FLOOD FREQUENCY ANALYSIS USING PL-MOMENTS APPROACH

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To my beloved parents

My father, Zakaria bin Omar

My mother, Allahyarhamah Ramlah binti Udin

To my lovely brothers and sisters

Salahudin bin Zakaria

Muhammad Saifuldin bin Zakaria

Mohd Shaharuddin bin Zakaria

Zahrahtul Hazwani binti Zakaria

Norfadillah binti Hassan Basuri

Siti Ariffah binti Mustajap

Alhamdulillah, I thank God for blessing me with all your presence.

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After all "success is a journey, not a destination".

ABSTRACT

Estimation of flood magnitude is a crucial component in planning, designing, and managing of water resources projects. Flood frequency analysis (FFA) provides a practical means of determining a robust probability distribution that fits streamflow data at a location of interest. The main focus in hydrology design is the estimation of high flow quantile. L-moments, popular among hydrologist in FFA is known to be oversensitive towards the lower part of the distribution and give insufficient weight to large sample values. As an alternative, the method of partial L-moments (PL-moments) is proposed to give weightage to the upper part of distribution and large values in censored sample. The aim of this study is to compare the performance of PL-moments and L-moments in FFA. The method of PL-moments was developed for generalized extreme value (GEV), generalized logistic (GLO), generalized pareto (GPA), extreme value type 1 (EV1) and logistic (LOG) distributions. Monte Carlo simulations from population distributions of known and unknown samples were conducted to assess the performance of PL-moments compared to L-moments. Simulation results showed that PL-moments give comparable and slightly better parameter estimates than those by L-moments particularly when estimating the high flow quantiles. In regional flood frequency analysis, new statistical tests based on PL-moments were developed to measure discordancy, regional homogeneity and identify a best regional distribution. The quantile estimates based on the regional distribution using PL-moments are more efficient than L-moments in estimating flood quantiles at higher return periods. The overall results strongly support that PL-moments method would improve the flood quantiles estimation particularly for higher quantiles and thus serves as a useful tool for application in flood frequency analysis.

ABSTRAK

Anggaran magnitud banjir adalah satu komponen yang penting dalam merancang, merekabentuk dan mengurus bagi projek sumber air. Analisis frekuensi banjir (FFA) menyediakan satu kaedah berguna dalam menentukan taburan kebarangkalian terbaik untuk dipadankan dengan data aliran sungai di lokasi yang diminati. Fokus utama dalam rekabentuk hidrologi adalah penganggaran kuantil aliran tinggi. L-momen yang popular di kalangan ahli hidrologi dalam FFA dikatakan terlalu sensitif terhadap bahagian bawah taburan dan tidak memberikan pemberat yang mencukupi untuk nilai sampel yang besar. Sebagai alternatif, kaedah separa L-momen (PL-momen) dicadangkan untuk memberi pemberat pada bahagian atas taburan dan nilai yang besar dalam sampel yang ditapis. Tujuan kajian ini adalah untuk membandingkan prestasi PL-momen dan L-momen dalam FFA. Kaedah PL-momen dibangunkan untuk taburan nilai ekstrim teritlak (GEV), logistik teritlak (GLO), pareto teritlak (GPA), nilai ekstrim jenis I (EV1) dan logistik (LOG). Simulasi Monte Carlo dari sampel taburan populasi yang diketahui dan tidak diketahui dilakukan untuk menilai keupayaan PL-momen berbanding L-momen. Hasil simulasi menunjukkan bahawa PL-momen memberikan anggaran parameter yang setara dan lebih baik sedikit berbanding L-momen apabila menganggar kuantil aliran tinggi. Dalam analisis frekuensi banjir serantau, ujian statistik baru berdasarkan PL-momen dibangunkan untuk mengukur keserasian data, kehomogenan rantau dan mengenalpasti taburan terbaik rantau. Anggaran kuantil berdasarkan taburan serantau menggunakan PL-momen didapati lebih efisyen berbanding L-momen apabila menganggar kuantil banjir pada tempoh ulangan yang panjang. Hasil keseluruhan menyokong sepenuhnya bahawa kaedah PL-momen berupaya memperbaiki anggaran kuantil banjir terutamanya pada kuantil tinggi dan oleh itu bertindak sebagai satu kaedah berguna dalam aplikasi analisis frekuensi banjir.

