PI/PISMC Control of Hydraulically Actuated Active Suspension System

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Abstract: The purpose of this paper is to present a robust strategy in controlling a hydraulically actuated active suspension system. The controller consists of the two controller loops namely inner loop controller for force tracking control of the hydraulic actuator and outer loop controller to reject the effects of road induced disturbances. The outer loop controller utilized a proportional integral sliding mode control (PISMC) scheme. Whereas, proportional integral (PI) control is used in the inner loop controller to track the hydraulic actuator in such a way that it able to provide the actual force as close as possible with the optimum target force produced by the PISMC controller. A quarter-car model is used in this study and the performance of the controller is compared with the state feedback controller and the existing passive suspension system. A simulation study is performed to prove the effectiveness and robustness of the control approach. Force tracking performance of the hydraulic actuator is also investigated.

1. INTRODUCTION

Active suspensions systems are widely studied over the last 30 years; with hundreds of research papers have been published (Hrovat, 1997). Most of the published works are focused on the outer-loop controller in computation of the desired control force as a function of vehicle states and the road disturbance (Shen and Peng, 2003). It is commonly assumed that the hydraulic actuator is an ideal force generator and able to carry out the commanded force accurately. Simulations of these outer-loop controllers were frequently done without considering actuator dynamics, or with the highly simplified hydraulic actuator dynamics.

In the real implementation, actuator dynamics can be quite complicated, and the interaction between the actuator and the vehicle suspension cannot be ignored. It is also difficult to produce the actuator force close to the target force without implementing inner-loop or force tracking controller. This is due to the fact that hydraulic actuator exhibits non-linear behavior resulted from servo-valve dynamics, residual structural damping, and the unwanted effects of back-pressure due to the interaction between the hydraulic actuator and vehicle suspension system. Some of the previous works on the force tracking controller of hydraulic actuator can be found in (Shen and Peng, 2003; Zhang and Alleyne, 2002; Chantranuwatana, 2001).

The development of a non-linear hydraulic actuator model including its force tracking controller for an active suspension system is investigated in this study. The non-linear hydraulic actuator model consists of servo-valve dynamics and the interaction of piston-cylinder. Force tracking control of the hydraulic actuator model is then performed using the Proportional Integral (PI) controller for a variety of the functions of target forces namely sinusoidal, saw-tooth, square and random functions. Once the inner loop controller of hydraulic actuator is able to track well the target forces with acceptable error, the hydraulic actuator model and the inner loop controller are then integrated with the outer loop of active suspension control. In this configuration, the inner loop controller must be able to track the optimum target force of hydraulic actuator calculated by the outer loop controller.

Various control strategies such as limited state feed back controller (Ahmadian and Vahdati, 2003; Ikenaga, 2001), optimal state-feedback (Alleyne and Hedrick, 1997; Esmailzadeh and Taghirad, 1996), back-stepping method (Lin and Kannelakopoulos, 1997), fuzzy control (D'Amato and Viassalo, 2000) and sliding mode control (Yoshimura *et al*., 2001) have been proposed in the past years for outer loop control of the active suspension system. From the previous investigations, it can be seen that the sliding mode control has relatively simpler structure and it guarantees the system stability. In this paper, newly developed sliding mode control schemes that can improve further the ride comfort and road

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handling of the active suspension system is considered. The proposed control scheme differs from the previous sliding mode techniques in the sense that the sliding surface is based on the proportional-integral (PI) sliding mode control. The additional integral in the proposed sliding surface provides one more degree of freedom and also reduce the steady state error.

This paper is organized as follows: the first section contains introduction, the second section describes the equations of motion of the quarter car model, the third section presents the dynamic equations and force tracking control of the hydraulic actuator model, the fourth section elaborates the switching surface and controller design of the active suspension system using PISMC, the fifth section contains the results of study and some discussions, and the last section presents some conclusion.

2. DYNAMIC MODEL OF A QUARTER CAR SUSPENSION SYSTEM

A quarter car model is considered in this study. The quarter car model for passive suspension system consists of one-fourth of the body mass, suspension components and one wheel as shown in Figure 1 (a). The quarter car model for active suspension system, where the hydraulic actuator is installed in parallel with the spring and the passive damper, is shown in Figure 1 (b). The assumptions for the quarter car modeling are as follows: the tire is modelled as a linear spring without damping, there is no rotational motion in wheel and body, the behavior of spring and damper are linear, the tyre is always in contact with the road surface and the effect of friction is neglected so that the residual structural damping is not considered into vehicle modeling.

