Methodology Of Rolling Horizon Scheduling Under Demand Uncertainty

Awanis Romli
Mohamed Ariff Ameendeen
Norasnita Ahmad
Kolej Universiti Kejuruteraan dan Teknologi Malaysia, Malaysia.
awanis@kuktem.edu.my, mohamedariff@kuktem.edu.my
asnita@fsksm.utm.my

Production planning and scheduling play a prominent role in any kind of manufacturing activities that require resources input in terms of men, materials, machines and money (capital). It is a process of developing good relationship between market demands and production capacity in such a way that customers demand are satisfied and at the same time production activities are carried out in an economic manner. A reliable and efficient production planning and scheduling is essential in order to manage the production operations effectively. In a rolling horizon setting, the frequency with which a master production schedule (MPS) is updated or replanned can have a significant impact on MPS stability, productivity, production and inventory costs and customer service. Hence, one of the important decisions in the design of a rolling horizon MPS is the frequency of replanning. In this paper, we propose the possibility to establish a method for planning the MPS under demand uncertainty. A stochastic lot sizing algorithm is used to test the effectiveness of the rolling horizon MPS construction and extension. Therefore, a computer model was built to simulate the MPS activities under rolling horizon requirement. This model use a combination of an autoregressive fractionally integrated moving average (ARFIMA) forecasting model and fractional differencing method. The advantages of the ARFIMA time series model with fractional differencing method will benefits in planning the MPS under demand uncertainty.

Production planning and scheduling play a prominent role in any kind of manufacturing activities that require resources input in terms of men, materials, machines and money (capital). It is a process of developing good relationship between market demands and production capacity in such a way that customers demand are satisfied and at the same time production activities are carried out in an economic manner. A reliable and efficient production planning and scheduling is essential in order to manage the production operations effectively.

Master Production Scheduling (MPS) is a very important activity in manufacturing planning and control. The MPS is essential in maintaining customer service levels and stabilizing production planning within a Manufacturing Resource Planning (MRP II) environment and a JIT based production systems.

Statement Of The Problem

Zhao, Xie and Jiang, (2001) studied the impact of MPS freezing parameters on the performance of multi item single level production systems with a single resource constraint and deterministic demand. They found that some of the conclusions without considering capacity constraints could not be generalized to the more realistic capacitated situations. However they did not examine the problem under demand uncertainty. In reality, many companies do not know their future demands and have to rely on demand forecasts to make production planning decision. They often use the same capacity to manufacture several products. Developing and maintaining MPS under capacity constraints and demand uncertainty is far more challenging because it may significantly influence the selection of the MPS freezing parameters.

In this paper, the researcher investigates the problems according to these questions:

i. What is a method for planning the MPS under demand uncertainty?
ii. What is a model to be used for replanning MPS assuming the lower-level schedule?

Objective Of The Study

The objectives of this research are as follows:

i. Establish a method for planning the MPS under stochastic demand based on Fractional ARIMA model.
ii. Provide a model for replanning MPS assuming the lower-level schedule change cost is known.

Literature Review

A rolling planning horizon is to replan MPS each period whenever information is updated. In industry, though, running a plant until it is empty is rare. Instead, plants usually contain many different orders, with new orders arriving as older ones are completed. Scheduling is often performed on some regular basis i.e. hourly, daily, weekly or monthly. The best schedule is implemented until the plant is rescheduled. Thus, scheduling occurs on a rolling horizon basis(Thoney et al 2002). In practice, a planner would probably deal with the situation described above on a ‘rolling horizon’ basis in that the planner would get an initial production plan based on current data and then after one time period the planner would update the linear programming and resolve production equation system to get a revised production plan.

Usually, an MPS planner will face the pressure to replan because of the changes of operational circumstances. Tang and Grubbstrom (2002) said that there are basically two conditions which lead to replanning. First, there is a rolling effect due to extension of the planning period. Secondly, when demand is uncertain, there is always a forecast error, and, therefore the old plan has to be modified to adapt a new information to keep the production cost low and maintain the service level.

