AN IMPROVISED THREE-DIMENSIONAL SLOPE STABILITY ANALYSIS BASED ON LIMIT EQUILIBRIUM METHOD BY USING PARTICLE SWARM OPTIMIZATION

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I dedicated this thesis to my beloved father and mother for their support and encouragement.

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ABSTRACT

The stability of slopes is a major concern in the field of geotechnical engineering. Two-dimensional (2D) limit equilibrium methods are usually implemented in this field due to their simplicity. However, these methods ignore the features of the third dimension of slopes. Although three-dimensional (3D) methods tried to remove the previous limitation, most of them assumed the direction of sliding, simplified or ignored the intercolumn forces, and avoided to search for location and shape of three-dimensional critical slip surface. This study was performed to overcome the mentioned limitations. In the present study, a new slope stability method was established based on the force and moment equilibrium in two vertical directions that was able to find the unique direction of sliding. Moreover, a modified Particle Swarm Optimization was developed by replacing the worst particle of each swarm with the previous global best particle and using a dynamic inertia weight to determine the 3D critical slip surface. Then, a computer program was established to model 3D slopes and perform the required calculations. Several benchmark problems were re-analyzed to verify the results of the study and good agreements were achieved with the results of previous studies when different failure mechanisms as ellipsoid, cylindrical, and composite slip surfaces were successfully applied in the analysis. The results indicated that the 3D factor of safety of a slope is always greater than its corresponding 2D factor. Moreover, the end effect in 3D analysis was found to be more significance in the problems with lower ratio of length to the width of the sliding mass. It was also found that the presence of water and weak layer enlarged this effect. Through the verification study, it was observed that different sliding directions produce different factors of safety, while the lowest value of factor of safety and 3D critical slip surface is only reachable through the real direction of sliding. Finally, case studies of actual stability problems were analyzed to find their critical slip surfaces. Achieving the minimum factor of safety of 0.977 for the critical slip surface of a failed slope demonstrated the validity of performance of presented computer code. Based on the obtained results, this study successfully overcame the mentioned limitations of the previous methods. The results of this study provided a better understanding of the actual failure mechanism and helped to enhance the safety and reduced the economic and health costs due to slope failure by a more detailed analysis than before.

ABSTRAK

Kestabilan cerun kini menjadi kebimbangan utama dalam bidang kejuruteraan geoteknik. Penggunaan had dua dimensi (2D) yang merupakan kaedah keseimbangan biasanya dijalankan kerana penyerderhanaan pengaplikasian. Walau bagaimanapun, kaedah ini mengabaikan dimensi ciri ketiga cerun. Kaedah tiga dimensi (3D) dihasilkan bagi menghapuskan keterbatasan sistem aplikasi sebelumnya. Kebanyakan pengkaji menganggap arah gelongsor, dipermudah atau diabaikan daya intercolumn, menghalang untuk mencari lokasi dan bentuk tiga dimensi permukaan slip critical. Kajian ini telah dijalankan untuk mengatasi keterbatasan tersebut. Dalam kajian ini, satu kaedah baru telah diperolehi berdasarkan kestabilan cerun dalam mengimbangi tenaga dua arah menegak dapat mencari hala tuju unik gelongsor. Pengubahan zarah sekumpulan pengoptimuman dibangunkan bagi menggantikan kekurangan kumpulan zarah terdahulu, dengan penggunaan berat inersia dinamik untuk menentukan permukaan slip critical 3D. Program komputer telah diperolehi untuk model cerun 3D dalam melaksanakan pengiraan yang tepat. Beberapa masalah penanda aras dianalisis untuk mengesahkan keputusan kajian dan satu kesepakatan telah dipersetujui berkaitan dengan penemuan kajian terdahulu, akibat terdapat kegagalan mekanisma yang berbeza di permukaan slip ellips, silinder, dan kegagalan permukaan komposit telah berjaya digunakan dalam analisis kajian. Keputusan menunjukkan bahawa faktor keselamatan cerun 3D adalah sentiasa lebih besar berbanding faktor 2D. Hasil akhir dalam analisis kajian 3D didapati lebih penting dalam penyataan masalah dengan nisbah yang lebih rendah jisim panjang gelongsor. Kajian juga mendapati bahawa kehadiran air yang diperbesarkan akibat dari kesan lapisan yang lemah. Melalui kajian pengesahan, Dapat diperhatikan bahawa gelongsor arahan berbeza menghasilkan keselamatan faktor yang berbeza, manakala faktor keselamatan yang lebih rendah dan 3D kegagalan permukaan kritikal hanya dapat dicapai melalui gelongsor arah sebenar. Kajian kes masalah kestabilan sebenar telah dianalisis bagi mencari kegagalan permukaan kritikal. Faktor minimum keselamatan 0.977 di kegagalan permukaan kritikal kegagalan cerun menunjukkan kesahihan prestasi kod komputer dipaparkan. Berdasarkan keputusan yang diperolehi, kajian ini telah berjaya mengatasi keterbatasan aplikasi yang terdahulu. Hasil keputusan kajian memberikan pemahaman yang lebih baik terhadap mekanisme kegagalan sebenar dan dapat membantu untuk meningkatkan keselamatan bagi mengurangkan kos ekonomi dan kesihatan akibat kegagalan analisis cerun secara lebih terperinci berbanding sebelumnya.