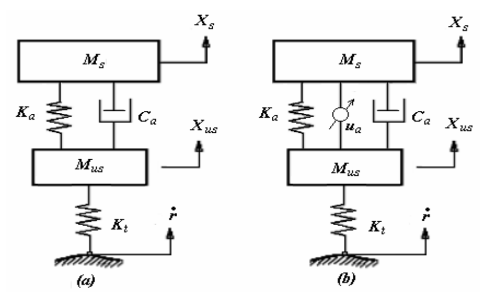

Figure 1: Quarter car model

The equations of motion for the sprung and unsprung masses of the passive and active quarter car model are given by the following state space representation

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & -1\\ -K_a/M_s & -C_a/M_s & 0 & C_a/M_s\\ 0 & 0 & 0 & 1\\ K_a/M_{us} & C_a/M_{us} & -K_t/M_{us} & -C_a/M_{us}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 1/M_s\\ 0\\ -1/M_s\end{bmatrix}u_a+\begin{bmatrix}0\\ 0\\ -1\\ 0\end{bmatrix}\dot{r}$$
(1)

where u_a is the control force from the hydraulic actuator and assumed as the control input. Eq. (1) can be written as

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t) \tag{2}$$

In the case of passive suspension system, u_a is set to be zero. M_s and M_{us} are the masses of car body and wheel respectively, x_s and x_{us} are the displacements of car body and wheel respectively, K_a is the spring coefficient of the suspension system, K_t is the spring constant of the tyre, C_a is the damping coefficient and $\dot{r}$ is the road disturbance. The following terms are defined as the state variables: $x_1 = x_s - x_{us}$ for suspension travel, $x_2 = \dot{x}_s$ for car body velocity, $x_3 = x_{us} - r$ for wheel deflection and $x_4 = \dot{x}_{us}$ for wheel velocity.

3. DYNAMICS MODEL OF HYDRAULIC ACTUATOR

A complete set of a hydraulic actuator consists of five main components namely electro hydraulic powered spool valve, piston-cylinder, hydraulic pump, reservoir and piping system as shown in Figure 2a. The power supply is needed to drive the hydraulic pump through AC motor and to control the spool valve position. The hydraulic pump will keep the supply pressure at the optimum level of pressure. The spool valve position will control the fluid to come-in or come-out to the piston-cylinder that determines the amount of force produced by the hydraulic actuator.

The hydraulic actuators are governed by electro hydraulic servo valve allowing for the generation of forces between the sprung and unsprung masses. The electro hydraulic system consists of an actuator, a primary power spool valve and a secondary bypass valve. As seen in Figure 2b, the hydraulic actuator cylinder lies in a follower configuration to a critically centered electro hydraulic power spool valve with matched and symmetric orifices. Positioning of the spool u_1 directs high pressure fluid flow to either one of the cylinder chambers and connects the other chamber to the pump reservoir. This flow creates a pressure difference P_L across the piston. This pressure difference multiplied by the piston area A_p is the active force F_A for the suspension system. The derivative of F_A is given by Eq. (3).

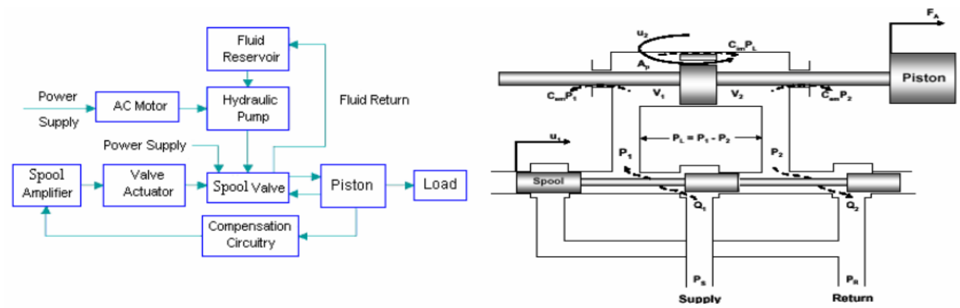

Figure 2: Diagram and the physical schematic of a complete set of hydraulic actuator.

