $\lambda$ when $K = 1, MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, S_r = 0.2, D_f = 0.2, \chi = 1, \xi = 0.2$	121
7.4	The temperature profiles for various values of $\lambda$ when $K = 1, MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, S_r = 0.2, D_f = 0.2, \chi = 1, \xi = 0.2$	122
7.5	The concentration profiles for various values of $\lambda$ when $K = 1, MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, S_r = 0.2, D_f = 0.2, \chi = 1, \xi = 0.2$	122
7.6	The velocity profiles for various values of $S_r$ when $K = 1, MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, D_f = 0.2, \chi = 1, \xi = 0.8$	123
7.7	The microrotation profiles for various values of $S_r$ when $K = 1,$	

- $MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, D_f = 0.2, \chi = 1,$   
 $\xi = 0.8$  123
- 7.8 The temperature profiles for various values of  $S_r$  when  $K = 1,$   
 $MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, D_f = 0.2, \chi = 1,$   
 $\xi = 0.8$  124
- 7.9 The concentration profiles for various values of  $S_r$  when  $K = 1,$   
 $MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, D_f = 0.2, \chi = 1,$   
 $\xi = 0.8$  124
- 7.10 The velocity profiles for various values of  $D_f$  when  $K = 1,$   
 $MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, S_r = 0.2, \chi = 1,$   
 $\xi = 0.8$  125
- 7.11 The microrotation profiles for various values of  $D_f$  when  $K = 1,$   
 $MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, S_r = 0.2, \chi = 1,$   
 $\xi = 0.8$  125
- 7.12 The temperature profiles for various values of  $D_f$  when  $K = 1,$   
 $MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, S_r = 0.2, \chi = 1,$   
 $\xi = 0.8$  126
- 7.13 The concentration profiles for various values of  $D_f$  when  $K = 1,$   
 $MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, S_r = 0.2, \chi = 1,$   
 $\xi = 0.8$  126
- 7.14 Variation of the skin friction with  $\lambda$  for different values of  $MD$   
when  $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1, S_r = 0.2,$   
 $D_f = 0.2, \chi = 1, \xi = 0.6$  127
- 7.15 Variation of the Nusselt number with  $\lambda$  for different values of  
 $MD$  when  $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1,$   
 $S_r = 0.2, D_f = 0.2, \chi = 1, \xi = 0.6$  127
- 7.16 Variation of the Sherwood number with  $\lambda$  for different values of  
 $MD$  when  $K = 1, M = 1, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1,$   
 $S_r = 0.2, D_f = 0.2, \chi = 1, \xi = 0.6$  128
- 7.17 Variation of the skin friction with  $\lambda$  for different values of  $S_r$   
when  $K = 1, MD = 1.5, Pr = 0.71, Sc = 0.94, n = 0.5, \Omega = 1,$   
 $D_f = 0.2, \chi = 1, \xi = 0.6$  128

- 7.18 Variation of the Nusselt number with  $\lambda$  for different values of  $S_r$  when  $K = 1$ ,  $MD = 1.5$ ,  $Pr = 0.71$ ,  $Sc = 0.94$ ,  $n = 0.5$ ,  $\Omega = 1$ ,  $D_f = 0.2$ ,  $\chi = 1$ ,  $\xi = 0.6$  129
- 7.19 Variation of the Sherwood number with  $\lambda$  for different values of  $S_r$  when  $K = 1$ ,  $MD = 1.5$ ,  $Pr = 0.71$ ,  $Sc = 0.94$ ,  $n = 0.5$ ,  $\Omega = 1$ ,  $D_f = 0.2$ ,  $\chi = 1$ ,  $\xi = 0.6$  129
- 7.20 Variation of the skin friction with  $\lambda$  for different values of  $D_f$  when  $K = 1$ ,  $MD = 1.5$ ,  $Pr = 0.71$ ,  $Sc = 0.94$ ,  $n = 0.5$ ,  $\Omega = 1$ ,  $S_r = 0.2$ ,  $\chi = 1$ ,  $\xi = 0.6$  130
- 7.21 Variation of the Nusselt number with  $\lambda$  for different values of  $D_f$  when  $K = 1$ ,  $MD = 1.5$ ,  $Pr = 0.71$ ,  $Sc = 0.94$ ,  $n = 0.5$ ,  $\Omega = 1$ ,  $S_r = 0.2$ ,  $\chi = 1$ ,  $\xi = 0.6$  130
- 7.22 Variation of the Sherwood number with  $\lambda$  for different values of  $D_f$  when  $K = 1$ ,  $MD = 1.5$ ,  $Pr = 0.71$ ,  $Sc = 0.94$ ,  $n = 0.5$ ,  $\Omega = 1$ ,  $S_r = 0.2$ ,  $\chi = 1$ ,  $\xi = 0.6$  131

## LIST OF SYMBOLS

### Roman Letters

$a, b$	-	positive constants
$b^*$	-	induced magnetic field
$B$	-	magnetic field vector
$B_0$	-	externally imposed magnetic strength in the $y$ – direction
$c_s$	-	concentration susceptibility
$\bar{C}$	-	fluid concentration
$C$	-	nondimensional fluid concentration
$C_\infty$	-	external concentration
$C_w$	-	surface concentration
$C_p$	-	specific heat at constant pressure
$C_{fx}$	-	skin friction coefficient
$\Delta C$	-	concentration difference
$Da^{-1}$	-	inverse Darcy number
$D_f$	-	Dufour number
$D_m$	-	mass diffusivity
$E$	-	electric field vector
$f$	-	non-dimensional velocity
$f_0$	-	suction or injection parameter
$\bar{\mathbf{F}}$	-	body force
$\bar{F}_x$	-	scalar force in $x$ – component
$g$	-	magnitude of the acceleration due to gravity
$Gc$	-	concentration Grashof number

$Gr$	-	thermal Grashof number
$h$	-	non-dimensional microrotation
$j$	-	microinertia density
$J$	-	electric current density vector
$\tilde{k}$	-	thermal conductivity
$k_1$	-	permeability of porous medium
$k_1^*$	-	vortex viscosity
$k_T$	-	thermal diffusion ratio
$K$	-	micropolar material parameter
$L$	-	characteristic length
$L_1$	-	slip length
$m_w$	-	non-dimensional mass flux from the surface of the wall
$M$	-	magnetic parameter
$MD$	-	effective Darcy number
$n$	-	ratio of the microrotation vector component to the fluid skin friction at the wall
$\bar{N}$	-	component of the microrotation vector normal to $x - y$ plane
$N$	-	non-dimensional component of the microrotation vector normal to $x - y$ plane
$Nu_x$	-	Nusselt number
$O$	-	order of magnitude
$\bar{p}$	-	pressure
$p$	-	non-dimensional pressure
$\bar{p}_d$	-	dynamic pressure
$\bar{p}_h$	-	hydrostatic pressure
$Pr$	-	Prandtl number
$q_w$	-	non-dimensional heat flux from the surface of the wall
$Re$	-	Reynolds number
$Sc$	-	Schmidt number
$Sh_x$	-	Sherwood number

