ONE DIMENSIONAL SOLUTE TRANSPORT IN HOMOGENEOUS POROUS DOMAIN

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Specially dedicated to my beloved family who inspires me throughout my journey in education. Thank you for everything.

Cikgu Munirah binti Sabran
Firdaus Sizzy

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ABSTRACT

Analytical solutions are obtained for dispersion of pollutants along groundwater flow in a longitudinal direction through semi-infinite aquifers which is porous. The solute dispersion is considered temporally dependent while the seepage velocity uniform. The dependency of the solute dispersion to time will indicate that the solute dispersion will change in certain times as the groundwater's parameters change due to monsoon season and in normal season. Analytical solutions are obtained for uniform pulse type input point source. The Laplace transformation technique is employed to get the analytical solutions of the present problem. The solutions obtained predict the time and distance from the location at which an input concentration is introduced at which the pollution concentration becomes harmless.

ABSTRAK

Sifat pengangkutan bahan pencemar di dalam air bawah tanah di dalam paksi x, separuh infiniti diselesaikan dengan menggunakan penyelesaian analisis. Sifat pengangkutan bahan pencemar adalah bergantung kepada masa. Namun kelajuan air bawah tanah adalah sekata. Ini untuk menunjukkan sifat bahan pencemar adalah berlainan pada masa tertentu selaras dengan parameter air bawah tanah yang bergantung dengan musim hujan dan musim biasa. Penyelesaian analisis ini diselesaikan untuk keadaan bahan pencemar yang dilepaskan dari satu tempat secara sekata. Kaedah transformasi Laplace telah digunakan untuk mendapatkan penyelesaian analisis bagi merungkai permasalahan ini. Penyelesaian yang diperolehi dapat menjangka atau meramal masa dan tempat bahan pencemar sudah tidak berbahaya dari lokasi di mana bahan pencemar itu dilepaskan.

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LIST OF SYMBOLS/NOTATIONS

Roman Letters

C, F	-	Pollutant/ solute concentration
x	-	Distance in *x* direction
x'	-	New distance *x*
Δx	-	Difference of distance in *x* direction
t	-	Time
t'	-	New time *t*
t_0	-	Time at which concentration stop being released
q	-	Solute concentration flux
C_i	-	Initial concentration along the porous domain
C_0	-	Concentration at source during the release
X, T	-	New independent variable
f, f_1, f_2	-	Function of distance, *x* and time, *t*
m	-	Unsteady parameter or flow resistance
K_h	-	Hydraulic head
h_i	-	Initial head
h_f	-	Final head
Δh	-	Head loss
L	-	Length
Q	-	Discharge
A	-	Area
n	-	Porosity
K_1, K_2	-	Empirical constant
D	-	Longitudinal dispersion/ diffusion coefficient
u	-	Velocity
D_0	-	Constant

M,L,T - Unit of mass, length and time

R_d - Retardation factor

K - Function of distance, *x* and time, *t*

$\bar{K}$ - K that has been transformed by Laplace transform

p - *t*, time that has been transformed by Laplace transform

T_0 - Independent variable involving time, t_0

y_H - Homogeneous solution

y_B - Particular solution

a, b, c - Constants

F_1, F_2 - Constants

∞ - Infinity

$erfc$ - Error function

exp - Exponent

Greek Letters

$\mathcal{L}$ - Laplace transform

$\propto, \beta$ - Arbitrary constant

μ - Zero order production

γ - First order decay rate

μ_0, u_0, γ_0 - Constant

τ, ε, ξ - Dummy variables

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CHAPTER 1

INTRODUCTION

1.1 Research Background

In late 19th centuries health officials from England and France have recognized the importance of soil and groundwater contamination and its effect to human health (Colten *et al.*, 1996). In the modern days, Love Canal tragedy in the City of Niagara, USA has become the main reference of soil and groundwater contamination. The long term exposure of contamination has revealed more than 248 types of chemicals in the Love Canal dump site, hence shows the critical problem of such contamination (Fletcher, 2002)

Groundwater and soil pollution in Malaysia for the past has not been identified as key environmental issue in Malaysia. This is true since not many cases of environmental and human health incidents have been reported. However with increasing demand for agricultural and drinking water use, groundwater and soil vulnerability has become an important environmental and human health issue.

Mohamed *et al.* (2009) stated that Langat Basin ecosystem is experiencing increasing pressure from urbanization and industrialization for the past three decades. The development process has resulted to increase the vulnerability of groundwater and soil quality. The increasing growth population and agricultural activity has increased the demand for good quality water.

One of the factors that contribute to groundwater pollution is leachate from landfill. Mohamed *et al.* (2009) studied on the leachate from Ampar Tenang landfill which is located very close to the Labu River, which is part of main tributaries of the Langat river basin. The study revealed that there is migration of leachate through the clay probably due to advection and diffusion transport mechanisms. Hence this illustrate that the leachate from landfill has been polluting the groundwater and soil as well as Labu River.

1.2 Problem Statement

It is found that leachate from Ampar Tenang landfill has been polluting groundwater, soil including Labu River through advection and diffusion transport mechanism (Mohamed *et al.*, 2009). On the other hand, Sirajudeen *et al.* (2012) were studying the effect of seasonal variation on the pollutant concentration in the groundwater. Therefore the early hypothesis is the pollutant concentration can be predicted at certain time and location using the advection diffusion/dispersion equation by considering the seasonal variation.

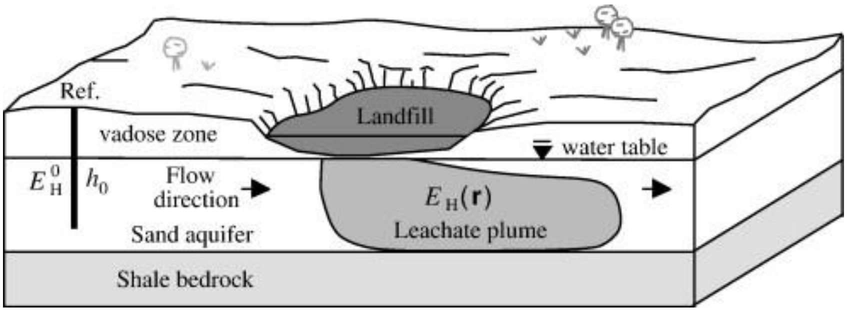

Figure 1.1 Landfill leachate is polluting the groundwater

1.3 Objectives of the Study

The main objective is to obtain the analytical solution for the one dimensional solute transport using Laplace transformation technique. Another objective is to discuss the solute concentration distribution against time (seasonal variation).

1.4 Scope of the Study

In order to achieve the objective of the research, it is important to set clear scopes for this research. Firstly, the solute transport described by one dimensional advection dispersion equation and is in horizontal direction. Laplace transformation technique is used to get the analytical solutions. The medium is considered semi-infinite homogeneous in longitudinal direction.

1.5 Significance of the Study

The pollutant transport in porous domain which is governed by the advection dispersion can be used to predict the pollutant concentration in the aquifer or groundwater. Therefore, the amount of pollutant release at certain time can be regulated to ensure the groundwater quality is under the standard.

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