

ENHANCED METHODS FOR BENCHMARKING AND RANKING IN
DATA ENVELOPMENT ANALYSIS

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To my beloved spouse and children

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I am thankful to my great supervisor Prof. Dr. Shaharuddin Salleh, and appreciative of my lovely wife, children and family. I am truly appreciative the people who supported me during this level of study.

ABSTRACT

Data Envelopment Analysis (DEA) is a non-parametric method in operations research for estimating production frontier, and benchmarking and ranking Decision Making Units (DMUs). The current DEA techniques are not suitable for these assessments, thus, this study proposes novel robust methods to improve the capabilities of DEA for approximating the production frontier while simultaneously benchmarking and ranking DMUs. Firstly, the shortcomings in the DEA techniques are illustrated with several counter examples followed by new proposed methods to remove the shortcomings. Then, the techniques are combined and used in a linear programming model called Kourosh and Arash Model (KAM). KAM estimates the production frontier and allows decisions within the target regions instead of points in the benchmark of DMUs. In this study, KAM produces three efficiency indexes, namely: the lowest, technical and highest efficiency scores for each DMU. These efficiency indexes provide a sensitivity index for each DMU and rank DMUs completely. KAM is also able to measure the efficiency scores of DMUs inclusive of integer and real data. To sum up, the proposed techniques in this study have improved the capabilities of DEA to assess the production frontier, as well as benchmark and rank DMUs.

ABSTRAK

Analisis Pensampulan Data (APD) ialah suatu kaedah tak berparametrik dalam penyelidikan operasi untuk menganggar sempadan pengeluaran, membanding dan menyisih Unit Penjana Keputusan (UPK). Kaedah APD semasa adalah tidak begitu sesuai untuk penilaian. Oleh itu, kajian ini mencadangkan kaedah sejati dan tahan lasak untuk membaiki keupayaan APD bagi menganggar sempadan pengeluaran di samping membanding dan menyisih UPK. Pertamanya, segala kekurangan dalam teknik UPK dicerahkan dengan beberapa contoh berlawanan, diikuti dengan kaedah cadangan untuk menghapus kekurangan mereka. Kemudian, teknik digabungkan dan digunakan dalam model pengaturcaraan dipanggil Kourash dan Arash (KAM). KAM memberi anggaran terhadap sempadan pengeluaran dan menjana keputusan dalam lingkungan kawasan sasaran, bukan sebagai titik dalam perbandingan UPK. Dalam kajian ini, KAM menghasilkan tiga indeks keberkesanan, iaitu skor keberkesanan terendah, teknikal dan tertinggi bagi setiap UPK. Indeks keberkesanan ini menghasilkan indeks kesensitifan bagi setiap UPK dan menyisih UPK sepenuhnya. KAM juga berupaya menghitung skor keberkesanan UPK yang meliputi data integer dan terapung. Ringkasnya, teknik yang dicadangkan dalam kajian ini telah berjaya membaiki keupayaan APD dalam menilai sempadan pengeluaran di samping membanding dan menyisih UPK.

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LIST OF SYMBOLS

DEA	-	Data envelopment analysis
DMU	-	Decision making unit
CCR	-	Charnes, Cooper and Rhodes model
ADD	-	Additive model
SBM	-	Slack-based measure model
BCC	-	Banker, Charnes and Cooper model
ERM	-	Enhanced Russell measure model
AP	-	Andersen and Petersen model
LP	-	Linear programming
CF	-	Cost-efficiency model
RF	-	Revenue-efficiency model
PF	-	Profit-efficiency model
NCN	-	Non-controllable model
PPS	-	Production possibility set
CRS	-	Constant returns to scale
VRS	-	Variable returns to scale
NFD	-	Near and far data
AM	-	Arash Method/model
KM	-	Kourosh Method/model
KAM	-	Kourosh and Arash Method/model
n	-	Number of DMUs
m	-	Number of DMU inputs
p	-	Number of DMU outputs
i	-	Index of DMUs
j	-	Index of DMU inputs
k	-	Index of DMU outputs
l	-	Index of specific DMU whose efficiency is being assessed
x_{ij}	-	Observed value of input j of DMU $_i$