$S_r$	-	Soret number
$\bar{t}$	-	time
$t$	-	non-dimensional time
$\bar{T}$	-	fluid temperature
$T$	-	non-dimensional fluid temperature
$T_m$	-	mean fluid temperature
$T_r$	-	reference temperature
$T_w$	-	surface temperature
$T_\infty$	-	external temperature
$\Delta T$	-	temperature difference
$\bar{u}_e(\bar{x})$	-	dimensional external velocity
$u_e(x)$	-	non-dimensional external velocity
$\bar{u}_w(\bar{x})$	-	dimensional velocity along the sheet
$u_w(x)$	-	non-dimensional velocity along the sheet
$\bar{u}, \bar{v}$	-	velocity components along $\bar{x}, \bar{y}$ axes
$u, v$	-	non-dimensional velocity components along $x$ and $y$
$U_\infty$	-	reference velocity
$\bar{V}$	-	dimensional velocity vector
$\bar{V}_T$	-	dimensional thermophoretic velocity
$v_0$	-	velocity suction or injection
$\bar{x}, \bar{y}$	-	cartesian coordinates along the plate and normal to it, respectively
$x, y$	-	non-dimensional cartesian coordinates along the wall and normal to it, respectively
$\Delta$	-	gradient

### Greek Letters

$\alpha$	-	thermal diffusivity
$\beta$	-	coefficient of thermal expansion

$\beta^*$	-	coefficient of concentration expansion
$\partial$	-	partial derivative
$\gamma$	-	spin gradient viscosity
$\xi, \eta, \zeta$	-	transformed coordinate
$\delta$	-	slip parameter
$\delta^*$	-	boundary layer thickness
$\theta$	-	non-dimensional temperature
$\kappa$	-	thermal conductivity
$\kappa^*$	-	thermophoretic coefficient
$\mu$	-	dynamic viscosity
$\mu_m$	-	magnetic permeability
$\nu$	-	kinematic viscosity
$\rho$	-	density
$\rho_\infty$	-	fluid density in the ambient medium
$\varepsilon$	-	shrinking parameter
$\varepsilon^*$	-	convergence tolerance
$\phi$	-	non-dimensional concentration
$\psi$	-	stream function
$\tau$	-	thermophoretic parameter
$\tau_w$	-	non-dimensional wall shear stress or skin friction
$\lambda^*$	-	concentration buoyancy parameter
$\lambda$	-	mixed convection parameter
$\sigma$	-	electrical conductivity
$\chi$	-	velocity ratio parameter
$\Omega$	-	buoyancy ratio

### Subscripts

$\infty, e$	-	far field or free stream condition
$w$	-	wall condition
$d$	-	dynamic pressure

$p$  - constant pressure condition

### Superscripts

$k$  - number of iteration

' - differentiation with respect to  $\eta$



**LIST OF APPENDICES**

<b>APPENDIX</b>	<b>TITLE</b>	<b>PAGE</b>
<b>A</b>	The Keller-Box Method	150
<b>B</b>	FORTTRAN Program for the Problem of the Thermophoresis and Suction or Injection Effects on the Stagnation Point Flow towards a Horizontal Sheet	164
<b>C</b>	Status of Publications	180

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Research Background**

The large volume of published studies on the boundary layer flow indicates the importance of the subject in engineering applications, such as hot rolling, skin friction drag reduction, grain storage, glass fiber and paper production. The importance of unsteady boundary layer flow is increasing in the field of fluid mechanics especially in the area of convective double diffusion. One of the main reason for such a importance is result of the complexity of the problem by including extra independent time variable. Generally, the ideal flow environment around the device is steady, although there are numerous situations for instances self-induced motions of the body, fluctuations, non-uniformities in the surrounding fluid, where undesirable unsteady effects arises. The study of unsteady boundary layer owes its importance to the fact that all the boundary layers, are, in sense, unsteady. Unsteady flows are generally observed in technological and environmental situations. Some of these observed in geophysical and biological flows, the processing of the materials, and in the spread of pollutants and fires. In the same vein, notable serious discussion on the unsteady boundary layer flow were previously carried out by researcher's like Riley (1975) and Telionis (1981).

The study of fluids uses two lenses to explain the concept at length. The two categories are Newtonian and non-Newtonian. The Newtonian fluid indicates that when the fluids shear stress is directly and linearly proportional to the rate of angular deformation, whereas in the non-Newtonian fluid, shearing stress is not related to the

rate of shearing strain. The equations which govern the flow of Newtonian fluid are the Navier-Stokes equations. There are few exceptions where Newtonian fluid fails to describe the properties of the fluid, these includes industrial colloids fluid, polymeric suspensions, liquid crystals and dust in air and blood flow in arteries and capillaries. Limitation in the explanation of these fluids led development of theories on non-Newtonian fluids. A classic example of the non-Newtonian fluids, that became centre of researcher's attention, is of micropolar fluid.

Eringen (1964) earlier developed the fluid mechanics of deformable microelements, which were termed as Simple Microfluids. Eringen defined the simple microfluid as: "A viscous medium whose behavior and properties are affected by the local motion of particles in its microvolume". These fluids are characterized by 22 viscosity and material constants and when applied to flow problems the result is a system of 19 partial differential equations with 19 unknown that may not be amenable to be solved. Eringen (1966) subsequently introduced a subclass of fluids which he named micropolar fluids that ignores the deformation of the microelements but still allows for the particle micromotion to take place. The theory of micropolar fluids, which consist of rigid, randomly oriented particles suspended in a viscous medium. These special features of micropolar fluids were discussed comprehensively by Ariman *et al.* (1973). In general, as part of the momentum is lost in rotating of particles, the flow of a micropolar fluid is less prone to instability than that of a classical fluid. The stability of micropolar fluids problems have been investigated by Lakshmana Rao (1970) as well as Sastry and Das (1985).

The study of micropolar fluid with heat and mass transfer has many engineering applications. These applications include refrigerator coils, power generators, metal and plastic extrusion, paper production, crystal growing, electric transformers and transmission lines. Eringen (2001) earlier demonstrated the adequacy of applying micropolar fluid theory in order to describe the liquid crystal behavior. Eringen in-addition indicated, other possible substances that may be modeled by micropolar fluid, these are magnetic fluids, clouds with dust, anisotropic fluids and biological fluids. It seems worth to mention importance of the study of

non-Newtonian fluid as a result of the behavior that is not described by the Newtonian relationships.

Recent evidences highlight increasing interest of researchers in the stagnation point flow in micropolar fluid. A review of fluid dynamics history reveals stagnation point flows as one of the unique issue of the field. These problems can take any form, such as steady or unsteady, viscous or inviscid, two dimensional or three-dimensional, forward or reverse, and normal or oblique. The impetus for studying convective flows near the stagnation point region is due to the fact that the heat transfer is maximum at the stagnation point. Similarly, Sharidan (2005) further added that solutions at stagnation point may also serve as a starting solution for the solution over the entire body.

