A NEW STATISTIC TO THE THEORY OF CORRELATION STABILITY TESTING IN FINANCIAL MARKET

SHAMSHURITAWATI SHARIF

UNIVERSITI TEKNOLOGI MALAYSIA

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SHAMSHURITAWATI SHARIF

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To my beloved AYAH, MAK , and CIKGU

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ABSTRACT

Testing the stability of correlation structures is an active research area involving the applications of multivariate analysis in financial market such as stock market analysis, risk management, market equity, general financial and economic studies, and real estates. In the financial market, the number of variable *p* is usually large and might reach thousands. As a consequence, the standard stability test Box's M and Jennrich's statistic are not capable to handle it. This condition makes the computation of the statistical tests quite cumbersome and tedious because the computational efficiency of finding the determinant and inverse of the correlation matrix becomes low. In order to solve these problems, this thesis introduces T^*-statistic for testing the stability of correlation structure in an independent sequence of sample correlation matrices from a *p*-variate normal distribution based on a repeated test approach. For this purpose, the asymptotic distribution of the test under the null hypothesis is derived mathematically using the *vec* operator and commutation matrix. The power of T^*-statistic is computed and compared with existing ones under certain conditions of the alternative hypothesis. It is found that, if *p* is large, then the power of T^*-statistic dominates the power of the *J*-statistic for all shifts. On the other hand, when the shift is small, its power is equal to that the *M*-statistic. The second problem is to diagnose and find an explanation when the null hypothesis is rejected. For that purpose, by considering correlation matrix as representing a complex network, network topology approach is used to demonstrate to what extent that two or more correlation structures are different from each other. To interpret the filtered network topology, four popular centrality measures have been used. Moreover, to enrich the economic interpretation, average of weights is introduced as another measure of centrality.

ABSTRAK

Pengujian kestabilan struktur korelasi menjadi satu bidang penyelidikan yang aktif melibatkan penggunaan analisis multivariat di dalam pasaran kewangan termasuk analisis pasaran bursa, pengurusan risiko, pasaran ekuiti, kewangan am dan pengajian ekonomi serta pasaran hartanah. Di dalam pasaran kewangan, bilangan pembolehubah p biasanya besar dan mungkin boleh mencapai ribuan, Oleh sebab itu, statistik ujian kestabilan sedia ada iaitu Box's M dan Jennrich tidak mampu untuk mengendalikannya. Keadaan tersebut membuatkan pengiraan kedua-dua ujian statistik itu menjadi sukar dan membosankan kerana kecekapan komputasi untuk mengira matrik penentu dan matrik songsang menjadi rendah. Untuk menyelesaikan masalah tersebut, tesis ini memperkenalkan statistik T^* untuk menguji kestabilan satu barisan matrik korelasi yang saling tidak bersandar daripada p-variat bertaburan normal berdasarkan kepada pendekatan ujian berulang. Bagi tujuan tersebut, taburan asimptot di bawah hipotesis nul diperoleh secara matematik dengan menggunakan operator *vec* dan matrix komutasi. Kuasa statistik dihitung dan dibandingkan bersama-sama dengan statistik sedia ada di bawah beberapa keadaan tertentu bagi hipotesis alternatif. Hasil menunjukkan, bagi p besar, kuasa statistik T^* adalah lebih berbanding kuasa statistik J untuk semua perubahan. Selainnya, apabila perubahan adalah kecil, kuasa statistik T^* juga sehebat statistik M. Masalah kedua adalah untuk mengenalpasti dan mendapatkan penerangan lanjut apabila hipotesis nul di tolak. Bagi tujuan tersebut, dengan mengambil kira matrix korelasi sebagai mewakili satu jaringan yang kompleks, kaedah topologi jaringan telah digunakan bagi menghuraikan tentang dua atau lebih struktur korelasi yang berbeza di antara satu sama lain. Untuk menghuraikan topologi jaringan, empat ukuran pemusatan yang terkenal telah digunakan. Selebihnya, untuk memperkayakan pengukuran, purata berpemberat telah diperkenalkan sebagai salah satu ukuran pemusatan.

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LIST OF ABBREVIATIONS

MSPC	-	Multivariate Statistical Process Control
PCA	-	Principle Component Analysis
MST	-	Minimum Spanning Tree
SDU	-	Sub Dominant Ultrametric
FOREX	-	Foreign Exchange Rates
LRT	-	Likelihood Ratio Test
XAU	-	Gold Ounces

LIST OF SYMBOLS

∂	-	partial differentiation operator
$\otimes$	-	kronecker product
I_p	-	identity matrix
$K_{p,p}$	-	commutation matrix
$\left(K_{p,p}\right)_d$	-	diagonal elements of commutation matrix
r	-	sample correlation coefficient
ρ	-	population correlation coefficient
R	-	sample correlation matrix
R_{pooled}	-	pooled sample correlation matrix
R_u	-	upper-off-diagonal elements of sample correlation matrix
Ω	-	population correlation matrix
Ω_u	-	upper-off-diagonal elements of population correlation matrix
S	-	sample covariance matrix
S_d	-	diagonal elements of sample covariance matrix
Σ	-	population covariance matrix
Σ_d	-	diagonal elements of population covariance matrix
σ^2	-	population variance
T	-	transformation matrix
$V^2(R_u)$	-	vectorization of R_u
$V^2(\Omega_u)$	-	vectorization of Ω_u
$vec(R)$	-	vectorization of R
$vec(\Omega)$	-	vectorization of Ω