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LIST OF SYMBOLS

D_i	-	Discordancy measure
$E[X]$	-	Expectation of order statistic
F	-	Non-exceedance Probability
f_i	-	Plotting position
F_0	-	Level of censoring
$F(x)$	-	Cumulative distribution function
$f(.x)$	-	Probability distribution function
H_i	-	Heterogeneity measure
N	-	Number of sites in a region
n_i	-	Record length of a site
Q_T	-	Magnitude of flood at *T*-year
T	-	Return period, year
x	-	Random variable
x_0	-	Censored data point
$x(F)$	-	Quantile function
Z^{DIST}	-	Goodness-of-fit measure, Z-test
μ	-	Population mean
σ	-	Population standard deviation
ν	-	Population coefficient of variation
δ	-	Population coefficient of skewness
κ	-	Population coefficient of kurtosis
α	-	Scale parameter
ξ	-	Location parameter

k, h	-	Shape parameter
α^R	-	Regional scale parameter
ξ^R	-	Regional location parameter
k^R	-	Regional shape parameter
μ_r	-	r^{th} moments
β_r	-	r^{th} probability weighted moments
$\beta_r^{'}$	-	r^{th} partial probability weighted moments
m_r	-	r^{th} sample moments
b_r	-	r^{th} sample probability weighted moments
$b_r^{'}$	-	r^{th} sample partial probability weighted moments
λ_r	-	r^{th} L-moments
$\lambda_r^{'}$	-	r^{th} PL-moments
l_r	-	r^{th} sample L-moments
$l_r^{'}$	-	r^{th} sample PL-moments
C_V	-	Coefficient of variation (CV)
C_S	-	Coefficient of skewness (CS)
C_K	-	Coefficient of kurtosis (CK)
τ	-	L-coefficient of variation (L-CV)
τ_3	-	L-coefficient of skewness (L-CS)
τ_4	-	L-coefficient of kurtosis (L-CK)
$\hat{\tau}_r$	-	r^{th} sample L-moment ratios
υ	-	PL-coefficient of variation (PL-CV)
υ_3	-	PL-coefficient of skewness (PL-CS)
υ_4	-	PL-coefficient of kurtosis (PL-CK)
$\hat{\upsilon}_r$	-	r^{th} sample PL-moment ratios
$\eta^{(i)}$	-	Coefficient of variation for site i
$\eta_3^{(i)}$	-	Coefficient of skewness for site i
$\eta_4^{(i)}$	-	Coefficient of kurtosis for site i
η_r^R	-	r^{th} regional average L-moment ratios

γ	-	Euler's constant
E(.)	-	Exponential Integral function
$\Gamma(.)$	-	Gamma function
$\gamma(.)$	-	Incomplete Gamma function
$B_{1-F_0}(.)$	-	Incomplete Beta function
GEV	-	Generalized extreme value distribution
GLO	-	Generalized logistic distribution
GPA	-	Generalized Pareto distribution
EV1	-	Extreme value type 1 distribution
LOG	-	Logistic distribution
PWMs	-	Probability Weighted Moments
PPWMs	-	Partial Probability Weighted Moments
RBIAS	-	Relative bias
RRMSE	-	Relative root mean square error
RFFA	-	Regional flood frequency analysis
WA	-	Wakeby distribution

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CHAPTER 1

INTRODUCTION

The main interest of this research is the application of flood frequency analysis based on L-moments and Partial L-moments approaches. Therefore, this chapter basically introduces the background of flood frequency analysis and highlights the problems arising in the analysis. This chapter also includes the objectives, scope, contribution of the research and organization of the thesis.

1.1 Research Background

A flood is an unusually high stage of water levels that generally happened in river that overflows and submerges land and inundates the adjoining area. The basic cause of river flooding is the incidence of heavy rainfall (monsoon or convective) and the resultant large concentration of runoff, which exceeds river capacity Flooding can caused damages of private and public properties, loss of life and economic problems. In terms of the number of population affected, frequency, area extent, duration and social economic damage, flooding is the most natural hazard in Malaysia. The flood events occur in various states in Peninsular Malaysia.

Several efforts have been done to overcome the floods including flood protection projects such as construction of barrages, dams, water reservoirs and widening or deepening the rivers. The barrages which act as an embankment have been constructed along a body of water to prevent water flooding onto the land from the sea or river. Dams and water reservoirs which are generally built to store water can also be used to prevent floods. However, the constructions of barrages and dams require high financial cost from the government while the process of widening and deepening the rivers have high potential in destroying the ecosystems of the river itself. Thus, these flood protection projects without proper planning and designing will only create drawbacks. In order to reduce the drawbacks, information related to these aspects need to be carefully considered. First, how long and high should the barrages be built? Second, how big should the water reservoirs and dams be constructed? Third, how wide and deep should the rivers be excavated?