The study of dynamics of electrically conducting fluid is known as magnetohydrodynamics (MHD). The study of MHD flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. There has been a great interest in the study of MHD flow with heat and mass transfer in any medium. This is largely, because of, the effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. This type of flow has attracted the interest of numerous researchers. One of the most significant reason of this importance is its applications in engineering problems such as MHD generators, plasma studies, nuclear reactors, geothermal energy extractions. However as a result of the application of magnetic field, hydromagnetic techniques are used for the extraction of pure molten metals from non-metallic inclusions. This is why; the type of problems that are dealing with is very useful for polymer technology and metallurgy.

Despite of the importance of MHD micropolar fluid flow near the stagnation point, recent efforts in this regard help identify new effects on the flow such as Soret and Dufour effects, slip effect and thermophoresis effect. Critical review of thermophoresis causes small particles to deposit on cold surfaces. Thermophoresis plays a central role in the fiber optical synthesis. Kishan and Maripala (2012) found

that this importance is the result of its identification as the principal mechanism of mass transfer which used in the technique of modified chemical vapor deposition (MCVD). Similarly, the mathematical modeling of the deposition of silicon thin films, using MCVD methods, has been accelerated by the quality control measures enforced by the micro-electronics industry. These topics involve variety of complex fluid dynamical processes including thermophoretic transport of particles deposits, heterogeneous/homogenous chemical reactions, homogenous particulate nucleation and coupled heat and energy transfer. Other notable example relating to thermophoresis is the blackening of glass globe of kerosene lanterns, chimneys and industrial furnace walls by carbon particles, corrosion of heat exchanger, which reduces heat transfer coefficient, and fouling of gas turbine blades (Kandasamy *et al.* 2010).

In case of heat and mass transfer, Soret and Dufour effects are significant when the temperature and concentration gradients are high. Thermal diffusion (thermo diffusion or Soret effect) corresponds to species differentiation developing in an initial homogeneous mixture submitted to a thermal gradient (Soret, 1980) while the energy flux caused by a composition gradient is called Dufour (diffusion-thermo) effect. These effects are considered as second order phenomena, on the basis that they are of smaller order of magnitude than the effects described by Fourier's and Fick's laws, but they may become significant in areas such as geosciences or hydrology (Benano-Melly *et al.* 2001).

Whilst discussing the field of fluid mechanics it is worth to mention important aspects like "partial slip condition". One of the important pillars on which the fluid mechanics is based is the no-slip condition. Although, there are situations where the conditions role is not significant or it is no more valid. These are the conditions where partial slip between the fluid and the moving surface may occur. Generally in case of no-slip condition it is noted that the molecule of the fluid flowing near the boundary sticks with the surface. In numerous practical situations it is important to replace the no-slip condition by the partial slip condition. This is because the no-slip condition at the solid fluid interface is no longer applicable when fluid flows in micro electro mechanical system (MEMS). The non-equilibrium

region near the interface is more accurately described by the slip flow model. In order to deal with the problem, Navier recommended general boundary condition which shows the fluid slip at the surface. Navier similarly suggested that, the difference of the fluid velocity and the velocity of the boundary is proportional to the shear stress at that boundary.

## 1.2 Problem Statements

Interest in the magnetohydrodynamic flow of micropolar fluid has increased substantially over the past few decades due to the occurrence of these fluids in many applications. The behavior of this flow near the stagnation point towards a stretching or shrinking sheet has been studied theoretically by many researchers. The phenomenon of this fluid affected by some important effects such as thermophoresis effect, suction or injection effect, slip effect, Soret and Dufour effects are not yet explore but interesting to be investigated. Therefore, this research is conducted to explore the following questions. What is the behavior of this fluid in nature near the forward stagnation point with the effect of thermophoresis and suction or injection towards a horizontal sheet? In-addition efforts are also required to see how do the micropolar fluids models compared with the Newtonian fluids models on the stagnation point flow towards a shrinking sheet with thermophoresis and slip effects? Apart from these need also exists to view how are the reduced skin friction, the reduced Nusselt number and the reduced Sherwood number affected due to the presence of magnetic parameter, slip effect and mixed convection parameter on stagnation point flow towards a permeable shrinking sheet? How are the skin friction, the Nusselt number and the Sherwood number affected due to the presence of Soret and Dufour effects towards a stretching sheet in a porous medium?

### 1.3 Objectives and Scope

The purpose of this study is to investigate theoretically the unsteady boundary layer flow of a micropolar fluid near the stagnation point towards a stretching or shrinking sheet. This involves with developing the mathematical formulation and numerical simulation for computation, in order to calculate the flow characteristics as well as analyzing the numerical results of the following MHD heat and mass transfer problems:

1. The thermophoresis and suction or injection effects on the stagnation point flow towards a horizontal sheet;
2. The thermophoresis and slip effects on the stagnation point flow towards a horizontal shrinking sheet;
3. The slip effect on the mixed convection stagnation point flow towards a permeable shrinking sheet;
4. The Soret and Dufour effects on the mixed convection stagnation point flow towards a stretching sheet in a porous medium.

This investigation examines the laminar two-dimensional incompressible flow of a micropolar fluid. These problems are solved numerically by using an implicit finite difference scheme, namely Keller's box. The Keller box method was earlier introduced by Keller (1970) and was used widely in solving the parabolic differential equations. The Newton's method can be used if the differential equations that need to be solved are nonlinear. The implicit nature of the Keller's Box method has generated a tridiagonal matrix, like other implicit method, however, the speciality of the Keller's Box method is, the entries are expressed in blocks rather than scalars. The work of Cebeci and Bradshaw (1984), Cebeci (2002), Nazar (2003), Sharidan (2005) and Lok (2008) provide details about the method. No real experiments have been conducted to validate the numerical results.

## 1.4 Significance of Study

The Newtonian fluids have few limitations, one of such shortcomings is its incapability to describe some engineering and industrial processes which are made up of materials having an internal structure. The theory of micropolar fluid model introduced by Eringen (1966) exhibits the local effects arising from the microstructure and micro motion of the fluid elements. The presence of smoke or dust particularly in gas may also modeled using micropolar fluid dynamics. Vogel and Patterson (1964) and Hoyt and Fabula (1964) conducted experiments with fluids that containing the amounts of minute polymeric additives. It was observed that the skin friction reduced near a rigid body. Gray and Hilliard (1966) in his invention, introduce relatively small amounts of a non-Newtonian fluid, a long-chain polymer such as polyethylene oxide, into the water adjacent the bow of the ship. This alters the shear characteristics of the fluid in boundary layer of the ship which decreases the overall frictional drag of the vessel. This leads to the increasing ship speed and it decreases the required power to maintain a given vessel speed. There are some advantages in the fields of aeronautics and submarine navigation.