CHAPTER 1

INTRODUCTION

This chapter begins with research background in the first section. It aims to understand why testing the stability of correlation structure and network analysis is important in financial industry. This section is followed by the details of problem statement in Section 1.2 and research objective in Section 1.3. Later on, in the fourth and fifth sections, the significant contributions and the scope of this research will be highlighted, respectively. In the last section, the organization of thesis will close this chapter.

1.1 Research Background

Since 1970's, a lot of studies involving correlation analysis can be found in many literatures, from simple correlation analysis to more advanced analysis such as correlation structure analysis. The former analysis can be seen, for example, in Makridakis and Wheelwright (1974), Haney and Lloyd (1978), Watson (1980), and Maldonado and Saunders (1981) who use it in testing hypothesis that two correlation coefficients are equal to each other. However, in real case, when dealing with high dimension datasets, understanding their interrelationships among several variables considered at one time is a complicated job. Therefore, the theory of correlation structure analysis becomes very important when dealing with related p variables where p is large.

In recent years, monitoring the stability of correlation structure becomes an imperative subject in economic development and financial industry. It has been extensively implemented by various researchers to understand the behaviour of a sequence of correlation structures based on independent samples in certain time periods. Its implementations can be seen, for example, in property and real estate analysis, equity market, stock market, corporate bonds market, credit risk management, asset business, portfolio analysis and several others financial industries.

Since three decades ago numerous studies on stability of correlation structure has been developed from time to time. For instance, Cho and Taylor (1987), Kaplanis (1988), Erb *et al.* (1994), Longin and Solnik (1995), Tang (1995a), Meric and Meric (1997), Sinclair *et al.* (1997), Lee (1998), Tang (1998), Rey (2000), Chesnay and Jondeau (2001), Chakrabati and Roll (2002), and Da Costa Jr. *et al.* (2005). All of them study the stability of correlation structure among stock returns.

Additionally, the same analysis can also be found in Eichholtz (1995) and Deblauwe and Le (1997) who stress on credit risk and portfolio analysis, respectively. It is followed by Goetzmann *et al.* (2005) who concern about global market correlation structure, Annaert *et al.* (2003) who focus their work on the stability of covariance and correlation structures based on small samples, and Ragea (2003) who shows that the correlation stability is of vital importance in market risk management and asset management. Moreover, Koch & Koch (1991), Gande and Parsley (2005) and Fischer (2007) are other examples that can be added into the list of implementation in testing the stability of correlation structure.

Basically, the theoretical perspective comes earlier than the application. There are so many theoretical aspects that have to be considered for the improvement concurrently with the development of technology and the demand to be working with high dimension database as well. See also Schott (2001) for some tests of the covariance stability when sample are independent and Cook *et al.,* (2002) for a study on the stability of correlation matrices based on time series data. Later on, in 2007, Schott present a discussion of the correlation structure stability when samples are dependent (Schott, 2007a). Another discussion by Schott can be found in Schott

(2007b) for a test of the equality of covariance matrices when the dimension is large relative to the sample size. It can also be discovered in Chilson *et al*. (2006) for the discussion on parallel computation of high dimensional robust correlation matrices. In this thesis, we consider the case where the sample correlation matrices are independent to each other.

Correlation structure can be considered as covariance structure when all variables have variance 1. Therefore, testing the stability of correlation structure is a special case of testing the stability of covariance structure. Both tests are very well established but in different field of study. The former is accepted in financial industries as we discuss, and the later is in manufacturing industries. However, the later came first into practice and it has been familiar to the practitioners. The classical statistical book, Anderson (1958), is among those which presents an early development of testing method for the stability of covariance structure. Incidentally, if the stability of correlation structure is rejected, it does not guarantee that the stability of covariance structure would also be rejected (Gibbons, 1981; Tang, 1995b; Lee, 1998; Djauhari and Herdiani, 2008).

To test the stability of covariance matrix, the most popular and widely used statistical tests are Box's M statistic and Jennrich's statistic. We call them as *M*-statistic and *J*-statistic. See, for example, Cho and Taylor (1987), Meric and Meric (1997), Sinclair *et al.* (1997), Tang (1998), Rey (2000), Meric *et al*. (2001), Da Costa Jr *et al*. (2005), Goetzmann *et al*. (2005), Annaert *et al*. (2006) and Lee (2006). Those tests apply to a sequence of independent samples of correlation matrices drawn from a *p*-variate normal distribution. They are constructed based on likelihood ratio test where their distributions under the null hypothesis were derived for asymptotic case. Furthermore, by construction, those tests are to test the correlation stability simultaneously.

