

**STUDY OF FLOATING BODIES IN WAVE BY USING SMOOTHED
PARTICLE HYDRODYNAMICS (SPH)**

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STUDY OF FLOATING BODIES IN WAVE BY USING SMOOTHED PARTICLE
HYDRODYNAMICS (SPH)

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A report submitted in partial fulfillment of the requirements for the award of degree of Master
of Science (Engineering Mathematics).

Faculty of Science
Universiti Teknologi Malaysia

JANUARY 2013

My lovely family, thank you for always being there for me.

I love you all.

ACKNOWLEDGEMENT

First of all, I am indebted to my supervisor, Dr. Yeak Su Hoe for guiding me throughout this research. Through his valuable support and advices, I was able to conduct my research without encountering difficulties. His efforts have proved to be very useful when I was able to finally complete this research.

I would also like to extend my gratitude to my family members. They have been an indispensable source of encouragement and motivation. Without them, I would not have the chance to conduct this research.

Last but not least, my appreciation is also extended to those who have directly or indirectly helped me in the completion of this research, especially my friends. They have been very kind and generous in providing assistance.

ABSTRACT

One of the problems is the environment area which involves the floating of body with effective microorganism (SM). As a result, we need a detail description as well as calculation of floating body is using finite difference method (FD). However, this method produces low accuracy results. Beside, FD method is based on regular grid problem which is not efficient especially in river layout. In recent years, Smoothed Particle Hydrodynamics (SPH) method is developed to solve mechanical problems as well as fluid flow problems. In order to simulate the floating body problem using irregular grid as well as higher order accuracy, SPH is the suitable method. The advantage of SPH is its meshless technique which avoid the time consuming remeshing process especially in finite element method. The above numerical methods are coded in Fortran program. SPH can be a useful hydrodynamics model for breaking waves and the nearshore zone, particularly for case where spray and splash are important. The advantage of SPH is to be able to accurately model the complex flows associated with breaking water waves, including instantaneous motions as well as (time-averaged) wave-induced flows, such as undertow, longshore currents, and rip currents. The results of the computations show that SPH can be used to simulate free surface flows without difficulty provided the density.

ABSTRAK

Salah satu masalah pencemaran alam adalah melibatkan apungan badan dengan mikroorganisma berkesan (EM). Jesteru, kita perlu membuat perangkaan secara terperinci termasuk pengiraan apungan jasad dengan menggunakan kaedah perbezaan terhingga (FD). Walau bagaimanapun, kaedah ini akan menghasilkan keputusan yang mempunyai ketepatan yang lebih rendah. Selain itu, kaedah FD adalah berdasarkan permasalahan grid biasa yang kurang sesuai dalam geometry yang kompleks, terutamanya dalam geometry sungai. Sejak kebelakangan ini, kaedah Rataan Zarah Hidrodinamik (SPH) digunakan untuk menyelesaikan masalah mekanikal serta aliran bendalir. SPH adalah satu kaedah yang sesuai untuk mensimulasikan masalah apungan jasad menggunakan teknik grid tidak teratur dengan ketepatan lebih tertinggi. Antara kelebihan SPH adalah tanpa penggunaan unsur teknik bilangan pecahan yang kecil, dan ini dapat mengelakkan proses penyusunan unsur dengan masa yang panjang dalam kaedah elemen terhingga. Kaedah SPH kemudiannya dikodkan dalam program Fortran. SPH juga menjadi model hidrodinamik yang berguna untuk pemecahan ombak dan zon yang berhampiran dengan pantai, terutamanya dalam penting bagi kes semburan dan percikan. Selain itu, SPH juga dapat menghasilkan model dalam bentuk aliran kompleks yang berkaitan dengan gelombang air terbuka, termasuk gerakan secara terus (serta-merta) serta (purata masa) gelombang yang disebabkan oleh aliran seperti arus bawah, arus panjang pantai dan rip semasa. Hasil pengiraan menunjukkan bahawa SPH boleh digunakan untuk mensimulasikan aliran permukaan bebas yang melibatkan apungan badan.

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CHAPTER 1

INTRODUCTION

1.1 Background of study

Mathematics in Industry Study Group (MISG) is a collaborative problem solving workshop where applied mathematics tackle real life problems shared by local companies. The workshop helps to rapidly determine the key scientific issues and mathematical challenges in the problems presented. MISG provide a unique opportunity for applied mathematicians to be exposed to industrial problems. It also helps to set up the link between industry and university, and in particular to encourage the greater use of mathematical modeling and analysis industry. MISG is first held/organized in Malaysia with the collaboration between Oxford University, UTM and MIMOS berhad in 2011 at UTM. One of the problems in MISG workshop is the environment area which involves the floating of body with effective microorganism (EM). As a result, we need a detail description as well as calculation on this floating body especially in river pollution.

Numerical simulation using computers has increasingly become a very important approach for solving problems in engineering and science. Grid or mesh based numerical methods such as the finite difference methods (FDM), and the finite element methods (FEM) have been widely applied to various areas computational fluid dynamics (CFD) and computational solid mechanics (CSM), and are currently the dominant methods in numerical simulations for solving problems in engineering and science.

In recent years, SPH method is developed to solve mechanical problems as well as fluid flow problems. The advantage of SPH is its meshless technique which avoid the time consuming remeshing process especially in finite element method. In possess individual material properties and move according to the governing conservation equations. Since its invention to solve astrophysical problems in three dimensional open spaces, SPH has been extensively studied and extended to dynamic response with material strength as well as dynamic fluid flows with large deformations.

1.2 Problem Statement

MISG is first held/organized in Malaysia with the collaboration between Oxford University, UTM and MIMOS berhad in 2011 at UTM. One of the problems in MISG workshop is the environment area which involves the floating of body with effective microorganism (SM). As a result, we need a detail description as well as calculation of floating body is using finite difference method (FD). However, this method produces low accuracy results. Beside, FD method is based on regular grid problem which is not efficient especially in river layout. In order to simulate the floating body problem using irregular grid as well as higher order accuracy, SPH is the suitable method.

1.3 Objective

- i) To understand the method of the Smoothed Particle Hydrodynamics (SPH) which is a useful hydrodynamics model for breaking waves.
- ii) To model the complex flows associated with breaking free surface of water waves by using SPH.
- iii) To apply SPH method by using Fortran program.

1.4 Significance of the Study

From this research, we know that Smoothed particle hydrodynamics (SPH) is possess individual material properties and move according to the governing conservation equation where the state of a system represented by a set of particles. Smoothed particle hydrodynamics, as a meshfree, Lgrangian, particle method , has its particular characteristics. SPH has been extensively studied and extended to dynamics response with material strength as well as dynamic fluid flows with large deformations. A simulation using the SPH method involves particle approximation. The particle approximation is an issue related to only the initial creation of the particle and it can be solved using the existing software packages commercially available.

1.5 Scope Of Study

Study SPH formulation is derived by discretizing the Navier Stoke equation spatially, leading to a set of ordinary differential equations (ODEs) with respect to time. Besides that, the incompressible fluid, irregular points, meshless method will be applied during the calculation floating body when using SPH. On the others hand, the particles of water will be calculate by using SPHysicsgen Fortran. The study will apply the momentum equation, energy equation and continuity equation to calculate the movement of the particles.

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