It is becoming extremely difficult to ignore importance of micropolar fluid with heat and mass transfer in areas like aeronautics and submarine. Similarly it have significant importance in engineering applications, such as exothermic reaction in packed-bed reactors, heat transfer associated with storage of nuclear waste, cooling metallic plate in a bath and heat removal from nuclear fuel debris. Besides, the study of micropolar fluids, there are other applications in several industrial and technical processes such as nuclear reactors cooled during emergency shutdown, solar central receivers exposed to wind current, electronic devices cooled by fan and heat exchangers placed in a low-velocity environment.

Moreover, due to the use of the micropolar fluid in different manufacturing and processing industries, considerable attention has been given towards the understanding the important phenomena involving in heat exchange devices (Elbarbary and Elgazery, 2005). This importance require scientists and engineers to



be familiar with the flow behavior and properties of such fluids or the way to use such kind of properties to predict flow behavior in the process equipment. In-addition it is also beneficial to know the nature of the flow, heat and mass transfer of micropolar fluid towards a stagnation point and the influence of the material properties to the stagnation point heat and mass transfer problems.

## 1.5 Thesis Outline

This thesis is comprised of eight chapters. The first chapter is introduction, which provides detail account of research background, problem statement, objectives, scope and significance of research. The study then moves to conduct critical review of the previous literature. Based on the efforts of chapter two in the Chapter 3 efforts was made for the mathematical formulation.

The fourth chapter is concerned with the thermophoresis and suction or injection effects on an unsteady MHD stagnation point flow in a micropolar fluid towards a horizontal sheet. The governing boundary layer equations are solved numerically by using Keller-box method. Graphical results presented includes velocity, microrotation, temperature and concentration profiles as well as the physical quantities, namely the skin friction, the Nusselt number and the Sherwood number, which significance in characterizing the heat and mass transfer. Apart from these different physical parameters such as MHD parameter, micropolar parameter, suction or injection parameter, thermophoresis parameter and time variable are also considered in this chapter. In-addition both weak concentration and strong concentration are considered in this chapter. The case  $n = 0$  represents concentrated particle flow in which micro-element to wall surface are unable to rotate and denote strong concentration;  $n = 0.5$  present the vanishing of the anti-symmetric part of the stress tensor and denote weak concentration while  $n = 1$  indicates the turbulent boundary layer flows. A FORTRAN program for the effect of thermophoresis and suction or injection on the stagnation point flow towards a horizontal sheet is given in Appendix B.

In Chapter 5, the unsteady MHD boundary layer flow near the stagnation point on a shrinking sheet with thermophoresis and slip effects is considered. Based on the explanation provided in Chapter 4, both weak ( $n = 0.5$ ) concentration and strong ( $n = 0$ ) concentration are considered. Graphs are plotted and discussed for various emerging parameters such as slip parameter, shrinking parameter, thermophoresis parameter, micropolar parameter and time variable.

The study, in Chapter 6, then looks at the numerical solution of the unsteady MHD mixed convection flow near the stagnation point on a shrinking sheet with slip effect. The effect of mixed convection parameter  $\lambda$  which is involved in the momentum equation is studied for both assisting ( $\lambda > 0$ ) and opposing flows ( $\lambda < 0$ ). The mixed convection parameter  $\lambda$  is a measure of the relative importance of free convection in relation to forced convection. When  $\lambda \cong 1$ , the free and forced convection are of the same order of magnitude. If  $\lambda \ll 1$ , flow is primary by forced convection while  $\lambda \gg 1$  free convection become dominant (Martynenko and Khramtsov, 2005).

In Chapter 7, the unsteady MHD mixed convection flow near the stagnation point in micropolar fluid on a stretching sheet in a porous medium with Soret and Dufour effects is considered. As in the Chapter 6, both cases of assisting ( $\lambda > 0$ ) and opposing ( $\lambda < 0$ ) flows are also considered. The novel aspect of this study is the focus on the porosity and Soret and Dufour effects. The numerical results have been plotted for the indispensable dimensionless parameters to show the influences on the velocity, microrotation, temperature as well as the three physical quantities.

Finally, the summary of this thesis and the suggestions for future research are given in Chapter 8. The list of the publication and current status are given in Appendix C.

## REFERENCES

- Abo-Eldahab, E.M. and Ghonaim, A.F. (2005). Radiation effect on heat transfer of a micropolar fluid through a porous medium. *Appl. Mathematics Computation* 169: 500–510.
- Ali, F.M., Nazar, R., Arifin, N.M. and Pop, I. (2011). Unsteady flow and heat transfer past an axisymmetric permeable shrinking sheet with radiation effect. *Int. J. Numer. Meth. Fluids* 67: 1310–1320.
- Ariman, T., Turk, M.A. and Sylvester, N.D. (1973). Microcontinuum Fluid Mechanics – A Review *Int. J. Eng. Sci.* 11: 905-930.
- Ashraf, M. and Bashir, S. (2011). Numerical simulation of MHD stagnation point flow and heat transfer of a micropolar fluid towards a heated shrinking sheet. *Int. J. Num. Meth. Fluids* DOI: 10.1002/flid.2564.
- Ashraf, M. and Shahzad, A. (2011). Radiation effects on MHD boundary layer stagnation point flow towards a heated shrinking sheet. *World Appl. Sci. J.* 13(7): 1748-1756.
- Attia, H.A. (2006). Heat transfer in a stagnation point flow of a micropolar fluid over a stretching surface with heat generation/absorption. *Tamkang J. Science Eng.* 9(4): 299-305.
- Attia, H.A. (2008). Stagnation point flow and heat transfer of a micropolar fluid with uniform suction or blowing. *J. Braz. Soc. Mech. Sci. Eng.* XXX(1): 51-55.
- Aziz, A. (2009). A similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. *Commun. Nonlinear Sci. Num. Simul.* 14: 1064-1068.
- Bayley, F.J., Owen, J.M. and Turner, A.B. (1972). *Heat Transfer*. London: Thomas Nelson and Sons Ltd.
- Bejan, A. (2004). *Convection Heat Transfer*. 3<sup>rd</sup> edition. New York: John Wiley.