Theoretically, both statistics can be used for arbitrary value of *p*. However, in financial market *p* is usually large and might be of order of thousands. This condition makes the computation of those statistical tests quite cumbersome and tedious because the computational efficiency of the determinant and inverse of correlation matrix becomes low (Djauhari, 2007; Djauhari and Herdiani, 2008).

Concerning the hypothesis formulation, the existing statistical tests are designed to test the equality of several correlation matrices simultaneously, i.e $H_0 : \boldsymbol{\Omega}_1 = \boldsymbol{\Omega}_2 = \ldots = \boldsymbol{\Omega}_m$ $(= \boldsymbol{\Omega}_0$, say) based on independent samples. However, based on the Multivariate Statistical Process Control (MSPC) approach, this test can be done as repeated tests, i.e $H_0 : \boldsymbol{\Omega}_k = \boldsymbol{\Omega}_0$. Thus, in this case the control charting procedure can be used as proposed by Alt and Smith (1988), Wierda (1994), Woodall and Montgomery (2005) and Djauhari (2005), but needs to construct a new test. This is one of the crucial problems to be explored in-depth and handled.

Next problem is to diagnose and find the root causes when the null hypothesis is rejected. One of the most widely used methods to study the change of correlation structure is principle component analysis (PCA) (Johnson and Wichern, 2002). See also Oyama (1997) and Meric *et al.* (2001), for the application of this methodology to study the changes in the co-movement patterns. In this thesis, by considering correlation matrix as representing a network, network topology approach will be employed. It can be used to explain to what extend that two or more correlation structures are different from each other (Mantegna and Stanley, 2000). This approach is the standard practise in network analysis and generally in complex system analysis where the information in the network is filtered by using minimum spanning tree (MST) which provides the filtered network topology, and sub-dominant ultrametric (SDU) which gives the hierarchical clustering of financial objects.

Nowadays, network analysis approach has been accepted to study financial objects. This approach has been investigated seriously by econophysicists who solve the economic problems based on the theory of physics. It starts with Mantegna (1999) who introduces MST and hierarchical tree as an approach to filter the economic information contained in a correlation matrix. Since then, there are numerous related studies have been developed including Djauhari (2012) who introduces the forest of all minimum spanning trees as a robust filter. Other examples include, Bonanno *et al.* (2002). Miccichè *et al.* (2003), Tumminello *et al.* (2007), Tabak *et al.* (2010), and Zhang *et. al* (2011) who investigate the network of stocks based on MST. Other application of MST can also be found in Naylor *et al.*

(2006), Mizuno *et al*., (2006), Gorski *et al.* (2008), Kwapień *et al*. (2009), and Jang *et al*. (2011) who study foreign exchange rate. Meanwhile, Onnela *et al*. (2003) has been working in portfolio analysis, Sieczka and Holyst (2009) in commodity market, and Di Matteo *et al*. (2010) in money and capital market interest rate.

Finally, before closing the thesis with some conclusions and outlooks, some illustrative examples in financial market will be presented to demonstrate the mechanism of the proposed statistical test.

1.2 Problem Statement

In practice, in financial market, p is large. In this area, p represents the number of stocks market which is not always less than the number of historical returns per stock. There can be over a thousand stocks, p to choose from, but rarely more than 10 years of monthly data, n data (i.e p =1000, n =120).

However, both M-statistic and J-statistic have the following properties:

i. They are to test the equality of some correlation matrices simultaneously based on independent samples.
ii. They involve the determinant or inverse of a matrix such as correlation matrix.

In financial market, the properties are not quite advantageous because:

i. Simultaneous test cannot provide the information about which correlation matrix (ces) are different from the other.
ii. The number of variables is usually large of order of thousands. Therefore the use of determinant or inverse of a correlation matrix is quite cumbersome.

This thesis is to handle these obstacles. Since the samples are independent, instead of using simultaneous test, according to MSPC approach we use repeated tests. Therefore the problem in this thesis consists of:

i. The construction of a new test.
ii. Mathematical derivation of the distribution of that test under the null hypothesis.
iii. The investigation of the power of test and compare with that of *M*-statistic and *J*-statistic.
iv. Finding the root causes if at certain repetition the null hypothesis is rejected.

1.3 Research Objective

According to the research problems mentioned in Section 1.2, the following objectives need to be accomplished:

i. Construct a new statistical test.
ii. Derive the asymptotic distribution of $V^2(\boldsymbol{R}_U)$.
iii. Perform a power of test by using simulation study in order to compare the sensitivity of the proposed statistic with the existing ones.
iv. Propose a procedure based on network topology approach for identifying the root causes of correlation structure instability.
v. Propose a new centrality measure to aid the interpretation of network topology.
vi. Perform the illustrative example using financial market case study.

1.4 Research Significance

This thesis is theoretical in nature without neglecting its application. The significance results of this research can be summarized as follows:

i. A statistical test has been introduced that can be used for large p.
ii. A mathematical derivation of the asymptotic distribution under null hypothesis.
iii. The power of test based on a simulation experiment for certain condition of alternative hypothesis.
iv. In case the null hypothesis is rejected, since p is large, network analysis using MST is very helpful to understand the shift of correlation structure.
v. Knowledge development presented in this thesis, in the theory of correlation analysis, will hopefully contribute to the area of financial market analysis.