Dynamics for the hydraulic actuator valve are given as the followings: the change in force is proportional to the position of the spool with respect to center, the relative velocity of the piston, and the leakage through the piston seals. A second input u_2 may be used to bypass the piston component by connecting the piston chambers.

$$\dot{F}_A = A_p \alpha \left[C_{d1} w u_1 \sqrt{\frac{P_s - \operatorname{sgn}(u_1) P_L}{\rho}} - C_{d2} u_2 \operatorname{sgn}(P_L) \sqrt{\frac{2 P_L}{\rho}} - C_{tm} P_L - A_p (\dot{x}_s - \dot{x}_{us}) \right] \quad (3)$$

The bypass valve u_2 could be used to reduce the energy consumed by the system. If the spool position u_1 is set to zero, the bypass valve and actuator will behave similar to a variable orifice damper. Spool valve positions u_1 and bypass area u_2 are controlled by a current-position feedback loop. The essential dynamics of the spool have been shown to resemble a first order system as the followings

$$\tau \dot{u} + u = kv \quad (4)$$

The structure of force tracking control of hydraulic actuator is shown in Figure 3. The hydraulic actuator model take two input namely spool valve position and real time piston speed. Proportional Integral control is implemented which takes force tracking error as the input and delivers control voltage to drive the spool valve. The target force is represented by sinusoidal, square, saw-tooth and random functions. The parameters of hydraulic actuator model are taken from Donahue (2001) as the followings: A_p= 0.0044 m^2 , α = 2.273e9 N/m^5 , C_{d1} = 0.7 , C_{d2} = 0.7 , w = 0.008 m , P_s = 20684 kN/m^2 , ρ =3500, C_{tm} = 15e-12 , τ = 0.001 sec^{-1} .

4. SWITCHING SURFACE AND OUTER LOOP CONTROLLER DESIGN

The controller structure adopted in this study is shown in Figure 3. Basically, the controller structure of an electronically controlled suspension system utilizes two controller loops namely outer loop and inner loop controllers which corresponds to vehicle controller and actuator controller. The similar terms, which are often used for outer and inner loop controllers, are global and local controllers. The controller structure was used for an active suspension system in (Chantranuwatana, 2001). The similar controller structure was used for semi-active suspension control in (Sims *et al.*, 1999; Hudha *et al.*, 2005).

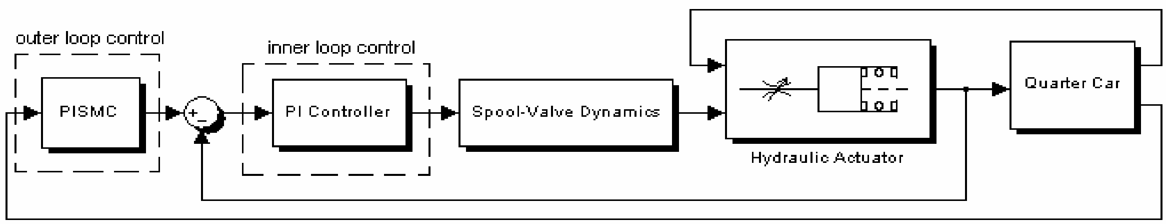

Figure 3: Controller structure of the active suspension system

The outer loop controller is used for disturbance rejection control to reduce unwanted vehicle's motions. The inputs of the outer loop controller are vehicle's states namely body velocity and wheel velocity, whereas the output of the outer loop controller is the target force that must be tracked by the hydraulic actuator. On the other hand, the inner loop controller is used for force tracking control of the hydraulic actuator in such a way that the force produced by the hydraulic actuator is as close as possible with the target force produced by the disturbance rejection control.

The disturbance rejection control utilized in this study is proportional integral sliding mode controller where the PI sliding surface is defined as follows (Sam et al., 2004; Sam et al., 2005)

$$\sigma(t) = Cx(t) - \int_0^t (CA + CBK)x(\tau)d\tau \qquad (5)$$

where $C \in \Re^{mxn}$ and $K \in \Re^{mxn}$ are constant matrices. The matrix K satisfies $\lambda(A + BK) < 0$ and C is chosen so that CB is nonsingular. It is well known that if the system is able to enter the sliding mode, hence $\sigma(t) = 0$. Therefore the equivalent control, $u_{eq}(t)$ can thus be obtained by letting $\dot{\sigma}(t) = 0$ (Itkis, 1976), i.e,