When the MPS is further used for material requirements planning (MRP), numerous changes in the MPS lead to schedule adjustments in the system. Such adjustments of plans also have an amplified effect in an assembly system, and this is often referred to as *system nervousness*. Nervousness may become an barrier in the implementation of MRP and even cause a breakdown of the whole system. Xie, Zhao and Lee (2003) studied that frequent adjustments to the MPS can induce major changes in the detailed MRP schedules. These changes can lead to increases in production and inventory costs and deterioration in customer service level. This phenomenon is called ‘‘schedule instability’’ or ‘‘MRP nervousness’’. Maintaining a stable MPS in view of changing customer requirements, adjustments in sales forecasts, and unforeseen suppliers or production problems is a difficult proposition for many firms (Sridharan, Berry and Udayabhanu, 1988). Therefore, to decrease the instability of the schedule becomes an important objective in planning the MPS.

Blackburn, Kropp and Millen (1986,) has suggested several methods to reduce schedule instability in MRP systems. One frequently used method involves the freezing of the MPS. Zhao and Lee (1993) examined the impact of different parameters for freezing the MPS upon total cost, schedule instability and service level in multi-stage systems. Zhao and Lam (1997), Zhao, Goodale and Lee (1995), Zhao and Xie (1998) also studied the impact of lot-sizing rules and forecasting models on the selection of MPS freezing parameters.

Although these studies address an important managerial issue in manufacturing planning and control and provide guidelines to help managers in their selection of MPS freezing parameters, they do not consider capacity constraints. Xie, Zhao and Lee (2003) examine even most production systems have capacity constraints and the master production scheduler has to take into consideration these constraints in developing the MPS, it is important to include capacity constraint in MPS studies. Therefore, it is of significant academic and practical value to know whether the conclusions and guidelines drawn under the assumption of unlimited capacity can be applied in the more realistic cases of having limited capacity. Investigations of the impact of capacity constraint on the selection of MPS freezing parameters will provide guidelines for practitioners to choose the proper set of MPS freezing parameters to enhance system performance.

Research Methodology

This section explains the methodology that the researcher intended to use in the research. It provides a description of the data sources and the instruments of data collection. The method of data analysis is described and the assumption made in the study are considered. The research methodology that will be used are in accordance with stages in Operational Research (OR) methodology as established by other OR practitioners such as Lucey (1994), Taha (1992) and Ravindran, Philips and Solberg (1987). The research methodology to be followed consists of the following phases:

Identifying the problems mathematically

The existence of a problem in developing models and solution methods for lot-sizing problems in general production systems with demand uncertainty in a rolling horizon environment. In a rolling horizon setting, the frequency with which a master production schedule (MPS) is updated or replanned can have a significant impact on MPS stability, productivity, production and inventory costs and customer service. In this project, the researcher will provide a model for replanning MPS assuming the lower-level schedule change cost is known. A method for planning the MPS under stochastic demand will be establish based on Fractional ARIMA model.

Data collection and Analysis

Relevant data is collected through observations and discussions with management staffs of the organization. The followings are the data that obtained :

i. Process flow and process sequence
ii. Production loading rules

iii. Production capacity
iv. Current system of creating production schedule
v. Current production and future demand forecast
vi. Analyze and identify product that is popular.

The main secondary resources for this research are printed and on-line journals. Journal and books review to identify previous and latest development in rolling horizon requirements and in particular the forecasting model development.

Model Building

(Fractional Differencing : ARIMA model) The modeling task is to come up with an autoregressive fractionally integrated moving average (ARFIMA) forecasting model, which combines the advantages of the ARIMA time series model and fractional differencing method. ARIMA models are homogenous nonstationery systems that can be made stationary by successively differencing observations. The more general ARIMA (p,d,q) model could also include autoregressive and moving average components, either mixed or separate. The differencing parameter, d, was always an integer value. Hosking (1981) further generalized the original ARIMa(p,d,q) value for fractional values. ARFIMA model can generate persistent and anti-persistent behavior in the manner of fractional noise.

Fractional differencing sounds strange. Conceptually, it is an attempt to convert a continuous process, fractional brownian motion into a discrete one by breaking the differencing process into smaller components. Integer differencing, which is only a gross approximation, often leads to incorrect conclusions when such a simplistic model is imposed on real process.

In addition, there is a direct relationship between the Hurst exponent and the fractional differencing operator, d:

$$d = H - 0.50 \qquad (3.0)$$

Thus, $0<d<0.50$ corresponds to a persistent black noise process, and $-0.50 < d < 0$ is equivalent to an antipersistent pink noise system. White noise corresponds to $d = 0$, and brown noise corresponds to $d = 1$ or an ARIMA (0,1,0) process, aw well known in the literature. Brown noise is the trail f a random walk, not the increments of random walk, which are white noise.