1.5 Scope of Research

The scope of this research was to construct the new statistical test for testing the stability of correlation structure when several independent samples are taken from multivariate normal distribution. In addition, the research study is delimited to:

i. a mathematical derivation based on its asymptotic distribution,
ii. a simulation study to show the performance of proposed test and the existing tests, and
iii. a practical aspect in financial market.

1.6 Organization of Thesis

The research progresses as follows. The following Chapter 2 is devoted to a review of correlation coefficient, stability test and its application in financial market, and the application on network topology. Chapter 3 describes the proposed statistic and its asymptotic distribution, and the computation of the covariance matrix of sample correlation matrix. Later, Chapter 4 discusses the power of the proposed test and existing tests. Next in Chapter 5, discussion begins with some preliminary, the details on network approach and centrality measure. The proposed measure of centrality also has been included in this chapter. In Chapter 6, the examples on 85 stocks market traded at Bursa Malaysia and 74 foreign exchange rates (FOREX) are delivered for illustration. At the end of this chapter, a conclusion and outlooks for future research has been concluded and the direction for further research will be delivered.

REFERENCES

Abbasi, A., and Altmann, J. (2010). On the correlation between research performance and social network analysis measures applied to research collaboration networks TEMEP (Discussion Paper No. 2010). Seoul National University, Korea.

Alt, F. B. and Smith, N. D. (1988). Multivariate Process Control. *Handbook of Statistics.* 7, 333-351.

Anderson, T. W. (1958). *Introduction to Multivariate Statistical Analysis.* New York: John Wiley & Sons, Inc.

Anderson, T. W. (2003). *Introduction to Multivariate Statistical Analysis.* New York: John Wiley & Sons, Inc.

Annaert, J., Claes, A. G. P. and De Ceuster, M. J. K. (2006). Intertemporal Stability of the European Credit Spread Co-movement Structure. *The European Journal of Finance.* 12(1), 23-32.

Annaert, J., Claes, A.G.P., and De Ceuster, M.J.K. (2003). Intertemporal stability of the European credit spread co-movement structure. Working Papers, Faculty of Applied Economics, University of Antwerp.

Apley, D. W. and Shi, J. (2001). A Factor-Analysis Method for Diagnosing Variability in Multivariate Manufacturing Processes. *Technometrics.* 43, 84–95.

Batagelj, V. and Mrvar, A. (2003). A Density Based Approaches to Network Analysis: Analysis of Reuters Terror News Network. *9th Annual ACM SIGKDD.* August 24-27, 2003. Washington, D.C, Retrieved from http://www.cs.cmu.edu/~dunja/LinkKDD2003/papers/Batagelj.pdf.

Batagelj, V. and Mrvar, A. (2011). PAJEK: Program for Analysis and Visualization of Large Networks, version 2.02. Retrieved from http://pajek.imfm.si/doku.php?id=download.

Benzécri, J. P. (1982). *Histoire et préhistoire de l'Analyse des Données.* Paris: DUNOD.

Bonacich, P. (1972). Factoring and Weighting Approaches to Status Scores and Clique Identification. *Journal of Mathematical Sociology.* 2, 113-120.

Bonanno, G., Caldarelli, G., Lillo, F. and Mantegna, R. N. (2003). Topology of Correlation-Based Minimal Spanning Trees in Real and Model Markets. *Physical Review E.* 68, 046-130.

Bondy, J. A. and Murty, U. R. S. (1976). *Graph Theory with Applications*. USA: American Elsevier Publishing Co. Inc.

Borgatti, S. P. (1995). Centrality and AIDS. *Connections.* 18(1), 112-114.

Borgatti, S. P., Carley, K. and Krackhardt, D. (2006). Robustness of Centrality Measures under Conditions of Imperfect Data. *Social Networks.* 28, 124–136.

Bowley, G. (2012, January 3). When investors rush in and out together. *New Straits Times*, pp. B16.

Box, G. E. P. (1949). A General Distribution Theory for a Class of Likelihood Criteria. *Biometrika.* 36(3/4), 317-346.

Browne, M. W. and Shapiro, A. (1986). The Asymptotic Covariance Matrix of Sample Correlation Coefficients under General Conditions. *Linear Algebra and Its Applications*. 82, 169-176.

Caldarelli, G., Battiston, S. and Garlaschelli, D. (2004). Emergence of Complexity in Financial Networks. *Lect Notes Phys.* 650, 399-423.

Chakrabarti, R. and Roll, R. (2002). East Asia and Europe during the 1997 Asian Collapse: A Clinical Study of a Financial Crisis. *Journal of Financial Markets,* 5(1), 1-30.

Chesnay, F. and Jondeau, E. (2001). Does Correlation Between Stock Returns Really Increase During Turbulent Periods?. *Economic Notes*, 30(1), 53—80.