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LIST OF ABBREVIATIONS

2D - Two-Dimensional
3D - Three-Dimensional
DP - Dynamic Programming
FOS - Factor of Safety
FEM - Finite Element Method
GA - Genetic Algorithm
KUR - Karaj Underground Railway
LEM - Limit Equilibrium Method
NATM New Austrian Tunneling method
PSO - Particle Swarm Optimization
RNG - Random Number Generation
SRM - Strength Reduction Method

LIST OF SYMBOLS

a - direction of sliding on base of column

a' - direction of sliding on x-y plane

a_x - inclination of column base along x-directions

a_y - inclination of column base along y-directions

ac - total number of active columns

a_L - distance of left side water force from center of rotation

a_R - distance of right side water force from center of rotation

A_e - percentage of error of area calculation

A_{half} - area of half-sphere

$A_{i,j}$ - base area of column

A_t - calculated area of half-sphere by the code

A_L - resultant water force at the left side

A_R - resultant water force at the right side

b - horizontal width of slice

b_c - width of tension crack

$bg_{n(i)}$ - best position in the swarm so far

$b_{i,j}$ - width of the column on x-y plane

$bn_{m(i)}$ - best position of mth neighbour of nth particle so far

$bp_{n(i)}$ - best position of nth particle so far

B - height of the side of column in y-direction

c' - average effective cohesion of the soil

C' - cohesion of the soil in terms of effective stress

d - distance of the external lineear load from the center of rotation

D - height of the side of column in x-direction

D_s - dimension of the particles of swarm

e - distance of the center of slice from the center of rotation

$e\omega_{i,j}$ - effective weight of column

E - deformation module

E_s - normal interslice force
E_x - normal intercolumn force in x-direction
E_y - normal intercolumn force in y-direction
E_L - normal interslice forces at left side of slice
E_R - normal interslice forces at right side of slice
F_w - horizontal force induced by water in tension crack
f - perpendicular distance of normal force and center of rotation
$f_{(x)}$ - interslice force function of Morgenstern and Price’s method
f_1, f_2, f_3 - unit vectors for base shear force
$f_{(X)}$ - objective function of optimization techniques
F_{eh} - horizontal force induced by earthquake
F_{ehx} - horizontal force induced by earthquake in x-direction
F_{ehy} - horizontal force induced by earthquake in y-direction
F_{ev} - vertical force induced by earthquake
F_m - overall moment equation
F_s - factor of safety of column
F_{sx} - directional factors of safety in x-directions
F_{sy} - directional factors of safety in y-directions
F_t - overall force equation
Fit - fitness value of particles
FOS - factor of safety
g_1, g_2, g_3 - unit vectors for base normal force
G_x - normal intercolumn forces along x-directions
G_y - vertical shear intercolumn forces along x-directions
G - resultant of intercolumn forces along x-directions
h_1 - height of right side of the column
h_2 - height of left side of the column
h_c - depth of tension crack
$h_{i,j}$ - central height of column
h - thickness soil layers
h_p - depth of point from water level
h_x - moment arm of normal intercolumn force in x-direction
h_y - moment arm of normal intercolumn force in y-direction
h_E - height of acting point of normal interslice force
H_u - coefficient of inclination of piezometric line