$$\dot{\sigma}(t) = C\dot{x}(t) - \{CA + CBK\}x(t) = 0 \qquad (6)$$

If the matrix C is chosen such that CB is nonsingular, this yields

$$u_{eq}(t) = Kx(t) - (CB)^{-1}Cf(t) \qquad (7)$$

The proposed control scheme is designed in such a way that drives the state trajectories of the system in Eq. (2) onto the sliding surface $\sigma(t) = 0$ and the system remains in it thereafter. For the uncertain system in Eq.(2), the following control law is proposed:

$$u(t) = -(CB)^{-1}[CAx(t) + \phi\sigma(t)] - k(CB)^{-1}\frac{\sigma(t)}{\|\sigma(t)\| + \delta} \qquad (8)$$

where $\phi \in \Re^{mxm}$ is a positive symmetric design matrix, k and $\hat{c}$ are the positive constants.

5. SIMULATION RESULTS AND DISCUSSION

This section contains the results of simulation studies in both inner loop and outer loop controllers. The parameters of inner loop controller must be optimized separately until the hydraulic actuator is able to provide the actual target force closely the same as the predefined target force. Then, the inner loop controller is integrated with the outer loop controller. In this configuration, the inner loop controller is used to track the optimum target force produced by the outer loop controller. This is due to the fact that the failure of inner loop controller in providing the actual force of hydraulic actuator as close as possible with the target force will degrade the performance of the outer loop controller. The performance of inner loop controller is characterized by its ability in tracking the target force with small amount of force tracking error. Whereas, the performance of outer loop controller is characterized by the four performance criteria namely body acceleration, body displacement, suspension travel and wheel displacement.

5.1 PERFORMANCE OF FORCE TRACKING CONTROLLER

The force tracking error of the hydraulic actuator model using Proportional Integral controller for sinusoidal, square, saw-tooth and random functions of the target force are shown in Figures 4(a), 4(b), 4(c) and 4(d) respectively. This is to check the controllability of the force tracking controller for a class of continuous and discontinuous functions. In this simulation study, the parameter of proportional gain *P* is set to 1.25 and for Integral gain *I* is set to 0.75. From this figure, it can be seen clearly that the hydraulic actuator model tracks the desired force well.

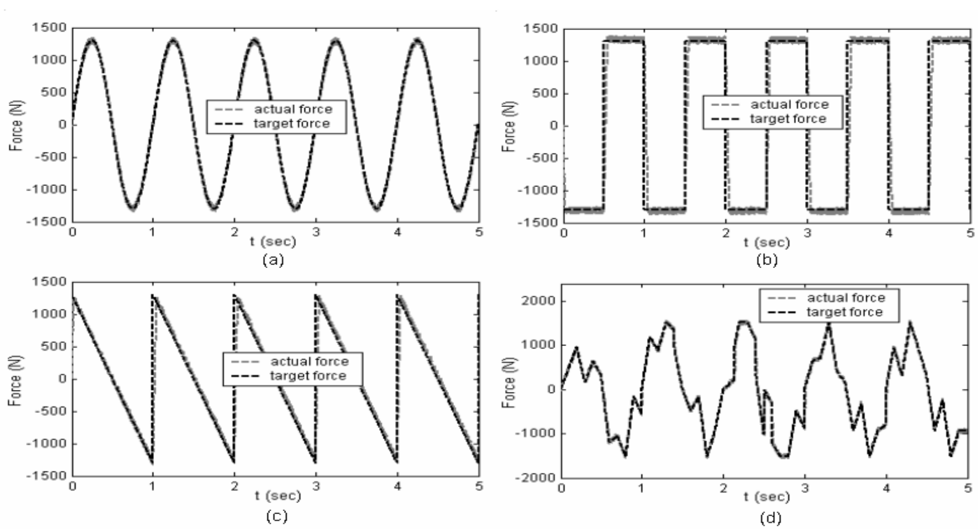

Figure 4: Force tracking controller of the hydraulic actuator

It is also noted that due to the rapid changes of force magnitude in the case of discontinuous function of target force such as saw-tooth and square functions, the performance of force tracking controller is slightly worse than that of continuous function of target force. This is caused by the response of the spool valve that fails to follow the target force without time delay particularly when the rapid change of force magnitude is occurred.