Consequently, a clear knowledge related to magnitude and frequencies of the flood occurrences are fundamentally needed to deal with all of those questions. A similar circumstance is applied in designing of hydraulic structures and water-related projects such as spillways, culvert, highways, etc. Information regarding accurate estimation of flood magnitudes and their frequency of occurrence are of great importance in the planning, designing and management of such structures at the location or station of interest. The structures need to be designed by considering the maximum flows that exceed certain level in a given return period. On the other hand, the flows below the critical value are less important since they do not negatively affect the structure.

Estimating flood magnitudes and their frequencies need knowledge related to distributions of flood flow series. Probability for future events can be predicted by fitting past observations to selected probability distributions. The primary objective is to relate the magnitude of these extreme events to their frequency occurrence through the use of probability distributions (Chow et al., 1988). In this case, flood frequency analysis is a most suitable method in order to determine a robust probability distribution that fit to streamflow data at a certain location of interest. The most widely used methods in predicting the flood magnitudes are at-site and regional flood frequency analyses. In general, flood frequency analysis is defined as an estimation of how often a specified event, in this case, flood will occur (Hosking and Wallis, 1997).

There are two important components in frequency analysis which are parameter estimation method (or estimator) and probability distribution being used for describing flood occurrences. Before a probability distribution can be fitted to the data, parameters of the particular distribution need to be estimated from the data samples. Many estimators have been introduced in estimating distributional parameters, but the most commonly used estimators in hydrology are method of moments (MOM), the maximum likelihood estimation (MLE) and the probability weighted moments (PWMs) or generally known as L-moments. The MOM is a conventional and relatively easy parameter estimation method. Although long established in statistics, this method is not always satisfactory. The MOM estimates are usually inferior in quality and generally not as efficient as the MLE and L-moments especially in the case where the distributions have a large number of parameters. The MLE often regarded as the most efficient method. However, the MLE might be hard to compute and involves numerical algorithm especially if the number of parameters is larger than three. This will in turn make it hard and might also be impossible to obtain MLE of small samples.

During the past three decades, major developments in flood frequency analysis revolved around the idea of probability weighted moments (PWMs) introduced by Greenwood et al. (1979) and the theory of L-moments proposed by Hosking (1986, 1990) as the parameter estimation method. L-moment estimators are an exact analogue to conventional MOM estimators, but are weighted linear sums of the expected order statistics. The L-moment estimates are comparable to the MLE estimates and in certain cases superior than the MLE particularly in small sample sizes. Recent studies on statistical analysis of annual maximum flood series have shown that L-moments provide simple and reasonably efficient estimators of characteristics of hydrologic data and of a distribution's parameters (Hosking, 1990; Hosking and Wallis, 1993; Stedinger et al., 1992).

The second component in frequency analysis is the probability distributions function being used. In determining a correct distribution function, the concern is to find the one that would be capable of describing the recorded sample and more importantly, extrapolating correctly to large return periods. Many distribution function forms have been proposed for describing flood occurrences. However, previous studies have been reported in the literature that there is no 'universal' distribution function that is best representing floods trend at all streamflow locations of interest. The extreme value Type I (EV1), II, and III distributions have recently gained considerable acceptance for describing annual maximum flows on the theoretical consideration that the distribution of the maximum of a sample tends to converge to one of the three extreme value distributions as the sample size increases (Wang, 1996). Other distributions also have been proposed to be used in flood frequency analysis including three-parameter distributions; generalized extreme value type I (GEV), generalized logistic (GLO), generalized Pareto (GPA), lognormal (LN3), Pearson Type III (P3) and log-Pearson Type III (LP3), and two-parameter distributions; extreme value Type I (EV1), logistic (LOG), normal and Pareto distributions.

Flood frequency analysis may suffer from sampling variability when applied to data for a single site, especially for estimating return periods that exceed the length of the observed record at a site (Hosking and Wallis, 1993; Cunnane, 1988). The observed flood data at a particular station are generally insufficient to obtain reliable estimates of the flood quantiles, especially in developing and undeveloped countries. This is due to lack of technology and other problems which affect the process of data collection. In a relatively young country like Malaysia, majority of stations having data record dated back from 1960 with the average record length of 35 years. This is inadequate to allow for reliable estimation of flood magnitudes especially for larger return period than the available length of data record.