It is common to express autoregressive processes in terms of a backward shift operator, B. for discrete time with white noise, $B(x_t) = x_{1-1}$, so that

$$\Delta x_1 = (1-B) * x_1 = a_t$$

where the a_t are IID random variables. Fractionally differenced white noise, with parameter, *d*, is defined by the following binomial series:

$$\Delta^d = (1-B)^d = \sum_{k=0}^{x} \binom{d}{k} (-B)^k$$

$$= 1 - d * B - \frac{1}{2} * d * (1-d) * B^2 - \frac{1}{6} * d(1-d) * (2-d) * B^3 - \ldots (3.1)$$

Characteristics of ARFIMA (0,d,0)

Hosking (1981) developed the characteristics of the ARFIMA equivalent of fractional noise processes, ARFIMA (0,d,0) an ARFIMA process with no short-memory effects from p and q. The relevant characteristics as follows:

Let { x_t } be an ARFIMA (0,d,0) process, where k is the time lag and a_t is a while noise process with mean zero and variance σ_a^2. These are the characteristics:

1. When d < 0.50, { x_t } is a stationary process and has the infinite moving average representation:

$$x_t = \Psi(B) a_t = \sum_{k=0}^{x} \Psi_k * a_{t-k} \qquad (3.2)$$

where:

$$\Psi_k = \frac{d(1+d)\ldots(k-1+d)}{k!} = \frac{(k+d-1)!}{k!(d-1)!}$$

As $k \to \infty, \Psi_k \sim \dfrac{k^{d-1}}{(d-1)!}$ (3.3)

2. When d > -0.50, { x_t } is invertible and has the infinite autoregressive representation:

$$\pi(B) x_t = \sum_{k=0}^{x} \pi_k * x_{t-1} \qquad (3.4)$$

where:

$$\pi_t = \frac{-d * (1-d)\ldots(d-1-d)}{k!} = \frac{(k-d-1)!}{k! * (d-1)!} \qquad (3.5)$$

As $k \to \infty, \pi_k \sim \dfrac{k^{-d-1}}{(-d-1)!}$

3. The spectral density of { x_t} is:

$$s(\omega) = (2 * \sin\frac{\omega}{2})^{-2*d} \quad (3.6)$$

for $0 < \omega \le \pi$

4. The covariance function of { x_t } is :

$$\gamma_k = E(x_t x_{t-k}) = \frac{(-1)^k (-2d)!}{(k-d)! * (-k-d)!} \quad (3.7)$$

5. The correlation of { x_t } is:

$$\rho_k \sim \frac{(-d)!}{(d-1)!} * k^{2*d-1} \quad (3.8)$$

as k approaches infinity

6. The inverse correlations of { x_t } are:

$$\rho_{inv,k} \sim \frac{d!}{(-d-1)!} * k^{2*d-1} \quad (3.9)$$

7. The partial correlations of { x_t } are:

$$\varphi_{kk} = \frac{d}{k-d}, (k = 1,2,......) \quad (3.10)$$

ARFIMA (p,d,q)

ARFIMA (p,d,q) process includes short memory AR and MA processes. The result is short frequency effects superimposed over the low-frequency or long memory process. Examining the simplest examples, ARFIMA (1,D,0) and ARFIMA (0,d,1) processes are good illustrate of the mixed systems. These are the equivalent of short memory AR(1) and MA(0,1) superimposed over a long memory process.

An ARFIMA (1,d,0) process is defined by:

$$(1 - \varphi * B)\Delta^d y_t = a_t \quad (3.11)$$

Where a_t is a white noise process. Include the fractional differencing process in equation (3.2), where $\Delta^a x_t = a_t$, so we have $x_t = (1 - \varphi * B) * y_t$. The ARIMA (1,d,0) variable, y_1, is a first order autoregression with ARIMA (0,d,0) disturbances; that is, it is an ARFIMA (1,d,0) process. y_1 will have short term behavior that depends on the coefficient of autoregression, φ , just like a normal AR(1) process. However, the long term behavior of y_1 will be similar to y_1 . It will be similar to x_t. It will exhibit persistence or antipersistence, depending on the value of d. For stationarity and invertibility, assume |d| < 0.50, and $|\varphi| < 1$

Of most value is the correlation function of the process, ρ_k^y . Using F (a,b,c,z) as the hypergeometric function, as k –> ∞

$$\rho_k^y \sim \frac{(-d)!}{(d-1)!} * \frac{(1+\varphi)}{(1-\varphi)^2} * \frac{k^{2*d-1}}{F(1,1+d;1-d;\varphi)} \qquad (3.12)$$

By comparing the correlation functions for the ARFIMA (1,d,0) and AR(1) processes for longer rags, we can see the differences after even a few periods. Remember that an AR(1) process is also an infinite memory process.