y_{ik}	-	Observed value of output k of DMU $_i$
λ_i	-	Multiplier used to compute linear combination of DMUs' inputs/outputs
s_j^-	-	Non-negative slack or potential reduction of input j of DMU $_i$
s_k^+	-	Non-negative slack or potential increase of output k of DMU $_i$
w_j^-	-	Positive specified weight or price for input j of DMU $_i$
w_k^+	-	Positive specified weight or price for output k of DMU $_i$
λ_i^*	-	Optimal multipliers to identify the reference sets for a DMU
s_j^{-*}	-	Optimal slack to identify an excess utilization of input j of DMU $_i$
s_k^{+*}	-	Optimal slack to identify a shortage utilization of output k of DMU $_i$
x_{ij}^*	-	Target of input j of DMU $_i$ after evaluation
y_{ik}^*	-	Target of output k of DMU $_i$ after evaluation
θ^*	-	Optimal technical efficiency score in input-oriented approach of CCR
φ^*	-	Optimal technical efficiency score in output-oriented approach of CCR
ρ^*	-	Optimal technical efficiency score of SBM
ρ_{in}^*	-	Optimal technical efficiency score of input-oriented SBM
R_e^*	-	Optimal technical efficiency score of ERM
γ^*	-	Optimal super-efficiency score of SBM
Γ_c^l	-	Optimal super-efficiency score of l_1 -norm
Cf^*	-	Optimal cost-efficiency score
Rf^*	-	Optimal revenue-efficiency score
Pf^*	-	Optimal profit-efficiency score
α	-	A positive real number
β	-	A positive real number
$\mathbb{Z}_{\geq 0}$	-	The nonnegative integer numbers set
$\mathbb{R}_{\geq 0}$	-	The nonnegative real numbers set
\mathbb{R}_+	-	The positive real numbers set
\emptyset	-	The empty set
M_{int}	-	A set of corresponding indexes of integer inputs
P_{int}	-	A set of corresponding indexes of integer outputs
C	-	A set of corresponding indexes of controllable data
NC	-	A set of corresponding indexes of non-controllable data
N_j	-	A nonnegative real number corresponding to input j
M_k	-	A nonnegative real number corresponding to output j
δ	-	A very small positive real number
\forall	-	For all

\exists	-	There is at least
\ll	-	Very less than
ε	-	A very small positive real number
$\boldsymbol{\varepsilon}$	-	A very small number in $\mathbb{R}_{\geq 0}^{m+p} - \{0\}$
ε_j^-	-	A very small positive real number corresponding to input j
ε_k^+	-	A very small positive real number corresponding to output k
A_ε^*	-	Optimal efficiency score of AM with ε degree of freedom
K_ε^*	-	Optimal efficiency score of KM with ε degree of freedom
KA_ε^*	-	Optimal efficiency score of KAM with ε degree of freedom
T_C	-	The CRS PPS
$T_C^{+\varepsilon}$	-	The CRS PPS with ε degree of freedom
$TT_C^{+\varepsilon}$	-	The efficient tape with 2ε diagonals
\widehat{x}_{lj}^*	-	Optimistic efficient target of input j of DMU_l after evaluation
\widehat{y}_{lk}^*	-	Optimistic efficient target of output k of DMU_l after evaluation
\widetilde{x}_{lj}^*	-	Pessimistic efficient target of input j of DMU_l after evaluation
\widetilde{y}_{lk}^*	-	Pessimistic efficient target of output k of DMU_l after evaluation
ε-AM	-	Arash model with ε degree of freedom
ε-KM	-	Kouros model with ε degree of freedom
ε-KAM	-	Kouros and Arash model with ε degree of freedom

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Data Envelopment Analysis (DEA) is a non-parametric technique in operations research to assess the performance of homogenous Decision Making Units (DMUs) such as factories, institutes and organizations. It was proposed by Charnes *et al.* (1978) based on earlier work by Farrell (1957). DEA has found amazing development during the last three decades. According to Emrouznejad *et al.* (2008) since the original DEA study by Charnes *et al.* to the year 2007, there have been over 7000 DEA references in the DEA database. In this research, the common DEA techniques and models are examined and their shortcomings to estimate production frontier, and benchmark and rank technically efficient and inefficient DMUs are illustrated. The study proposes new techniques in DEA which are able to rectify the shortcomings. A robust linear programming model is also proposed to measure the performance of DMUs inclusive integer and non-controllable data.

1.2 Research Background

One of the most important parts in management science is how to improve the efficiency of firms and organizations. DEA produces a fair benchmarking tool to measure the efficiency of DMUs. Once the input and output variables are identified for a set of DMUs, a Production Possibility Set (PPS) is produced by DEA axioms (Cooper *et al.*, 2007) and its frontier is considered to estimate the production frontier.

PPS contains all the correspondences of input and output points which are feasible in principle, even if not actually observed in practice. Then, the location of a DMU within the PPS is compared to the frontier of PPS in order to calculate its efficiency as well as benchmark and ranks DMUs. The first two definitions of DEA are as follows.

Definition 1.1: Efficiency of a DMU is the ratio of its produced output to its used input, that is, output/input (Cooper *et al.*, 2007).