H_x - horizontal intercolumn shear force in x-direction
H_y - horizontal intercolumn shear force in y-direction
k - coefficient of seismic dynamic force
K - number of the neighbours in swarm
K_h - pseudo-static horizontal seismic coefficient
K_v - pseudo-static vertical seismic coefficient
l - external linear load on slope
L - horizontal length of sliding surface
L_z - external vertical load
L_x - external horizontal load in x-direction
L_y - external horizontal load in y-direction
N - total normal force at base of column
$N`$ - effective normal force base at base of column
N_s - swarm size of particle swarm optimization
N_x, N_y, N_z - components of base total normal force in x- y- and z-directions
p - horizontal shear intercolumn forces
P - total normal force at base of slice
P_v - vertical external force
P_x - moment of external forces in x-direction
P_y - moment of external forces in y-direction
q - normal intercolumn forces along z-directions
Q - resultant of intercolumn forces
r - radius of sphere
r_u - pore water ratio
R - radius of moment arm of shear force
R_c - cohesion part of intercolumn shear force
R_x, R_y, R_z - semi-radiuses of ellipsoid in x- y- and z-directions
R_ϕ - cohesionless parts of intercolumn shear force
S - mobilized shear strength at base of column
S' - projection of mobilized shear strength on x-y plane
S_f - available shear strenght of soil
S_m - mobilized shear strenght at the base of slice
S_x, S_y, S_z - components of base shear strength along x- y- and z- directions
S_{xz}, S_{yzi} - components of base shear strength in x-z and y-z planes
T - base shear strength of column