Chilson, J., Ng, R., Wagner, A. and Zamar, R. (2006). Parallel Computation of High Dimensional Robust Correlation and Covariance Matrices. *Algorithmica,* 45(3), 403-431.

Cho, D. C. and Taylor, W. M. (1987). The Seasonal Stability of the Factor Structure of Stock Returns. *Journal of Finance*. 42, 1195-211.

Cook, W., Mounfield, C., Ormerod, P. and Smith, L. (2002). Eigen Analysis of the Stability and Degree of Information Content in Correlation Matrices

Constructed From Property Time Series Data. *The European Physical Journal, B*. 27, 189–195.

Coronnello, C., Tumminello, M., Lillo, F., Miccichè, S. and Mantegna, R. N. (2005). Sector Identification in a Set of Stock Return Time Series Traded at the London Stock Exchange. *Acta Physica Polonica B.* 36 (9), 2653-2679.

Costenbader, E. and Valente T. W. (2003). The Stability of Centrality Measures When Networks are Sampled. *Social Networks.* 25, 283-307.

Da Costa, Jr, N., Nunes, S., Ceretta, P. and Da Silva, S. (2005). Stock-Market Co-Movements Revisited. *Economics Bulletin.* 7(3), 1-9.

De Nooy, M., Mvrar, A. and Batagelj, V. (2004). *Exploratory Social Network Analysis with Pajek.* Cambridge: Cambridge University Press.

Deblauwe, F and Le, H. (1997). Stability of correlation between credit and market risk over different holding periods. Retrieved from http://users.telenet.be/rik.deblauwe/2%20artikelen_hier_gepubliceerd/Stability%20of%20correlation%20between%20credit%20and%20market%20risk%20over%20different%20holding%20periods%20%28FRD-LH%29.pdf

Di Matteo, T., Pozzi, F., Aste, T. (2010). The Use Of Dynamical Networks To Detect The Hierarchical Organization Of Financial Market Sectors. *The European Physical Journal B*. 73, 3–11.

Diestel, R. (2010). *Graph Theory*. (4th ed.) Heidelberg: Springer-Verlag.

Djauhari, M. A. (2005). Improved Monitoring of Multivariate Process Variability. *Journal of Quality Technology.* 37(1), 32-39.

Djauhari, M. A. (2007). A Measure of Multivariate Data Concentration. *Journal of Applied Probability.* 2(2), 139 – 155.

Djauhari, M. A. (2012). A Robust Filter in Stock Networks Analysis. *Physica A.* 391(20), 5049-5057.

Djauhari, M. A. and Herdiani, E. T. (2008). Monitoring the Stability of Correlation Structure in Drive Rib Production Process: An MSPC Approach. *The Open Industrial and Manufacturing Engineering Journal*. 1, 8-18.

Eichholtz, P. M. A. (1995). Regional Economic Stability and Mortgage Default Risk in the Netherlands. R*eal Estate Economics*, 23(4), 421 -439.

El Maache, H. (1997). Measures *D' Association Vectorielle Basées Sur Des Matrices De Corrélation.* Doctor Philosophy, Université de Montréal.

El Maache, H. and Lepage, Y. (1998). Mesures d'Association Vectorielle Basées sur une Matrice de Corrélation. *Revue de Statistique Appliquée*, 46(4), 27-43.

Eom, C., Oh, G. and Kim, S. (2006). Topological Properties of the Minimal. Spanning Tree in Korean and American Stock Markets. *Physica A*. 388(6), 900-906.

Eom, C., Oh, G. and Kim, S. (2008). Statistical Investigation on Connected Structures of Stock Networks in a Financial Time Series. *Journal of the Korean Physical Society.* 53(6), 3837-3841.

Erb, C. B., Harvey, C. R. and Viskanta, T. E. (1994). Forecasting international equity correlations. *Financial Analysts Journal. 5*0, 32–45.

Fenn, D. J., Porter, M. A., Mucha, P. J., McDonald, M., Williams, S., Johnson, N. F. and Jones, N. S. (2010). Dynamical Clustering of Exchange Rates. *Quantitative Finance*. 12(10), 1493-1520.

Fischer, M. (2007). Are correlations constant over time? Application of the CC-TRIG-Test to return series from different asset classes SFB 649 (Discussion Paper 2007-012). Retrieved from University of Erlangen-Nurnberg, Germany.

Freeman, L. C. (1977). A Set of Measures of Centrality Based on Betweenness. *Sociometry.* 40, 35-41.

Gande, A. and Parsley, D. (2005). News Spillovers in the Sovereign Debt Market. *Journal of Financial Economics 75*, 691–734.

Gibbons, M. R. (1981). Empirical Examination of the Return generating Process of the Arbitrage Pricing Theory. Working Paper, Stanford University, Stanford, CA.

Global Equity Derivatives and. Delta One Strategy. (2011, May 16). Rise of cross-asset correlations. Retrieved from http://www.cboe.com/Institutional/JPMCrossAssetCorrelations.pdf.

Goetzmann, W. N., Li, L. and Rouwenhorst, K. G. (2005). Long-Term Global Market Correlations. *Journal of Business.* 78(1), 1-38.