- Benano-Melly, L.B., Caltagirone, J.P., Faissat, B., Montel, F. and Costesque, P. (2001). Modelling Soret coefficient measurement experiments in porous media considering thermal and solutal convection. *Int. J. Heat Mass Transfer* 44: 1285–1297.
- Bhargava, R., Kumar, L. and Takhar, H.S. (2003). Finite element solution of mixed convection micropolar flow driven by a porous stretching sheet. *Int. J. Eng. Sci.* 4: 2161–2178.
- Bhargava, R., Sharma, S., Takhar, H.S., Bég, O.A. and Bhargava, P. (2007). Numerical solutions for micropolar transport phenomena over a nonlinear stretching sheet. *Nonlinear Analysis: Modelling Control* 12(1): 45–63.
- Bhargava, R. and Takhar, H.S. (2000). Numerical study of heat transfer characteristics of the micropolar boundary layer near a stagnation point on a moving wall. *Int. J. Eng. Sci.* 38: 383-394.
- Bhattacharyya, K. and Layek, G.C. (2011). Effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation. *Int. J. Heat Mass Transfer* 54: 302–307.
- Bhattacharyya, K. Mukhopadhyay, S., Layek, G.C. Pop, I. (2012). Effects of thermal radiation on micropolar fluid flow and heat transfer over a porous shrinking sheet. *Int. J. Heat Mass Transfer* 55: 2945-2952.
- Bhattacharyya, K. and Vajravelu, K. (2012). Stagnation-point flow and heat transfer over an exponentially shrinking sheet. *Commun. Nonlinear Sci. Numer. Simulat.* 17: 2728–2734.
- Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2006). *Transport Phenomena*. Second Edition: Wiley India Edition.
- Cebeci, T. (2002). *Convective Heat Transfer*. California: Horizon Publishing Inc.
- Cebeci, T. and Bradshaw, P. (1988). *Physical and Computational Aspects of Convective Heat Transfer*. New York: Springer.
- Chamkha, A.J. and Camille, I. (2000). Effects of heat generation/absorption and the thermophoresis on hydromagnetic flow with heat and mass transfer over a flat plate. *Int. J. Numerical Methods Heat Fluid Flow* 10(4): 432-438.
- Chen, C.K. and Hsu, T.H. (1991). Heat transfer of a thermomicropolar fluid past a porous stretching sheet. *Computers Math. Applic.* 21(8): 37-45.

- Das, K. (2012). Slip effects on MHD mixed convection stagnation point flow of a micropolar fluid towards a shrinking vertical sheet. *Comp. Math. Appl.* 63: 255-267.
- El-Arabawy, H.A.M. (2003). Effect of suction/injection on the flow of a micropolar fluid past a continuously moving plate in the presence of radiation. *Int. J. Heat Mass Transfer* 46: 1471–1477.
- Elbashbeshy, E.M.A. and Aldawody, D.A. (2010). Heat transfer over an unsteady stretching surface with variable heat flux in the presence of a heat source or sink. *Comp. Math. Appl.* 60: 2806–2811.
- Elbarbary, E.M.E. and Elgazery, N.S. (2005). Flow and heat transfer of a micropolar fluid in an axisymmetric stagnation flow on a cylinder with variable properties and suction (Numerical Study). *Acta Mechanica* 176: 213-229.
- Eringen, A.C. (1964). Simple microfluids. *Int. J. Eng. Sci.* 2: 205-217.
- Eringen, A.C. (1966). Theory of Micropolar fluid. *Journal Mathematics Mechanics* 16: 1-18.
- Eringen, A.C. (2001). *Microcontinuum Field Theories. II: Fluent Media*. New York, Springer.
- Fan, T., Hang X. and Pop, I. (2010). Unsteady stagnation flow and heat transfer towards a shrinking sheet. *Int. Commun. Heat Mass Transfer* 37: 1440–1446.
- Fang, T. and Zhang, Ji. (2009). Closed-form exact solutions of MHD viscous flow over a shrinking sheet. *Commun Nonlinear Sci Numer Simulat.* 14: 2853–2857.
- Fang, T.G., Zhang, Ji. and Yao, S.S. (2009). Viscous flow over an unsteady shrinking sheet with mass transfer. *CHIN. PHYS. LETT.* 26(1): 014703-1-4.
- Fang, T.G., Tao, H. and Zhong, Y.F. (2012). Non-Newtonian power-law fluid flow over a shrinking sheet. *CHIN. PHYS. LETT.* 29(11): 114703-1-5.
- Gorla, R.S.R. (1983). Micropolar boundary layer flow at a stagnation point on a moving wall. *Int. J. Eng. Sci.* 21: 25-33.
- Gorla, R.S.R. and Gorla, P. (1996). Unsteady planar stagnation point heat transfer in micropolar fluids. *Can. J. Phys.* 74: 77-80.
- Gray, W.O. and Hilliard, B.A. (1966). *U.S. Patent No. 3, 289, 623*. Washington DC: United States Patent Office.
- Guram, G.S. and Smith, A.C. (1980). Stagnation flows of micropolar fluids with strong and weak interactions. *Comp. Maths. Appls.* 6: 213-233.

- Hassanien, I.A. and Gorla, R.S.R. (1990). Combined forced and free convection in stagnation flows of micropolar fluids over vertical non-isothermal surfaces. *Int. J. Eng. Sci.* 28: 783-792.
- Hayat, T., Abbas, Z. and Javed, T. (2008). Mixed convection flow of a micropolar fluid over a non-linearly stretching sheet. *Physics Letters A* 372: 637-647.
- Hayat, T., Javed, T. and Abbas, Z. (2009). MHD flow of a micropolar fluid near a stagnation-point towards a non-linear stretching surface. *Nonlinear Analysis: Real World Appls.* 10(3): 1514-1526.
- Hayat, T., Hussain, M., Hendi, A.A. and Nadeem, S. (2012a). MHD stagnation point flow towards heated shrinking surface subjected to heat generation/absorption. *Appl. Math. Mech. -Engl. Ed.* 33(5): 631-648.
- Hayat, T., Awais, M. and Alsaedi, A. (2012b). Newtonian heating and magnetohydrodynamic effects in flow of a Jeffery fluid over a radially stretching surface. *Int. J. Physical Sci.* 7(21): 2838-2844.
- Hayat, T. and Qasim, M. (2010). Influence of thermal radiation and Joule heating magnetohydrodynamics flow of a Maxwell fluid in the presence of thermophoresis. *Int. J. Heat Mass Transfer* 53: 4780-4788.
- Heruska, M.W., Watson, L.T. and Kishore, K.S. (1986). Micropolar flow past a stretching sheet. *Computers Fluids* 14(2): 117-129.
- Hoyt, J.W. and Fabula, A.G. (1964). *The effect of additives on fluid friction*. US Naval Ordnance Test Station Report.
- Hsiao, K.L. (2012). Multimedia physical feature for unsteady MHD mixed convection viscoelastic fluid over a vertical stretching sheet with viscous dissipation. *Int. J. Physical Sci.* 7(17): 2515 – 2524.
- Ibrahim, F.S., Hassanien, I.A. and Bakr, A.A. (2004). Unsteady magnetohydrodynamic micropolar fluid flow and heat transfer over a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with a constant heat source. *Canadian J. Phys.* 82(10): 775-790.
- Iftikhar, A. (2008). *Solution of some unsteady flows over a stretching sheet using homotopy analysis method*. Quaid-i-Azam University: Ph.D Thesis.
- Ishak, A. (2010). Similarity solution for flow and heat transfer over a permeable surface with convective boundary condition. *Appl. Math. Comp.* 217: 837-842.