T_a/F - resultant of resisting forces

$T_{x,y}$ - resultant of base shear strength in x-y plane

T_z - shear strength along z-direction

u_a - pore air pressure

u_w - pore water pressure

$u(0, \vartheta)$ - vectors of random numbers for velocity equation of swarm

U - pore water force at the base of column

v - velocity of particle

V_e - percentage of error of volume calculation

V_{half} - volume of half-sphere

V - vertical intercolumn shear force

V_t - calculated volume of half-sphere by the code

w - inclination of the external linear load

W - weight of column

x_h - horizontal distance of center of slice and center of rotation

x_v - design variables in optimization techniques

x_{vmin} - lower bound of design variables in optimization techniques

x_{vmax} - upper bound of design variables in optimization techniques

x - position of particle in article swarm optimization

X_s - vertical interslice shear force

X_c, Y_c, Z_c - coordinates of center of ellipsoid in x- y- and z-directions

X_v - vector of design variables in optimization techniques

X_x - vertical intercolumn shear force in x-direction

X_y - vertical intercolumn shear forces in y-direction

X - vertical intercolumn shear forces

X_R - interslice shear forces at right side of slice

X_L - interslice shear forces at left side of the slice

α_s - inclination of base of slice

α_1 - inclinations of the left side of column base

α_2 - inclinations of the right side of column base

α - projection of direction of sliding at base of column

α'_i - direction of sliding on x-y plane

α_{xz} - inclination of column base along x-direction

α_{yz} - inclination of column base along y-direction

β - angle of resultant intercolumn force along x-direction

υ - poisson’s ratio

γ_w - unit weight of water

γ_t - total unit weight of overburden

γ - unit weight of soil layer

γ_s - saturated unit weight of soil layer

δ - angle of resultant interslice forces in Spencer’s method

Δy - width of the column in x-direction

Δy - width of the column in x-direction

ΔE_x - resultant normal intercolumn forces in x-direction

ΔE_y - resultant normal intercolumn forces in y-direction

ΔH_x - resultant horizontal intercolumn shear forces in x-direction

ΔH_y - resultant horizontal intercolumn shear forces in y-direction

ΔX_y - resultant vertical intercolumn shear forces in y-direction

ΔX_x - resultant vertical intercolumn shear forces in x-direction

ε - required accuracy of result

ϑ - constant in velocity equation of particle swarm optimization

ϕ' - angle of internal friction of soil in terms of effective stress

θ - angle between sides of column on base plane

θ_{xy} - rotation angle of ellipsoid in x-y plane

κ_x - total external forces in x-direction

κ_y - total external forces in y-direction

λ - coefficient of interslice force function

ξ - constant multiplier in velocity equation of swarm

ρ - direction of shear strength at base of column

σ_n - total normal stress

σ' - effective normal stress

χ - coefficient related to degree of saturation

ψ_x - inclination between action points of normal intercolumn forces

ψ_y - inclination between action points of normal intercolumn forces

ω - inertia weight of particles in velocity equation

[ ] - calculator of integer part

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

The stability of slopes is always an important concern in geotechnical engineering. Limit equilibrium method is commonly used in slope stability analysis (Morgenstern and Price, 1965; Fredlund and Krahn, 1977; Baker, 1980; Chen and Chameau, 1982; Fredlund, 1984; Ugai, 1995; Yu *et al.*, 1998; Kim *et al.,* 2002; Krahn, 2003; Cheng *et al.,* 2008; Askari and Farzaneh, 2008; Sun *et al.,* 2012; and Hongjun and Longtan, 2011). This method is established based on the principles of kinematics that does not consider displacement within the soil mass. Consequently, a kinematically admissible sliding surface is assumed for the failure. The soil mass above the sliding surface is considered a free body. Then disturbing and resisting forces along the sliding surface are estimated by using static equilibriums of force, moment, or both within the failure mass. It is assumed that the free body of the soil approaches the failure condition by reducing strength with a factor that is called the factor of safety (FOS). Although this solution provides quantitative information regarding the stability of slope as FOS, it is based on an assumed failure mass. Therefore, it has to be repeated for a number of probable slip surfaces to find the minimum FOS that is related to the critical slip surface. This surface is theoretically the critical slip surface, but the real failure surface may differ if the sliding occurs. This difference depends on how precisely the real slope idealized and what assumptions were used in the analysis.

The major classifications of slope stability analysis are 2D and 3D methods. In order to analyze the stability of a slope, 2D methods use its critical 2D section as a plain-strain problem, while 3D methods utilize its 3D model by using an appropriate mesh. These methods have similarities and differences in their basic theories and assumptions regarding the shape of the slip surface and dealing with the internal forces. The method of slices as a limit equilibrium method is commonly used to analyze complicated condition of 2D slopes. The equivalent of these methods in 3D analyses is called the method of columns. Since the internal forces of the failure mass are mainly unknown in this method, usually the problem becomes statically indeterminate. In this situation, the number of available equations to solve the problem is less than the number of unknowns. In order to overcome the mentioned indeterminacy, several assumptions and principles are applied in formulation of the limit equilibrium analysis. These assumptions are mainly related to determination of the shape and position of the sliding surface, and the magnitude, direction, and action point of the forces inside the sliding mass. The difference in theory behind the slope stability methods along with the applied assumption makes different processes to analyze the stability of slopes.