Górski, A. Z., Drożdż, S., Kwapień, J. and Oświęcimka, P. (2008). Minimal Spanning Tree Graphs and Power Like Scaling in FOREX Networks. *Acta Physica Polonica A.* 114(3), 531-538.

Graham, R. L. and Hell, P. (1985). On the History of the Minimum Spanning Tree *Problem. Annals of the History of Computing*. 7(1), 43-57.

Grudzinski, C. (2012, February 12). The rise of ETFs and stock market Correlation. *The Street*. Retrieved from http://www.thestreet.com/story/11644474/1/the-rise-of-etfs-and-stock-market-correlation.html.

Guerrero-Cusumano, J. L. (1995). Testing Variability in Multivariate Quality Control: A Conditional Entropy Measure Approach. *Information. Science.* 86, 179–202.

Haney, R. L. Jr. and Lloyd, W. P. (1978). An Examination of the Stability of the Intertemporal Relationships Among National Stock Market Indices. *Nebraska Journal of Economics and Business.* 17, 55-65.

Huang, F., Gao, P., and Wang, Y. (2009). Comparison of Prim and Kruskal on Shanghai and Shenzhen 300 index hierarchical structure tree. *International Conference on Web Information Systems and Mining*, November 7-8, 2009. Shanghai, China. 237-241.

Jang, W., Lee, J. and Chang, W. (2011). Currency Crises and the Evolution of Foreign Exchange Market: Evidence from Minimum Spanning Tree. *Physica A.* 390, 707-718.

Jayawant, P. and Glavin, K. (2009). Minimum Spanning Trees. *Involve a Journal of Mathematics.* 2(4), 439–450.

Jennrich, R. I. (1970). An Asymptotic Test for the Equality of Two Correlation Matrices. *Journal of the American Statistical Association*. 65(330), 904-912.

Johnson, R. A. and Wichern, D. A. (2002). *Applied Multivariate Statistical Analysis.* (5th ed.) New Jersey: Prentice Hall.

Kamada, T. and Kawai, S. (1989). An Algorithm for Drawing General Undirected Graphs. *Information Processing Letters (Elsevier).* 31(1), 7–15.

Kaplanis, E. C. (1988). Stability and Forecasting of the Comovement Measures of International Stock Market Returns. *Journal of International Money and Finance*. 7, 63-76.

Khoo, M. B. C. and Quah, S. H. (2003). Multivariate Control Chart for Process Dispersion Based on Individual Observations. *Quality Engineering.* 15(4), 639-642.

Khoo, M. B. C. and Quah, S. H. (2004). Alternatives to the Multivariate Control Chart for Process Dispersion. *Quality Engineering.* 16(3), 423-435.

Krichel, T. and Bakkalbasi, N. (2006). A Social Network Analysis of Research Collaboration in the Economics Community. *The International Workshop on Webometrics, Informetrics and Scientometrics and Seventh COLLNET Meeting*. May 12, 2006. France.

Kruskal, J. B. (1956). On the Shortest Spanning Subtree and the Travelling Salesman Problem. *Proceedings of the American Mathematical Society*. 7(1), 48-50.

Kullback, S. (1967). On Testing Correlation Matrices. *Applied Statistics.* 16(1), 80-85.

Kwapień, J., Gworek, S., Drożdż, S. and Górski, A. (2009). Analysis of a Network Structure of the Foreign Currency Exchange Market. *Journal of Economic Interaction and Coordination.* 4(1), 55-72.

Kwon, O. and Yang, J. S. (2008). Information Flow Between Stock Indices. *Europhysics Letters.* 82, 68003-68007.

Larntz, K. and Perlman, M.D. (1985). A simple test for the equality of correlation matrices. Technical Report (No.63). May, 1985. University of Washington, Washington.

Lee, S. (2006). The stability of the co-movements between real estate returns in the UK. *Journal of Property Investment and Finance*, 24 (5), 434-442.

Lee, S. L. (1998). The inter-temporal stability of real estate returns: an empirical investigation. Working Papers in Land Management and Development, April, 1998. International Real Estate Conference. Maastricht, the Netherlands.

Longin, F. and Solnik, B. (1995). Is the correlation in international equity returns constant: 1960–1990?. *Journal of International Money and Finance.* 14, 3–26.

Magnus, J. R. and Neudecker, H. (1979). The Commutation Matrix: Some properties and its applications, *Annals of Statistics* 7 (2), 381-394.

Makridakis, S. G. and Wheelwright, S. C. (1974). An Analysis of the Interrelationships Among the Major World Stock Exchanges. *Journal of Business Finance and Accounting.* 1, 195-215.

Maldonado, R. and Saunders, A. (1981). International Portfolio Diversification And The Intertemporal Stability Of International Stock Market Relationships, 1957-78. *Financial Management.* 10, 54-63.

Mantegna, R. N. (1999). Hierarchical Structure in Financial Markets. *European Physical Journal B.* 11, 193-197.

Mantegna, R. N. and Stanley, H. E. (2000). *An Introduction to Econophysics: Correlations and Complexity in Finance.* Cambridge UK: Cambridge University Press.