- Ishak, A. and Nazar, R. (2010). Effects of suction or injection on the stagnation point flow over a stretching sheet in a micropolar fluid. *Proc. 2<sup>nd</sup> Int. Conf. Mathematical Sciences* 1-7.
- Ishak, A., Nazar, R. and Pop, I. (2008a). Mixed convection stagnation point flow of a micropolar fluid towards a stretching sheet. *Meccanica* 43: 411–418.
- Ishak, A., Nazar, R. and Pop, I. (2008b). Magnetohydrodynamic (MHD) flow of a micropolar fluid towards a stagnation point on a vertical surface. *Comput. Math. Appls.* 56: 3188-3194.
- Ishak, A., Lok, Y. Y. and Pop, I. (2010). Stagnation-point flow over a shrinking sheet in a micropolar fluid. *Chem. Eng. Comm.* 197: 1417-1427.
- Ishak, A., Yacob, N.A. and Bachok, N. (2011). Radiation effects on the thermal boundary layer flow over a moving plate with convective boundary condition. *Meccanica* 46: 795-801.
- Jadidi, M., Moallemi, N. Shafieenejad, I. and Alaei, J. (2011). An analytical approximation of stagnation point flow and heat transfer of a micropolar fluid in a porous medium. *Adv. Theor. Appl. Mech.* 4(2): 81- 90.
- Jena, S.K. and Mathur, M.N. (1981). Similarity solution for laminar free convection flow of a thermomicropolar fluid past a nonisothermal flat plate. *Int. J. Eng. Sci.* 19: 1431-1439.
- Joshi, N., Kumar, M. and Saxena, P. (2010). Chemical reaction in steady mixed convection MHD viscous Flow over shrinking sheet. *Int. J. Stability Fluid Mech.* 1(1): 155-161.
- Kandasamy, R., Muhaimin, I. and Hashim B.S. (2010). Lie group analysis for the effect of temperature-dependent fluid viscosity with thermophoresis and chemical reaction on MHD free convective heat and mass transfer over a porous stretching surface in the presence of heat source/sink. *Commun. Nonlinear Sci. Numer. Simulat.* 15: 2109–2123.
- Katagiri, M. (1969). Magnetohydrodynamic flow with suction or injection at the forward stagnation point. *J. Phy. Soc. Japan* 27(6): 1677-1685.
- Kays, W.M., Crawford, M.E. and Weigand, B. (2005). *Convective Heat and Mass Transfer*. 4<sup>th</sup> ed. New Jersey: McGraw-Hill.
- Keller, H.B. (1970). A New Difference Scheme for Parabolic Problems. In: Bramble, J. *Numerical Solution of Partial Differential Equations*. New York: Academic Press.

- Keller, H.B. and Cebeci, T. (1972). Accurate numerical methods for boundary layer flows, II: Two dimensional turbulent flows. *AIAA Journal* 10: 1193-1199.
- Kelson, N.A. and Farrell, T.W. (2001). Micropolar flow over a porous stretching sheet with strong suction or injection. *Int. Comm. Heat Mass Transfer* 28(4): 479-488.
- Khan, Md.S., Karim, I. and Biswas, Md.H.A. (2012). Non-Newtonian MHD mixed convective power-law fluid flow over a vertical stretching sheet with thermal radiation, heat generation and chemical reaction effects. *Natural Appl. Sci.* 3(2): 80-92.
- Kim, J.K. (2001). Unsteady convection flow of micropolar fluids past a vertical porous plate embedded in a porous medium. *Acta Mechanica* 148: 105-116.
- Kishan, N. and Deepa, G. (2012). Viscous dissipation effects on stagnation point flow and heat transfer of a micropolar fluid with uniform suction or blowing. *Advances Applied Science Research* 3(1): 430-439.
- Kishan, N. and Maripala, S. (2012). Thermophoresis and viscous dissipation effects on Darcy-Forchheimer MHD mixed convection in a fluid saturated porous media. *Advances Applied Science Research* 3 (1): 60-74.
- Koichi, A. (2006). *Mass Transfer*. Wiley-VCH: Japan.
- Kumari, M. and Nath, G. (1984). Unsteady incompressible boundary layer flow of a micropolar fluid at a stagnation point. *Int. J. Eng. Sci.* 22(6): 755-768.
- Kumari, M. and Nath, G. (1986). Unsteady self-similar stagnation point boundary layers for micropolar fluids. *Indian J. Pure Appl. Math.* 17(2): 231-244.
- Kumar, L., Singh, B., Kumar, L. and Bhargava, R. (2011). Finite element solution of MHD flow of micropolar fluid towards a stagnation point on a vertical stretching sheet. *Int. J. Appl. Math. Mech.* 7(3): 14-30.
- Lakshmana Rao, S.K. (1970). Stability of micropolar fluid motions. *Int. J. Eng. Sci.* 8: 753-762.
- Leal, L.G. (1992). *Laminar Flow and Convective Transport Process: Scaling, Principles and Asymptotic Analysis*. London: Heinemann.
- Lienhard IV, J.H. and Lienhard V, J.H. (2011). *A Heat Transfer Textbook*. 4<sup>th</sup> Edition, Mineola NY: Dover Publications.
- Lok, Y.Y. (2008). *Mathematical Modelling of a Micropolar Fluid Boundary Layer near a Stagnation Point*. Universiti Teknologi Malaysia: Ph.D Thesis.



- Lok, Y.Y., Amin, N. and Pop, I. (2003a). Unsteady boundary layer flow of a micropolar fluid near the rear stagnation-point of a plane surface. *Int. J. Thermal Sci.* 42: 995-1001.
- Lok, Y.Y., Phang, P., Amin, N. and Pop, I. (2003b). Unsteady boundary layer flow of a micropolar fluid near the forward stagnation-point of a plane surface. *Int. J. Eng. Sci.* 41: 173-186.
- Lok, Y.Y., Amin, N., Campean, D. and Pop, I. (2005). Steady mixed convection flow of a micropolar fluid near the stagnation point on a vertical surface. *Int. J. Num. Meth. Heat Fluid Flow* 15(7): 654- 670.
- Lok, Y.Y., Amin, N. and Pop, I. (2007). Unsteady boundary layer flow of a micropolar fluid near a stagnation point with uniform suction or injection. *Jurnal Teknologi* 46(C): 15-32.
- Lok, Y. Y., Ishak, A. and Pop, I. (2011a). MHD stagnation point flow with suction towards a shrinking sheet. *Sains Malaysiana* 40(10): 1179–1186.
- Lok, Y. Y., Ishak, A. and Pop, I. (2011b). MHD stagnation-point flow towards a shrinking sheet. *Int. J. Num. Meth. Heat Fluid Flow* 21(1): 61-72.
- Łukaszewicz, G. (1999). *Micropolar Fluids: Theory and Applications*. Basel: Birkhäuser.
- Magyari, E. and Chamkha, A.J. (2010). Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface: The full analytical solution. *Int. J. Thermal Sci.* 49(9): 1821-1828.
- Mahapatra, T.R., Dholey, S. and Gupta, A.S. (2007). Heat Transfer in Oblique Stagnation-point Flow of an Incompressible Viscous Fluid towards a Stretching Surface. *Heat Mass Transfer* 43: 767-773.
- Mahapatra, T.R. and Nandy, S.K. (2011). Unsteady stagnation-point flow and heat transfer over an unsteady shrinking sheet. *Int. J. Appl. Math. Mech.* 7(16): 11-26.
- Makinde, O.D. and Olanrewaju, P.O. (2010). Buoyancy effects on thermal boundary layer over a vertical plate with a convective surface boundary condition. *J. Fluids Eng.* 132: 1-4.
- Makinde, O.D., Zimba, K. and Bég, O.A. (2012). Numerical study of chemically-reacting hydromagnetic boundary layer flow with Soret/Dufour effects and a