One way of providing more reliable estimation is to use several records from a region with identical behavior of flood, rather than only single site information (Hussain and Pasha, 2009). This is known as regional flood frequency analysis (RFFA). In RFFA, estimates at a single site can be enhanced by pooling the data from other sites which confirmed to have similar frequency distribution. The information from other sites however, only can be appropriately transferred within a homogenous region. Studies have shown that, even though a region may be moderately heterogeneous, regional frequency analysis will still yield much more accurate quantile estimates than at-site frequency analysis (Lettenmaier et al., 1987).

Recent advance in RFFA involves the use of L-moment estimators as reported by Hosking and Wallis (1997). In RFFA, the objectives are to identify a robust regional distribution for each identified homogenous region and to estimate the quantiles at the station of interest for a given return period. Therefore, the following procedures of RFFA based on L-moments approach have been employed to attain the goals. The procedures include the detection of outliers, identification and verifying of homogenous regions, identification and testing of regional frequency distribution, and estimation of flood quantiles at stations of interest. This methodology has been applied successfully in modeling floods in a number of case studies from Malaysia (Lim and Lye, 2003; Zin et al., 2009; Shabri et al., 2011), New Zealand (Pearson, 1991, 1995; Madsen et al., 1997), South Africa (Kachroo et al., 2000; Kjeldsen et al., 2002), China (Jingyi and Hall, 2004), UK (Fowler and Kilsby, 2003), Pakistan (Hussain and Pasha, 2009; Hussain, 2011), Turkey (Saf, 2009a, 2009b), Iran (Abolverdi and Khalili, 2010) and Italy (Norbiato et al., 2007; Noto and Loggia, 2009).

1.2 Problems Statement

The purpose of analyzing hydrological extreme events such as annual maximum series of floods is, in most cases, to predict magnitude of flood of relatively large return period such as 100 years and above (Wang 1990). Hence, it is actually advantageous to intentionally censor (or eliminate) low-value observations because using only the larger value flood ensures that the extrapolation to large return periods flood is carried out by exploring the trend of these larger flows only. Cunanne (1987) suggested that in such cases a censored sample should be used and the analysis will be based on only those floods whose magnitudes have exceeded a certain threshold.

Since L-moments were first introduced by Hosking (1990) as a parameter estimation method, it has been widely applied in many fields of hydrology. Although L-moments result in quite efficient estimate in parameter estimation, this may not be so for predicting large return period events. The question arose whether L-moments are oversensitive to the lower part of distributions and give insufficient weight to large data values that actually contain useful information on the upper distribution tail (Wang, 1997; Bobee & Rasmussen, 1995).

Wang (1990) has introduced the L-moments method based on the concept of partial probability weighted moments (PPWMs), which are called partial L-moments (PL-moments) for fitting distribution functions to censored samples. PL-moments are variants of L-moments and also analogous to the PPWMs. PL-moments are introduced for characterizing the upper part of distributions and larger events in data. Using PL-moments reduce undesirable influences that small sample events may have on the estimation of large return period events.

However, there is no further research investigating on flood frequency analysis of censored sample thus far. Hence, this research will provide further investigation and more comprehensive evaluation of censored sample based on PL-moments approach in flood frequency analysis especially on evaluating the performance of PL-moments compared to L-moments.

1.3 Research Objectives

In this research, a comprehensive evaluation of PL-moments estimator in flood frequency analysis will be investigated particularly on evaluating the performance of PL-moments compared to L-moments. The PL-moments at various levels of censoring, F_0 will be considered in this research. Five probability distributions with three-parameter; GEV, GLO and GPA distributions and two-parameter; EV1 and LOG distributions will be used in flood frequency analysis of this research.

The main objectives of this research are:

i. To derive the parameters estimation models of PL-moments approach for GLO, GPA, EV1 and LOG distributions and to enhance the parameters estimation for GEV distribution.
ii. To evaluate the sampling properties of PL-moments compared to L-moments in characterizing larger events in sample using Monte Carlo simulation data generated from known and unknown parent distribution function.
iii. To develop the PL-moments approach in regional flood frequency analysis based on L-moments approach in modeling the annual maximum streamflow over stations in Peninsular Malaysia.
iv. To assess performances of PL-moments compared to L-moments in all stages of regional flood frequency analysis.