Mardia, K. V., Kent, J. T. and Bibby, J. M. (1979). *Multivariate Analysis.* London: Academic Press.

Mason, R. L., Chou, Y. M. and Young, C. J. (2010). Decomposition of Scatter Ratios Used in Monitoring Multivariate Process Variability. *Communication in Statistics – Theory and Methods.* 39, 2128-2145.

Mc Bride, E. (2011, September 3). Understanding the recent rise in correlations and how you can turn it to your advantage. Retrieved from http://www.forbes.com/sites/riabiz/2011/03/09/understanding-the-recent-rise-in-correlations-and-how-you-can-turn-it-to-your-advantage

McDonald, M., Suleman, O., Williams, S., Howison, S., Johnson, N. F. (2005). Detecting a currency's dominance or dependence using foreign exchange network trees. *Physical Review E*, 72, 046106.

Meric, G., Leal, R. P. C., Ratner, M. and Meric, I. (2001). Co-Movements of U.S. and Latin American Equity Markets Before and After the 1987 Crash. *International Review of Financial Analysis.* 10, 219–235.

Meric, I. and Meric, G. (1997). Co-Movements of European Equity Markets Before and After the 1987 Crash. *Multinational Finance Journal.* 1(2), 137–152.

Miccichè, S., Bonanno, G., Lillo, F. and Mantegna, R. N. (2003). Degree Stability of a Minimum Spanning Tree of Price Return and Volatility. *Physica A.* 324, 66–73.

Mizuno, T., Takayasu, H. and Takayasu, M. (2006). Correlation Networks Among Currencies. *Physica A.* 364, 336-342.

Monárrez-Espino, J. and Caballero-Hoyos, J. R. (2010). Stability of Centrality Measures in Social Network Analyses to Identify Long-Lasting Leaders from an Indigenous Boarding School of Northern Mexico. *Estudios sobre las Culturas Contemporaneas.* 16(32), 155-171.

Montgomery, D. C. (2001). *Introduction to Statistical Quality Control.* (4th ed.) New York: John Wiley & Sons, Inc.

Montgomery, D. C. (2005). *Introduction to Statistical Quality Control.* (5th ed.) New York: John Wiley & Sons, Inc.

Muirhead, R. J. (1982). *Aspect of Multivariate Statistical Theory*. New York: John Wiley & Sons, Inc.

Naylor, M. J., Rose, L. C. and Moyle, B. J. (2007). Topology of Foreign Exchange Markets using Hierarchical Structure Methods. *Physica A*. 382, 199-208.

Neudecker, H. and Wesselman, A. M. (1990). The Asymptotic Covariance of Sample Correlation Matrix. *Linear Algebra and Its Applications*. 127, 589–99.

Newman, M. E. J. (2010). *Networks: An Introduction.* Oxford: Oxford University Press.

Noguchi, K., and Gel, Y.R. (2009). Combination of Levene-type tests and a finite-intersection method for testing equality of variances against ordered alternatives. *Journal of Nonparametric Statistic*, 22, 897–913.

Okamoto, K., Chen, W. and Li, X. Y. (2008). *Ranking of Closeness Centrality for Large Scale Social Networks.* In F. P. Preparata, X. Wu, and J. Yin. (Eds), Frontiers in algorithmics. (pp 186–195). Berlin Heidelberg: Springer-Verlag.

Onnela, J. -P., Chakraborti, A., Kaski, K., Kert'esz, J. and Kanto, A. (2003). Dynamics of Market Correlations: Taxonomy and Portfolio Analysis. *Physical Review E*. 68, 056110(1)-056110 (12).

Oyama, T. (1997). Determinants of stock prices. The Case of Zimbabwe, IMF Working Paper, (Paper No. 97/115). 1-44. Retrieved from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=882653.

Park, K. and Yilmaz, A. (2010). A Social Network Analysis Approach to Analyze Road Networks. *Paper presented at the ASPRS Annual Conference.* Date. San Diego, CA. Retrieved from http://dpl.ceegs.ohio-state.edu/papers/park_asprs_2010.pdf

Ragea, V. (2003). Testing Correlation Stability During Hectic Financial Markets. *Financial Market and Portfolio Management*. 17(3), 289-308.

Rey, D.M. (2000). Time-varying stock market correlations and correlation breakdown. Swiss Institute of Banking and Finance: University of St. Gallen. 387-412. Retrieved from http://www.fmpm.ch/files/2000_04_Rey.pdf.

Sauder School of Business. (2011). PACIFIC Exchange Rate Service. Retrieved from http://fx.sauder.ubc.ca.

Schäfer, J. and Strimmer, K. (2005). A Shrinkage Approach to Large-Scale Covariance Matrix Estimation and Implications for Functional Genomics. *Statistical Applications in Genetics and Molecular Biology*, 4 (1), 1-30.

Schott, J. R. (1997). *Matrix Analysis for Statistics*. New York: John Wiley & Sons.

Schott, J. R. (2001). Some Tests for the Equality of Covariance Matrices. *Journal of Statistical Planning and Inference.* 94, 25-36.