A number of 3D methods have established to consider the third dimension of slopes after the method of Anagnosti was proposed in 1969. However, the majority of them are limited in practice because of assumptions and limitations. A major assumption that is commonly made in 3D slope stability analyses is assuming a plane of symmetry for the sliding mass (Hovland, 1977; Chen and Chameau, 1983; Dennhardt and Forster, 1985; Leshchinsky et al., 1985; Ugai, 1985; Leshchinsky and Baker, 1986; Baker and Leshchinsky, 1987; Hungr, 1987; Leshchinsky and Mullet, 1988; Ugai, 1988; Xing, 1988; Hungr *et al.*, 1989; Leshchinsky and Huang, 1992; Cavounidis and Kalogeropoulos, 1992; Lam and Fredlund, 1993; and Jiang and Yamagami, 2004). Those methods that use the plane of symmetry assume the direction of sliding and consider only half of the sliding mass.

It is expected that the sliding mass move in a cross-sectional direction in symmetric slopes, but there is no guarantee to find the direction of sliding in asymmetric slopes without precise calculation. Consequently, the application of a large number of 3D methods is limited to symmetric problems. In order to generalize the application of 3D methods some researchers have tried to find the real sliding direction in asymmetric problems (Yamagami and Jiang, 1996 and 1997; Huang and Tsai, 2000; Huang *et al.*, 2002; and Cheng and Yip, 2007). However, these methods still have limitations in basic theories and practice.

Accordingly, the slope stability analysis within the framework of limit equilibrium needs to determine the critical slip surface. Although many 2D studies were involved with the 2D critical slip surface, only a few well-known studies tried to find the 3D critical slip surface (e.g. Yamagami and Jiang; 1997; Jiang, Yamagami, and Baker, 2003; Mowen, 2004; and Mowen *et al.*, 2011). All these studies still have limitations and assumptions in their objective functions and/or in the applied searching techniques that are discussed further. Consequently, an effective method is required to be able to find the 3D critical slip surface.

1.2 Statement of the Problems

The slope stability is applied in 2D and 3D analyses; both of which have shortcoming in modelling the real slope and sliding procedure. Two-dimensional analyses mainly simplify the real condition of the slope. As a mutual assumption, all 2D methods consider infinite width for the slope. The significance of this assumption is to reduce the unknowns related to the third dimension of the slope and simplify the calculations.

In reality, natural slopes are generally limited in the third dimension and performing a 3D analysis best presents the longitudinal changes of the slip surface, especially when the geometry of the failure mass is complex. Consequently, the results of 2D methods may largely differ from the real condition. Overestimated and underestimated results as the probable outcomes of using inappropriate method may cause economic and safety issues. These issues are more significant when asymmetric slopes with complex geometries or failure mechanisms are considered by 2D methods. Moreover, the real direction and shape of sliding is not determinable in

2D analysis. Although, some quasi-3D approximately evaluates the 3D FOS based on 2D factors, the accuracy of these methods is not guaranteed (Chen *et al.*, 2006). Therefore, performing 3D analysis provides closer results to the actual condition than 2D approaches.

The mentioned inadequacies of 2D analysis were partially solved by some existing 3D slope stability analyses. However, many of the available methods cannot provide an appropriate 3D model in asymmetric slopes, due to their assumptions and simplifications. In addition, the existing 3D methods did not offer a well-defined process to find the direction of sliding. Moreover, an effective search strategy is still required to determine the 3D critical slip surface.

Researchers avoid using 3D analysis because it is difficult to consider complex analytic equations, determine the static condition of the problem, and find the 3D critical slip surface. In order to overcome the mentioned problems, the limitations of the existing methods have to be removed to make 3D analysis feasible for slopes stability analysis. The main constraints of existing limit equilibrium methods in analyzing the 3D slopes are maintaining the required accuracy in calculation, calculating the direction of sliding, making an appropriate model of the real slope, and applying an effective search technique to find the 3D critical slip surface. Limit equilibrium method contributes to the establishment of a stability analysis technique. An optimization process can help to determine the position of 3D critical slip surface. This research attempted to eliminate the mentioned limitations by developing a 3D limit equilibrium method, a PSO technique, and a computer code. The proposed 3D method calculated the 3D FOS and direction of sliding. The developed PSO technique determined the 3D critical slip surface. Lastly, the developed computer code modeled the 3D slope and performed the calculations.