- convective surface boundary condition. *Int. J. Thermal Environmental Eng.* 4(1): 89-98.
- Martynenko, O.G. and Khramtsov, P.P. (2005). *Free-Convective Heat Transfer: With Many Photographs of Flows and Heat Exchange*. New York: Springer
- Merkin, J.H. and Kumaran, V. (2010). The unsteady MHD boundary-layer flow on a shrinking sheet. *European J. Mechanics B/Fluids* 29: 357-363.
- Miklavčič, M. and Wang, C.Y. (2006). Viscous flow due to a shrinking sheet. *Quart. Appl. Math.* 64: 283–290.
- Mohammad, H.Y., Abdullah, S., Hashim, I. and Sopian, K. (2011). Effects of viscous dissipation on the slip MHD flow and heat transfer past a permeable surface with convective boundary conditions. *Energies* 4: 2273-3394.
- Mohamed, R.A. and Abo-Dahab, S.M. (2009). Influence of chemical reaction and thermal radiation on heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium with heat generation. *Int. J. Thermal Sci.* 48: 1800-1813.
- Mohammadein, A.A and Gorla, R.S.R. (2001). Heat transfer in a micropolar fluid over a stretching sheet with viscous dissipation and internal heat generation. *Int. J. Num. Meth. Heat Fluid Flow* 11(1): 50-58.
- Mohamed, M.K.A., Salleh, M.K., Nazar, R. and Ishak, A. (2012). Stagnation point flow over a stretching sheet with Newtonian heating. *Sains Malaysiana* 41(11): 1467–1473.
- Mohanty, A.K. (2002). *Fluid Mechanics*. New Delhi: Prentice-Hall of India.
- Mostafa, A.A.M., Abd-Elaty, M.M. and Waheed, S.A. (2006). Hydromagnetic boundary layer micropolar fluid flow over a stretching surface embedded in a non-Darcian porous medium with radiation. *Math. Probl. Eng.* 2006: DOI: 10.1155/MPE/2006/39392.
- Mostafa, A.A.M. and Shima, E.W. (2011). MHD flow and heat transfer of a micropolar fluid over a nonlinear stretching surface with variable surface heat flux and heat generation. *Can. J. Chem. Eng.* 89:1408–1415.
- Mostafa, A.A.M. and Waheed, S.E. (2012). MHD stagnation point flow of a micropolar fluid towards a moving surface with radiation. *Meccanica* 47:1119–1130.
- Mucoglu, A. and Chen, T.S. (1978). Mixed convection about a sphere with uniform surface heat flux. *J. Heat Transfer* 100: 542-544.

- Muhaimin, Kandasamy, R., Hashim, I. and Khamis, A.B. (2010a). On the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer past a porous shrinking sheet with suction. *Theoret. Appl. Mech.* 36(2): 101-117.
- Muhaimin, Kandasamy, R., Hashim, I. (2010b). Effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction. *Nuclear Engineering Design* 240: 933–939.
- Mutlag, A.A. (2012). Scaling group transformation under the effect of thermal radiation heat transfer of a non-Newtonian power-law fluid over a vertical stretching sheet with momentum slip boundary condition. *Appl. Math. Sci.* 121(6): 6035 – 6052.
- Nadeem, S. and Hussain, A. (2009). MHD flow of a viscous fluid on a nonlinear porous shrinking sheet with homotopy analysis method. *Appl. Math. Mech. – Engl. Ed.* 30(12): 1569-1578.
- Nadeem, S., Hussain, M. and Naz, M. (2010). MHD stagnation flow of a micropolar fluid through a porous medium. *Meccanica* 45: 869–880.
- Nazar, R. (2003). *Mathematical Models for Free and Mixed Convection Boundary Layer Flows of Micropolar Fluids*. Universiti Teknologi Malaysia: Ph.D Thesis.
- Nazar, R., Amin, N., Filip, D. and Pop, I. (2004). Stagnation point flow of a micropolar fluid towards a stretching sheet. *Int. J. Non-Linear Mechanics* 39: 1227-1235.
- Nik Long, N.M.A., Suali, M., Ishak, A., Bachok, N. and Arifin, N.M. (2011). Unsteady stagnation point flow and heat transfer over a stretching/shrinking sheet. *J. Appl. Sci.* 11(20): 3520-3524.
- Odejide, S.A. (2012). Flow of a viscoelastic fluid over a stretching sheet using method of weighted residuals. *J. Modern Math. Stat.* 5(1): 1-2.
- Ozisik, M.N. (1985). *Heat Transfer: A Basic Approach*. New York: McGraw- Hill Book Company.
- Olanrewaju, P.O. and Adesanya, A.O. (2011). Effects of radiation and viscous dissipation on stagnation flow of a micropolar fluid towards a vertical permeable surface. *Australian J. Basic Appl. Sci.* 5(9): 2279-2289.
- Ojha, S.K., Mathur, M.N. and Banerjee, A.K. (1979). Longitudinal surface curvature effects on boundary layer of a micropolar fluid. *Acta Mechanica* 34. 215-231.