1.4 Research Scope

This research covers the following aspects:

i. This research covers the derivation of the parameters estimation models for GLO, GPA, EV1 and LOG distributions and to enhance the parameters estimation for GEV distribution based on PL-moments approach. The parameter estimation models for GEV, GLO, GPA, EV1 and LOG distributions based on L-moments are revisited.

ii. Two types of data are utilized in this study. The first data are synthetic "flood-like" data obtained from Monte Carlo simulation data. The Monte Carlo simulation generates synthetic flows from various background distributions of known and unknown parent distribution function. The GEV, GLO, GPA, EV1 and LOG distributions are assumed as known parent distribution function while six Wakeby distributions are assumed as unknown parent distribution function.

iii. The second data are used in regional flood frequency analysis. Data of annual maximum streamflow over stations located throughout Peninsular Malaysia which ranges from 1960 to 2009 has been used. Records of daily streamflow from 56 stations with record lengths of 15 to 50 years were acquired from the Department of Irrigation and Drainage, Ministry of Natural Resources and Environment, Malaysia.

iv. PL-moments with various levels of censoring, F_0 are investigated in Monte Carlo simulation study ranging from F_0 = 0.1, 0.2, 0.3, 0.4 and 0.5.

1.5 Research Contribution

This research offers several contributions. The main contributions are:

i. This research contributes to the development of the several three-parameter estimation models of GEV, GLO, GPA distributions and two-parameter estimation models of EV1 and LOG distributions based on PL-moments method to be used in application of flood frequency analysis. The PL-moments method is developed as similar to L-moments method in estimating parameters of various probability distributions for extreme events in hydrology.

ii. Since the choice of censoring values, F_0 are still under discussion, by investigating PL-moments with censoring levels, F_0 ranging from 0.1 to 0.5, the readers will have some ideas in choosing the suitable censoring value to improve the estimation of extreme events particularly in high return period events in frequency analysis studies.

iii. This research also contributes to the development of regional flood frequency analysis (RFFA) based on PL-moments approach in each stages of RFFA. These include the process of screening out the data, verifying the homogenous region using statistical measure, selecting suitable regional probability distribution and estimating regional parameters and flood quantiles according to L-moments approach.

iv. The results of this study give benefits to hydrological studies. The direct beneficiaries of the study are the engineers and hydrologists working in the research areas of applications from the result of specifying the probability distribution of extreme events which in this case is flood. By knowing the information regarding flood magnitudes and corresponding frequencies of occurrence, engineering projects such as dams, spillways, highways, etc can be planned, designed and managed effectively. Thus, this also helps our country from unnecessary cost and economic losses as well as preventing possible danger due to overflow of water in the country.

1.6 Organization of Thesis

The rest of the thesis is organized as follows:

Chapter 2 reviews main subjects used in the study that includes flood frequency analysis, parameter estimator, simulation of the estimator, censored data in hydrology, regional flood frequency analysis, regional homogeneity measure and selection of regional probability distribution.

Chapter 3 describes in detail the related theories and methodologies for the development of flood frequency analysis. The main ideas behind the building of flood frequency model are also discussed. The background of L-moments and Partial L-moments are defined by explaining their population and sample theories.

Chapter 4 discusses on flood frequency analysis and quantile estimation using probability distribution function. Several distributions, namely GEV, GLO, GPA, EV1 and LOG distributions are considered to be used as possible candidates in this study. The details of each distribution will be presented including their probability distribution function (pdf), cumulative distribution function (cdf) and quantile function. The parameter estimation using the methods of L-moments is revisited and parameter estimation using the methods of PL-moments is derived for each distribution.

Chapter 5 presents the results of Monte Carlo simulation study to investigate the sampling properties of the proposed parameter estimation methods of L-moments and PL-moments. The analyses of the simulations are for the cases of known parent distribution function and unknown parent distribution function.

Chapter 6 develops the procedures of regional flood frequency analysis (RFFA) for PL-moments based on the L-moments approach. The procedures include four stages of RFFA such as screening of the data, identification of homogeneous regions, identification and testing of regional frequency distributions, and estimation of flood quantiles at recurrence intervals of interest.

Chapter 7 presents the analysis of the regional flood frequency analysis (RFFA) based on L-moments and PL-moments for study data of daily streamflow from 56 stations located throughout Peninsular Malaysia. Finally, capabilities of the L-moments and PL-moments estimators in estimation of design flood quantiles are evaluated at specific recurrence intervals.

Chapter 8 summarizes the procedures and analysis in the research, draws some conclusions of the research and provides suggestions and recommendation for future research.

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