The efficiency also means “doing the jobs right” and can be measured with comparing between observed and optimal values of input and output (Fried *et al.*, 2008).

Definition 1.2: A DMU is to be rated as fully (100%) efficient (referred to as ‘technical efficiency’ in economics) on the basis of available evidence if and only if the performances of other DMUs do not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs (Cooper *et al.*, 2011).

Definition 1.2 is usually called Pareto-Koopmans definition in DEA. From this definition, DMUs on the frontier of PPS are efficient and otherwise they are inefficient (Cooper *et al.*, 2011).

Charnes, Cooper and Rhodes in 1978 proposed a radial model called CCR which is able to measure the technical efficiency of DMUs. CCR in input oriented case assigns a technical efficiency score less than one to an inefficient DMU which means that a linear combination of observed DMUs could produce the same value of outputs using a smaller value of inputs. Then it provides a technical efficiency rating of inefficient DMUs, because the score reflects the radial distance from the estimated production frontier to the DMU under evaluation, that is, the minimum proportional decrease in inputs yielding technical efficiency. Charnes *et al.* (1985) also proposed a non-radial model called Additive model (ADD) which considers the possibility of input decreases and/or output increases simultaneously. Some other basic DEA models can be recognized as Banker, Charnes and Cooper (BCC) (Banker *et al.*,

1984), Slack Based Measure (SBM) (Tone, 1997) and Enhanced Russell Measure (ERM) (Fare and Lovell, 1978; Pastor *et al.*, 1999).

The basic DEA models divide DMUs into inefficient and technically efficient DMUs and only assign identical score for technically efficient ones, that is, they are not able to arrange technically efficient DMUs. The most popular method to compare technically efficient DMUs that leads to a concept called ‘super-efficiency’ proposed by Andersen and Petersen (1993). The basic idea of Andersen and Petersen was to compare the DMU under evaluation with a linear combination of all other DMUs in the sample, that is, the DMU itself is excluded. It is conceivable that a technically efficient DMU may increase its input vector proportionally while preserving efficiency with a technical efficiency score more than one. The score of Andersen and Petersen model (AP) reflects the radial distance from the DMU under evaluation to the production frontier estimated with that DMU excluded from the sample, that is, the maximum proportional increase in inputs preserving efficiency. Some other super-efficiency models can also be recognized as Mehrabian, Alirezaee and Jahanshahloo (MAJ) (Mehrabian *et al.*, 1999), SBM (Tone, 2002) and l_1 -norm (Jahanshahloo *et al.*, 2004).

The Pareto-Koopmans definition of efficiency (Definition 1.2) and the basic and super-efficiency DEA models avoid the need for recourse to prices or other assumption of weights between input and output of DMUs. In order to identify some types of inefficiency when cost information and unit price are available for DMUs, allocation and assurance regions models have been also proposed in DEA (Cooper *et al.*, 2007). On the other side, the basic DEA models may benchmark a DMU to some targets which in general some of their inputs or outputs are not integer, whereas they should be specified in the set of natural numbers, such as the number of books or vehicles. Therefore, Lozano and Villa (2006) proposed a model based on CCR to rectify the shortcomings. Soon after, Kuosmanen and Kazemi Matin (2009) proposed the DEA integer axioms and improved Lozano and Villa’s model.

DEA has been constructed on the above techniques and increasingly developed in many various fields. Full details on the description of DEA techniques, and the short history of DEA in last three decades can be found in Cooper *et al.*

(2007) and Cook and Seiford (2009), respectively. In this research, the shortcomings in DEA are illustrated and novel techniques are produced to remove the shortcomings. The proposed methods improve the capabilities of DEA to measure the efficiency scores of DMUs as well as benchmark and rank them.

1.3 Motivation for Research

The Pareto-Koopmans definition of efficiency is able to characterize the technically efficient and inefficient DMUs, however, it is not able to identify the efficient DMUs, that is, the DMUs which do the jobs right. In other words, there may be some technically efficient DMUs which have the worst performance in comparison with all observed DMUs. In other words, Definition 1.2 is not equivalent with Definition 1.1. The basic DEA models are not able to distinguish technically efficient DMUs. They cannot arrange both inefficient and technically efficient DMUs at the same time, and always arrange inefficient DMUs after technically efficient ones. However, an inefficient DMU may be more efficient than a technically efficient one. They cannot also benchmark technically efficient DMUs. Therefore, the basic DEA models are not almost always valid to benchmark and rank DMUs. The Andersen and Petersen technique is not a harmless technique to arrange technically efficient DMUs as it is depicted in this research. Therefore, all super-efficiency DEA models based on this method are not valid. The basic DEA models are not almost always able to measure the cost-efficiency, revenue-efficiency or profit-efficiency of DMUs and users requires using allocation models.