1.3 Objectives of the Study

The present study was aimed at performing 3D slope stability analysis based on limit equilibrium method by using PSO. In line with the main goals of the research, the following are the objectives of the study:

i. To determine the limitations of the existing three-dimensional slope stability analyses based on limit equilibrium methods
ii. To develop the equation of three-dimensional factor of safety based on limit equilibrium method
iii. To develop a particle swarm optimization to determine the three-dimensional critical slip surface
iv. To establish a computer code to model three-dimensional slopes and perform the calculations of the slope stability analysis
v. To verify and validate the performance of the present study

1.4 Research Questions

In order to reach the objectives of the present research, the research questions of the study were defined as following:

i. What are the limitations of existing three-dimensional slope stability analyses based on limit equilibrium method?
ii. How to develop the equation of three-dimensional factor of safety based on limit equilibrium method?
iii. How to develop a particle swarm optimization to find the three-dimensional critical slip surface?
iv. How to establish a computer code to model three-dimensional slopes and perform the calculations of the slope stability analysis?
v. How to verify and validate the performance of the present study?

1.5 Significance of the Study

In many cases of construction in urban areas or on industrial fields, the whole project or a part of it has to be established on top of slope, on slope, or at the toe of the slope. Consequently, the construction, design, remediation, and maintenance of slopes have always been important to geotechnical engineers. On the other hand, the importance of study on the stability of slopes is engaged with the safety and economic aspects of human life. Over the past decades, the frequency and consequences of landslides has increased significantly and this trend continues (Petely *et al.*, 2005; Petely, 2012). In addition to enormous economic losses resulting from slope failures and landslides, considerable loss of human life and injury also occur as a result of this events. Landslides were reported responsible for more than 100,000 deaths for the period of 1980 to 2000 for the main continencial area (Petely *et al.*, 2005). Moreover, based on the recent published information more than 32,000 people lost their lives around the world only in the period of 2004 to 2010 as the direct act of landslides (Petley, 2012). With increasing frequency and adverse impact of slope failures, the increasing requirement of better understanding of hilly urban areas and constructions related to slopes is revealed.

Since the shape of slopes are naturally asymmetric, 2D analyses have to simplify the real condition. This simplification is not satisfactory in many cases, especially when a slope with structural asymmetry is considered. In order to perform realistic analysis of these cases, three-dimensional slope stability studies are needed. Furthermore, finding the critical slip surface is an important task in a 3D analysis, because it defines the location and shape of the probable failure mass. Defining this surface can lead to estimate of the most hazardous area of the slope. Another important issue in stability analysis of the slope that is confined to 3D analyses is the calculation of direction of sliding that determines the direction of probable failure. The present research sets its goals to prepare a realistic model of 3D slopes, find the direction of sliding together with the corresponding 3D FOS, and determine the 3D critical slip surface of the slopes.

The outcomes of the present study help to better understanding the behavior of soil slopes. This is also possible to find the probable instability of a 3D slope. These significances can avoid the consequences of overestimated and underestimated results in geotechnical designing or assessing of a slope that are the main causes of economic lost and safety issues. Therefore, two direct benefits that can be achieved from the results of this research are preventing the unnecessary stabilization costs and enhancing the safety related to the slope environment.

1.6 Scope of the Study

In order to improve the 3D equation of FOS, this study choses limit equilibrium method as the most common method among all soil slope stability methods to focus on. Some of the reasons of this selection are as follows:

i. Limit equilibrium method is able to consider almost all of the engaged conditions in a slope stability problem such as external and internal forces of the soil mass, pore water pressure, and multi layered slopes (Morgenstern and Price, 1965).

ii. Limit equilibrium method has a simple theoretical approach that considers the major effective factors on the shearing resistance and is reliable in modelling the practical cases (Fredlund and Krahn, 1977; Chen and Chameau, 1982; and Askari and Farzaneh, 2008).

iii. The results of limit equilibrium method are similar to the results of more rigorous methods, while the required input parameters are reduced and achieved much easier than the mentioned methods (Spencer, 1967; Wright, Kulhawy, and Duncan, 1973; Spencer, 1973; Yu *et al.*, 1998; Duncan, 1996; Hongjun and Longtan, 2011).