5.2 PERFORMANCE OF DISTURBANCE REJECTION CONTROL

The typical road disturbance considered in this simulation study is shown in Figure 10. This type of road disturbance has been used in (Amato and Viassalo, 2000; Sam et al., 2004; Sam et al., 2005) and set in the following form where, *a* denote bump amplitude which is set to be ± 8 cm.

$$r = \begin{cases} a\dfrac{(1-\cos(8\pi t))}{2} & if \quad 0.50 \le t \le 0.75 \quad and \quad 3.00 \le t \le 3.25 \\ 0 & otherwise \end{cases} \tag{9}$$

The simulation was performed for a period of 10 second using Heun solver with a step size of 0.001 second. The numerical values of quarter car model parameters are set based on the quarter car characteristics of Malaysian National car as in Hudha (2005) as the following: M_s = 282 kg; C_s = 1500 Nsec/m; M_u = 45 kg; K_t =165790 N/m and K_s = 17900 N/m.

The mathematical model of the system as defined in Eq. (2) and the proposed proportional integral sliding mode controller (PISMC) in Eq. (8) were simulated on MATLAB-Simulink. For the comparison purposes, the performance of the PISMC is compared to the limited state feed back control approach. In the design of the limited state feed back controller, the designed matrix gains of the limited state feed back controller are selected to be K_s = [900 250 500]. The values of the matrix K for the PISMC is $K = [0.6565 \quad -0.2366 \quad 0.3722 \quad 1.0711]$ such that $\lambda(A + BK)$ = {-8.708 ± 58.197i, -1.4717 ± 7.201i, -0.020}. In this simulation study, the following values namely $C = [400 \quad 10 \quad 7500 \quad 15000]$, $\phi = 100$, k=1 and $\delta = 10$ are also selected for the PISMC controller parameters. The sliding surface obtained from the simulation is shown in Figure 5.

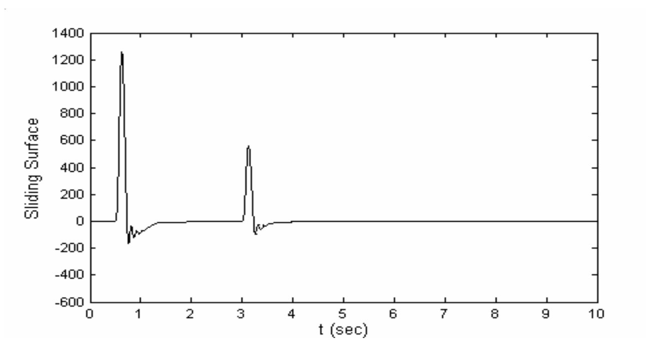

Figure 5: Sliding surface of PISMC

From the simulation results, the body acceleration and body displacement performances of PISMC compared to state feedback along with passive system are shown in Figure 6(a) and 6(b) respectively. From the figures, it is clear that the active system with PISMC is able to significantly reduce both amplitude and the settling time of unwanted body motions in the forms of body acceleration and body displacement as compared with the counterparts.