Schott, J. R. (2007a). A Test for the Equality of Covariance Matrices When the Dimension is Large Relative to the Sample Sizes. *Computational Statistics and Data Analysis.* 51, 6535 – 6542.

Schott, J. R. (2007b). Testing the Equality of Correlation Matrices When Sample Correlation Matrices Are Dependent. *Journal of Statistical Planning and Inference.* 137, 1992 – 1997.

Sharif, S. and Djauhari, M. A (2011a). Analysis of an Out-of-Control Signals in Process Variability Monitoring using Network Topology Approach. *2011 IEEE International Conference on Quality and Reliability (ICQR). September, 14-17 2011*. Thailand, 590-594.

Sharif, S. and Djauhari, M. A (2011b). An Application of Network Topology to Understand the Signals in Process Variability: A Case Study in Petrochemical Industry. *IEEE International Conference on Industrial Engineering Management (IEEM). December 6-9, 2011.* Singapore, 41-45.

Sharif, S. and Djauhari, M. A. (2012). A Proposed Centrality Measure: The Case of Stocks Traded at Bursa Malaysia. *Modern Applied Science*. 6(10), 62-29.

Sieczka, P. and Holyst, J. A. (2009). Correlations in Commodity Markets. *Physica A.* 388, 1621-1630.

Sinclair, C. D., Power, D. M., Lonie, A.A. and Avgoustinos, P.A. (1997). A Note on the Stability of Relationships Between Returns from Emerging Stock Markets. *Applied Financial Economics* 7(3), 273–280.

Situngkir, H. and Surya, Y. (2004). Hierarchical Taxonomy in Multi-Party System Working Paper WPM 2004. Bandung Fe Institute. 139–155.

Situngkir, H. and Surya, Y. (2004). On Stock Market Dynamics through Ultrametricity of Minimum Spanning Tree. Working Paper WPH2005, Bandung Fe Institute.

Sullivan, J. H., Stoumbos, Z. G., Mason, R. L. and Young, J. C. (2007). Step-Down Analysis for Changes in the Covariance Matrix and Other Parameters. *Journal of Quality Technology.* 39(1), 66–84.

Tabak, B. M., Serra, T. R. and Cajueiro, D. O. (2010). Topological Properties of Stock Market Networks: The Case of Brazil. *Physica A.* 389, 3240-3249.

Tang, G. Y. N. (1995a). Covariance and Correlation Stationarity: Experiences from Seven Asian Emerging Markets. *Financial Engineering and the Japanese Markets.* 2, 219-231.

Tang, G. Y. N. (1995b). Intertemporal Stability in International Stock Markets Relationships: A Revisit. *The Quarterly Review of Economics and Finance*. 35, 579-593.

Tang, G. Y. N. (1998). The Intertemporal Stability of the Covariance and Correlation Matrices of Hong Kong Stock Return. *Applied Financial Economics*. 8, 359-365.

Tang, P. F. and Barnett, N. S. (1996a). Dispersion Control for Multivariate Processes. *Australian Journal of Statistic*s. 38(3), 235–251.

Tang, P. F. and Barnett, N. S. (1996b). Dispersion Control for Multivariate Processes-Some Comparisons. *Australian Journal of Statistics*. 38(3), 253–273.

Tumminello, M., Aste, T., Matteo, T. D. and Mantegna, R. N. (2005). A Tool for Filtering Information in Complex Systems. *Proceedings of the National Academy of Science USA,* 102, July 26, 2005.USA, 10421-10426.

V´ertes, P., Nicol, R., Chapman, S., Watkins, N. and Bullmore, E. (2011). Topological isomorphisms of human brain and financial market networks. *Frontiers in Systems Neuroscience*, 5, 75.

Watson, J. (1980). The Stationarity of Inter-Country Correlation Coefficients: A Note. *Journal of Business Finance and Accounting.* 1, 297-303.

Werner, M. (2003). *Identification of Multivariate Outliers in Large Data Sets.* Doctor Philosophy, University of Colorado at Denver.

Wierda, S. J. (1994). Multivariate Statistical Process Control – Recent Results and Directions for Future Research. *Statistica Neerlandica.* 48(2), 147-168.

Woodall, W. H. and Montgomery, D. C. (1999). Research Issues and Ideas in Statistical Process Control. *Journal of Quality Technology*. 31(4), 376-386.

Xu, Y., Ma, J. Sun, Y., Hao, J., Sun, Y. and Zhao, Y. (2009). Using Social Network Analysis as a Strategy For E-Commerce Recommendation. *Pacific Asia Conference on Information Systems (PACIS).* July 10-12, 2009. India.

Yeh, A. B., Lin, D. K. and McGrath, R. N. (2006). Multivariate Control Charts for Monitoring Covariance Matrix: A Review. *Quality Technology and Quantitative Management*. 3, 415–436.

Zhang, Y., Lee, G. H. T., Wong, J. C., Kok, J. L., and Prusty, M. (2011). Will the US economy recover in 2010? A minimal spanning tree study. *Physica A*. 390(11), 2020-2050.