1.4 Problem Statement

This study proposes new DEA methods with definitions and models to improve DEA capabilities, and remove the shortcomings of current DEA techniques to estimate the production frontier, and rank and benchmark DMUs.

1.5 Objective of the Study

The objectives of this research are as follows:

1. To identify the shortcomings of DEA definitions and techniques to assess production frontier, and benchmark and rank DMUs.
2. To propose new definitions and techniques to estimate production frontier, benchmark and rank DMUs inclusive integer and real data, where cost information and unit price are available or unknown.
3. To produce a new linear programming model for approximating production frontier while simultaneously benchmarking and ranking DMUs.

1.6 Research Questions

The research questions in this study are as follows:

1. What are the shortcomings of DEA models for estimating production frontier?
2. Are the DEA techniques valid to benchmark and rank DMUs?
3. What method is able to approximate production frontier accurately?
4. What model can simultaneously benchmark and rank DMUs inclusive integer and real values?
5. What technique is able to measure the efficiency score of DMUs where cost information and unit price are available or unknown?
6. What model is able to produce the mentioned properties together?

1.7 Significance of the Study

This study removes the shortcomings of current DEA models to estimate production frontier, and benchmarking and ranking DMUs. It eliminates the computational complexity of using most of DEA models. The study produces a

flexible linear programming model to assess production frontier as well as benchmark and rank DMUs inclusive integer and real data. It also suggests researchers to avoid using the basic DEA models for assessing the performance of DMUs. The proposed techniques and models cover many subjects regarding DEA.

1.8 Scope of the Study

The scope of this research is Linear Programming (LP), DEA, concepts of efficiency, and the software to solve linear and non-linear programming models such as Microsoft Excel Solver and Lingo software. In addition, in order to compare and examine the results of the proposed models and previous DEA models, some data in previous researches are used, and some data and examples are also designed.

1.9 Research Methodology

The research is started on the various concepts and properties of DEA in order to review the capabilities and shortcomings of basic DEA models and techniques. In the first step, simple examples are proposed to depict the weaknesses of Pareto-Koopmans definition and AP model. Moreover, the shortcomings of radial and non-radial basic DEA models to benchmark inefficient DMUs are illustrated. In the second step, new definitions, techniques and models are proposed to rectify the mentioned shortcomings. Next, the models are improved to measure the efficiency score of DMUs inclusive integer and real data. At the end, the proposed models are combined in order to suggest a robust model to assess production frontier while simultaneously benchmark and rank DMUs.

1.10 Thesis Organization

The first chapter of this study illustrates the introduction part of this thesis which includes the concepts of DEA and its shortcomings.

Chapter 2 represents DEA and depicts its basic techniques and models such as radial, non-radial, super-efficiency, allocation, non-controllable and mixed integer models to assess the performance of DMUs where cost information and unit price are available or unknown.

Chapter 3 clearly depicts the shortcomings in basic and super-efficiency DEA techniques and proposes a method to rectify the weaknesses. The method is able to measure cost-efficiency of DMUs, distinguishes between efficient and technically efficient DMUs, arranges both inefficient and technically efficient DMUs at the same time, and characterizes the reasonable score for each DMU. The technique is also improved to estimate the efficiency score of DMUs with non-controllable data. In order to measure all the inefficiencies that the model can identify, the proposed model is improved to a non-linear model which can easily be solved by transforming to linear programming.

In Chapter 4, firstly a technique is proposed to measure revenue-efficiency of DMUs and examine the instabilities of DMUs' technically efficiency score where a small error is introduced in their output values. After that, the previous proposed techniques are combined together for having a complete model to evaluate DMUs' efficiency score. It is also depicted how the combined proposed model is able to measure profit-efficiency of DMUs where the cost information and unit price are available.

In the next chapter, the shortcomings in integer DEA axioms and techniques are presented. In order to rectify the shortcomings, the integer DEA axioms are improved and the previous proposed method is also extended to measure the efficiency score of DMUs with both real and integer valued data.

Chapter 6 illustrates a technique to estimate the DEA efficient frontier based on the previous proposed techniques in a way different from the statistical inferences. The technique allows decisions in the targets regions instead of points to benchmark DMUs. It suggests three efficiency indexes, called the lowest, technical and highest efficiency scores, for each DMU where small errors are considered in both input and

output components of the Farrell frontier, even if the data is accurate. These efficiency indexes provide a sensitivity index for each DMU and arrange both inefficient and technically efficient DMUs together while simultaneously detecting and benchmarking outliers. Two numerical examples depict the validity of the proposed method and identify how the technique is able to restrain the curvature of production frontier.

In the last chapter, summary and conclusion of this research is illustrated with some suggestions for future research.

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