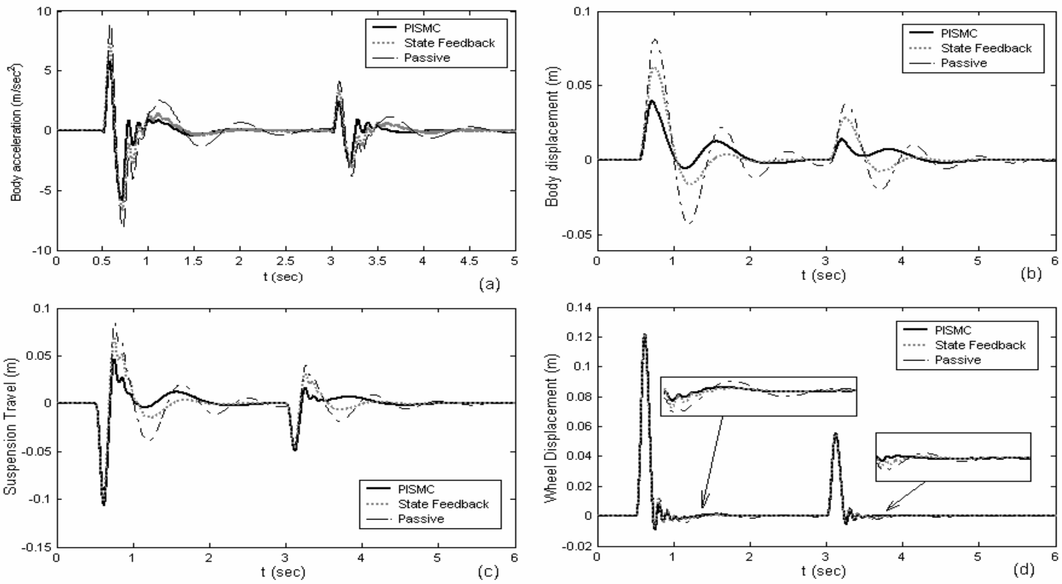

Figure 6: Performance of disturbance rejection control

The similar trend was found on the suspension deflection performance as shown in Figure 6(c), in which the active system with PISMC shows significant performance in reducing both amplitude and the settling time compared with the state feedback controller and the passive system. It is also noted that the active system with PISMC is able to improve the rattle-space dynamics of the suspension system. In term of the wheel displacement, it can be seen that the magnitude of the wheel displacement for the active system of both PISMC and state feedback controllers are slightly worse than the passive system as shown in Figure 6(d). Roughly, the magnitude of wheel displacement of the active system is about 1% larger than the passive system. But, it can be seen that the settling time of wheel-hop for the active system with PISMC is better than the counterparts.

The force tracking performances of the inner loop controller for the specified bump input are shown in Figures 7(a) and 7(b) respectively. It is clear that the hydraulic actuator is able to provide the actual force close to the optimum target force for both PISMC and state feedback controller. From the figures, it can be seen that when the tyre hits the bump with positive magnitude as shown in Figure 10, the hydraulic actuator produces negative force to prevent the vehicle body in moving upward and to lift up the wheel in following the bump profile.

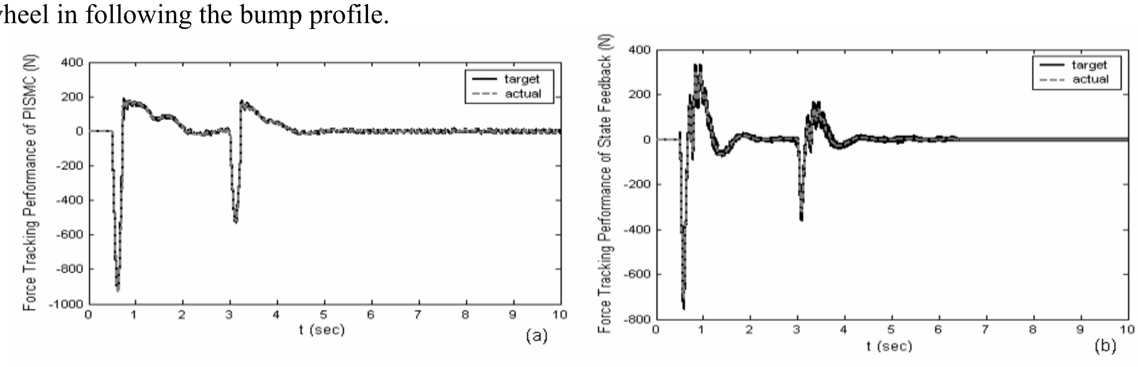

Figure 7: Force tracking performance of PISMC and state feedback controller

6. CONCLUSIONS

The paper presents a robust strategy in designing a controller for an active suspension system based on variable structure control theory. The result shows that the use of the proposed proportional integral sliding mode control technique is effective in controlling a vehicle and more robust compared to the counterparts. The PISMC was used to reject the effects of road disturbance to the vehicle dynamics performance. From the simulation results, it can be seen that the proposed controller shows significant improvement in reducing both magnitude and settling time of the body acceleration, body displacement and suspension displacement. In term of the wheel displacement, it is noted that even though the magnitude of the wheel displacement for the active system is slightly worse than passive system, the settling time of wheel-hop for the active system with PISMC is better than the counterparts. Force tracking performance of the non-linear hydraulic actuator model was also investigated. Proportional Integral control was implemented for force tracking control of the hydraulic actuator. The results of the study show that the hydraulic actuator is able to provide the actual force close to the target force with acceptable force tracking error for sinusoidal, square, saw-tooth and random functions of target forces. It can also be noted that the force tracking control is also able to closely follow the target force produced by PISMC and state feedback controller.

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