Hosking (1981) described an ARFIMA (0,d,1) process as a first order moving average of fractionally different white noise. The MA parameter , θ , is used such that $|\theta| < 1$, again |d|<0.50, for stationarity and invertibility. The ARFIMA (0,d,1) process id defined as :

$$y_t = (1-\theta * B) * x_t \qquad (3.13)$$

the correlation function is as follows, as k $\rightarrow \infty$:

$$\rho_k^y \sim \frac{(-d)!}{(d-1)!} * a * k^{2*d-1} \qquad (3.14)$$

where:

$$a = \frac{(1-\theta)^2}{(1+\theta^2 - (2*\theta*d/(1-d))} \qquad (3.15)$$

Procedure for ARFIMA (p,d,q) model

Hosking (1981) gave the following procedure for identifying and estimating an ARFIMA (p,d,q) model:

i. Estimate d in the ARIMA(0,d,0) model $\Delta^d y_t = a_t$
ii. Define $u_t = \Delta^d y_t$
iii. Using Box-Jenkins modeling procedure, identify and estimate the φ and θ parameters in the ARFIMA (p,0,q) model $\varphi * B * u_t = \theta * B) a_t$
iv. Define $x_t = (\theta * B)^{-1} * (\varphi * B * y_t)$
v. Estimate d in the ARFIMA(0.d.0) model $\Delta^d x_t = a_t$
vi. Check for the convergence of the d, φ and θ parameters, if not convergent, go to step 2.

Hosking specifically suggested using R/S analysis to estimate d in steps 1 and 5, using equation (3.0).

R/S Mehodology Analysis

This analysis consists from 2 processes. They are pre processing and calculation for H value. Data processing start by taken time series data with M length. For analysis that will be done in economic factor, data is more appropriate to change in algorithm close price. Peters (1994) suggested an old data will be changed to basic 10 algorithm with length for N = M-1 as follows:

$$N_i = \log(M_{(i+1)}/M_i),\ i = 1,2,3,\ldots,(M-1)$$

Result for processing data will be used to calculate value of H. this time series divided to sub time series A with length n, as A*n = N and for each sub time series, average will be calculated like the formula below.

$$e_a = (1/n) * \sum_{k=1}^{n} N_{k,a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.16)$$

after mean value have been calculate, the value of range for R(A) and variance S(A) for each sub time series will be calculated as follows:

with Y_j is an element data in sub period involve.

$$R(A) = (\mathrm{Max}\sum(Y_j - e_a) - \mathrm{Min}\sum(Y_j - e_a) \text{ dengan } s \geq j \geq 1 \quad \ldots\ldots\ldots\ldots\ldots(3.17)$$

$$S(A) = (1/n(\sum(Y_j - e_a)^2)^{0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.18)$$

Calculation for value R(A) and S(A) will be repeated for another sample to find average value for R(A)/S(A) as equation 3.19.

$$(R/S)_n = (1/A) * (\sum(R(A)/S(A))) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.19)$$

Equation 3.19 will be calculated with a small sample and one H value will be get. Calculation will be repeated with a big sample. This process will be repeated until value H achieve peak value and decrease. Value H will be find from equation 3.20

$$\log(R/S)_n = \log(c) + H*\log(n) \quad \ldots\ldots(3.20)$$

Each data will be assumed to have a long term memory structure when H in the range of $0.5<H<1$. If H in the range of $0<H<0.5$ so anti-persistence structure exist in that data. If $H=0.5$ so the habit for that data in white noise condition.

Developing heuristic Rolling Horizon Scheduling

The steps for constructing rolling horizon scheduling as follows:

Step 1: Smooth raw data and construct the MPS based on MRP lot size.

Step 2: Let product proceed according to step 1 until new requirement arrive at day n. Add new demand to current MPS. Expand MPS until T+n. Go to step 1.