- Olanrewaju, P.O., Okedayo, G.T. and Gbadeyan, J.A. (2011). Effects of thermal radiation on magnetohydrodynamic (MHD) flow of a micropolar fluid towards a stagnation point on a vertical plate. *Int. J. Applied Science Technology* 1(6): 219-230.
- Pal, D. (2009). Heat and mass transfer in stagnation-point flow towards a stretching surface in the presence of buoyancy force and thermal radiation. *Meccanica* 44: 145-158.
- Patil, P.M., Pop, I. and Roy, S. (2010). Unsteady heat and mass transfer over a vertical stretching sheet in a parallel free stream with variable wall temperature and concentration. *Num. Meth. Partial Diff. Equations* 2010: DOI 10.1002/num.20665.
- Prasad, K.V., Santhi, S.R. and Datti, P.S. (2012). Non-Newtonian power-law fluid flow and heat transfer over a non-linearly stretching surface. *Appl. Math.* 3: 425-435.
- Rahman, M.M., Rahman, M.A., Samad, M.A. and Alam, M.S. (2009a). Heat transfer in a micropolar fluid along a non-linear stretching sheet with a temperature-dependent viscosity and variable surface temperature. *Int. J. Thermophys.* 30: 1649–1670.
- Rahman, M.M., Eltayeb, I.A. and Rahman, S.M.M. (2009b). Thermo-micropolar fluid flow along a vertical permeable plate with uniform surface heat flux in the presence of heat generation. *Thermal Science* 13(1): 23-36.
- Ramachandran, N., Chen, T.S. and Armaly, B.F. (1988). Mixed convection in stagnation flows adjacent to vertical surfaces. *J. Heat Transfer* 110: 373-377.
- Raptis, A. (2000). Boundary layer flow of a micropolar fluid through a porous medium. *J. Porous Media* 3(1), 95-97.
- Rawat, S. and Bhargava, R. (2009). Finite element study of natural convection heat and mass transfer in a micropolar fluid-saturated porous regime with Soret/Dufour effects. *Int. J. Appl. Math. Mech.* 5(2): 58-71.
- Rawat, S., Kapoor, S., Bhargava, R. and Beg, O.A. (2012). Heat and mass transfer of a chemically reacting micropolar fluid over a linear stretching sheet in Darcy Forchheimer porous medium. *Int. J. Comput. Appl.* 44(6): 40-51.
- Reddy, M.G. (2012a). Magnetohydrodynamics and radiation effects on unsteady convection flow of micropolar fluid past a vertical porous plate with variable wall heat flux. *Int. Scholarly Research Network* doi:10.5402/2012/146263.

- Reddy, M.G. (2012b). Heat generation and thermal radiation effects over a stretching sheet in a micropolar fluid. *Int. Scholarly Research Network* doi:10.5402/2012/795814.
- Reddy, S.K., Kesavaiah, D.C. and Shekar, M.N.R. (2013). MHD heat and mass transfer flow of a viscoelastic fluid past an impulsively started infinite vertical plate with chemical reaction. *Int. J. Innovative Res. Sci. Eng. Tech.* 2(4): 973-981.
- Rees, D.A.S. and Bassom, A.P. (1996). The Blasius boundary-layer flow of a micropolar fluid. *Int. J. Eng. Sci.* 34: 113-124.
- Riley, N. (1975). Unsteady Laminar Boundary Layers. *SIAM Review* 17: 274-297.
- Rosali, H., Ishak, A. and Pop, I. (2011). Stagnation point flow and heat transfer over a stretching/shrinking sheet in a porous medium. *Int. Commun. Heat Mass Transfer* 38: 1029-1032.
- Rosali, H., Ishak, A. and Pop, I. (2012). Micropolar fluid flow towards a stretching/shrinking sheet in a porous medium with suction. *Int. Comm. Heat Mass Transfer* 39(6): 826-829.
- Sajid, M. and Hayat, T. (2009). The application of homotopy analysis method for MHD viscous flow due to a shrinking sheet. *Chaos, Solitons and Fractals* 39: 1317-1323.
- Salleh, M.Z., Nazar, R. and Pop, I. (2010). Boundary layer flow and heat transfer over a stretching sheet with Newtonian heating. *J. Taiwan Inst. Chem. Eng.* 41(6): 651-655.
- Sastry, V.U.K. and Das, T. (1985). Stability of Couette flow and dean flow in micropolar fluids. *Int J. Eng. Sci.* 23: 1163-1177.
- Sharidan, S. (2005). *Mathematical Modelling of g-Jitter Induced Free Convection*. Universiti Teknologi Malaysia: Ph.D Thesis.
- Sharidan, S., Mahmood, T. and Pop, I. (2006a). Similarity solution for the unsteady boundary layer flow and heat transfer due to a stretching sheet. *Int. J. Applied Mech. Eng.* 11(3): 647-654.
- Sharidan, S., Amin, N. and Pop, I. (2006b). Unsteady boundary layer due to a stretching sheet in a porous medium using Brinkman equation model. *Heat Tech.* 25(2): 111-117.
- Shercliff, J.A. (1965). *A text book of magnetohydrodynamics*. Pergamon Press: Oxford.

- Shit, G.C. and Haldar, R. (2012). Thermal radiation effects on MHD viscoelastic fluid flow over a stretching sheet with variable viscosity. *Int. J. Appl. Math. Mech.* 8(14): 14-36.
- Soret, C. (1880). Influence de la temperature sur la distribution des sels dans leurs solutions. *C R Acad Sci Paris* 91: 289–291.
- Srinivasacharya, D. and RamReddy, Ch. (2011). Soret and Dufour Effects on Mixed Convection in a Non-Darcy Micropolar Fluid. *Int. J. Nonlinear Science* 11(2): 246-255.
- Suali, M., Nik Long, N.M.A. and Ariffin, N.M. (2012). Unsteady stagnation point flow and heat transfer over a stretching/shrinking sheet with suction or injection. *J. Appl. Math.* doi:10.1155/2012/781845.
- Subhashini, S.V., Samuel, N. and Pop, I. (2011). Double-diffusive convection from a permeable vertical surface under convective boundary condition. *Int. Commun. Heat Mass Transfer* 38: 1183-1188.
- Takhar, H.S., Agarwal, R.S., Bhargava, R. and Jain, S. (1998). Mixed convection flow of a micropolar fluid over a stretching sheet. *Heat Mass Transfer* 34: 213-219.
- Telionis, D.P. (1981). *Unsteady Viscous Flows*. New York: Springer-Verlag.
- Tritton, D.J. (1985). *Physical Fluid Dynamics*. England: Van Nostrand Reinhold (UK) Co. Ltd.
- Vogel, W.M. and Patterson, A.M. (1964). *An Experimental Investigation of Additives Injected into the Boundary Layer of an Underwater Body*. Pacific Naval Lab. of the Defense Res. Board of Canada, Report 64-2.
- Wang, C.Y. (2008). Stagnation flow towards a shrinking sheet. *Int. J. Non-Linear Mech.* 43: 377-382.
- Xu, H., Liao, S.J. and Pop, I. (2006). Series solutions of unsteady boundary layer flow of a micropolar fluid near the forward stagnation point of a plane surface. *Acta Mechanica* 184: 87-101.
- Yacob, N.A. and Ishak, A. (2010). Flow over a stretching sheet in a micropolar fluid with radiation effect. *J. Quality Measurement Analysis* 6(1): 85-93.
- Yacob, N.A. and Ishak, A. (2012). Stagnation point flow towards a stretching/shrinking sheet in a micropolar fluid with a convective surface boundary condition. *Can. J. Chem. Eng.* 90(3): 621-626.