The scope of this research is limited to the soil slopes with both homogeneous and layered material; however, the present method may be useful in analyzing other soil-alike materials. The linear Mohr-Coulomb failure criteria was adopted to model the behavior of material at the verge of failure. Moreover, the linear relation of Terzaghi (1936) was adopted to determine effective stresses on the sliding surface. Consequently, unsaturated soils and nonlinear relationships of effective stresses was excluded from the scope of this research. The main failure mechanism of the present study is rotational. A 3D ellipsoid slip surface was adopted that includes the spherical shape as a specific condition. However, a cylindrical slip surface also employed as a part of this study. The procedure of finding the critical slip surface was established based on advance searching optimization techniques. A particle swarm optimization was developed and employed as a global search technique to find the critical slip surface. Finally, Matlab coding language was used in this research due to its capabilities as:

i. Using matrix based calculation that saves the calculation time
ii. Providing great graphical tools to better understanding the results
iii. Providing powerful graphic user interface (GUI) to input data
iv. Providing advanced geometrical and mathematical functions
v. Co working with other databases such as Microsoft Excel to manage data

1.7 Expected Outcomes

The main outcomes of the present study are expected as following:

i. An equation of factor of safety based on limit equilibrium method that is able to calculate the unique direction of sliding and factor of safety of 3D slopes.
ii. A searching process based on particle swarm optimization that is able to find the critical slip surface of 3D slopes.
iii. A computer program that is able to model 3D slopes and perform the calculations of factor of safety and critical slip surface.

1.8 Limitations of the Study

There were some unavoidable limitations to carry out the present research. Firstly, an elastic perfectly plastic behavior was adopted for the soil materials so the linear Mohr-Coulomb failure criteria were used to determine the shear strength on the slip surface. Secondly, the framework of limit equilibrium method was applied in the analysis, thus the failure mass was considered as a rigid body and the static equilibriums were used to establish the 3D equation of factor of safety. The third limitation was related to the modelling of the slope. The method of columns with a square grid was employed in the present study to discretize the sliding mass. Therefore, the 3D failure mass was simulated by vertical columns.

As the next limitation, soil materials were assumed saturated by Terzaghi's (1936) linear equation of effective stress. Consequently, the effects of negative pore pressure and nonlinear equations of effective stresses for unsaturated soils were not included in the present study. Moreover, a hydrostatic water condition was assumed to include the pore water pressure in the analysis, so the flow of moisture was not considered. The fifth limitation was related to the materials of the slope. Although the present study was able to handle different soil layers including weak layer and bedrock, an isotropic and homogeneous condition was assumed for the behavior of layers of the slope profile.

Finally, the mechanism of the failure was limited to 3D rotational or complex rotational surfaces, so the translational sliding mode was not in the scope of the analysis. Although a general ellipsoid shape was used in the analysis, other rotational slip surfaces were also applicable within the framework of the present study.

1.9 Definition of Terms

To provide a basis for discussion, following definitions are used in this study:

i. Method of slices: This method divides the sliding mass into a number of vertical slices to establish the equation of factor of safety.
ii. Method of columns: This method can be considered as an extension of method of slices into the third dimension that discretizes the failure mass into a number of columns.
iii. Plane of symmetry: A hypothetical vertical plane that divides the sliding mass into two symmetric parts.
iv. Graphical user interface: A graphic interface that is used to connect the computer code and the user.
v. Optimization technique: A procees that uses different search techniques among a series of probable answers to find the best suitable solution for a problem.

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