The steps in designing a heuristic rolling horizon scheduling is shown by the following table 1 :

Table 1: Rolling Horizon Scheduling

Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
MPS	21	0	31	4	12	3		
Product A	21	0	50	0	0	0		
Add A				5	7			
MPS			50	5	7	0	α	β
Product A			63	0	0	0	$\alpha + \beta$	0

3.5 Testing the Model Validity

After the rolling horizon scheduling is created, testing and validating are carried out to determine the feasibility and relevance of the system. Redesign is carried out whenever is required.

The assumption to be made in the research will only be known when the actual data are available and suitable models are to be constructed based on the data and information.

Model Implementation

Results of the study will be presented to the management and production department at MSG Berhad in the hope that it will be significant impact on

MPS stability, productivity, production and inventory costs and customer service in their organisation.

Instrumentation

Generally, the instrumentation involves in developing the system can be divided into two types: software and hardware.

All the pseudo codes and procedures are translated into working programs written in Java 2 utilizing jdk 1.3.1 Compiler. The integrated development environment of choice is a licensed version of Kawa CodeWright Enabled Version 4.01. Hardware platforms employed by the researcher is 1 personal computer with a Pentium III 450 MHz processor with 128MB of memory and running Windows 98 Second Edition.

Conclusion

The steps that are used to obtain the fluent production model that is going to be included inside the MPS are based on the methodology of the ARFIMA forecasting model. This method to be used is the Box Jenkins methodology.

References

Blackburn, J. D., Kropp, D. H. and Millen, R. A. (1986). "A Comparison Of Strategies To Dampen Nervousness in MRP Systems." *Management Science.* **32** (4). 413–429.

Hosking, J.R.M. (1981). *Fractional Differencing***.** Biometrica 68.

Hurst H.E. (1951). *The Long Term Storage Capacity of Reservoirs*. Transactions of The American Society of Civil Engineering.

Peters, E. (1994). Fractal Market Analysis, Applying Chaos Theory To Investment & Economics. John Wiley & Sons, Inc.

Ravindran, A., Phillips, Don, T. and Solberg, J. J. 1987. *Operation Research : Principles and Practise*. Singapore : John Wiley and Sons, Inc.

Sipper, D. and Robert, L. B. (1997). *Production Planning, Control and Integration.* New York : McGraw-Hill Book Co.

Sridharan, V., Berry, W. L. and Udayabhanu, V. (1988). "Measuring Master Production Schedule Stability Under Rolling Planning Horizons." *Decision Sciences*. **19** (1). 147–166.

Stonebraker, P. W. and Leong, G. K. (1994). *Operations Strategy*. Boston: Allyn and Bacon.

Taha, Hamdy A. 1992. *Operations Research : An Introduction*. Fifth Edition. New York: Macmillan Publishing Company.

Tang, O. and Grubbstrom, R. W. (2002). "Planning and Replanning The

Master Production Schedule Under Demand Uncertainty." *International Journal of Production Economics*. **78**. 323-334.

Thoney, K. A., Joines J.A., Manninagarajan,P. and Hodgson, T. J. (2002). "Rolling Horizon Scheduling In Large Job Shops." *Proceedings Of The 2002 winter Simulation Conference.* 1891-1896.

Xie, J., Zhao, X. and Lee, T. S. (2003). "Freezing The Master Production Schedule Under Single Resource Constraint and Demand Uncertainty." *International Journal of Production Economics*. **83**. 65-84.

Zhao, X. and Lee, T. S. (1993). "Freezing The Master Production Schedule For Material Requirements Planning Systems Under Demand Uncertainty." *Journal of Operations Management*. **11**. 185-205.

Zhao, X., Goodale, J. and Lee, T. S. (1995). "Lot-Sizing Rules And Freezing The Master Production Schedule In MRP Systems Under Demand Uncertainty." *International Journal of Production Research*. **33** (8). 2241–2276.

Zhao, X. and Lam, K. (1997). "Lot Sizing Rules And Freezing The Master Production Schedule In Material Requirements Planning Systems." *International Journal of Production Economics*. **53**. 281–305.

Zhao, X. and Xie, J. (1998). "Multilevel Lot-Sizing Heuristics And Freezing The Master Production Schedule In Material Requirements Planning Systems." *Production Planning and Control*. **9** (4). 371–384.

Zhao, X., Xie, J. and Jiang, Q. (2001). "Freezing The Master Production Schedule Under Single Resource Constraint and Deterministic Demand." *Production and Operations Management.* **10** (1). 45–67.