

IMPROVED EQUIVALENT CIRCUIT MODEL FOR HIGH CAPACITY LITHIUM
FERRO PHOSPHATE BATTERY

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*Specially dedicated to my beloved father, mother and friends for their encouragement
and support*

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ABSTRACT

Electric Vehicle (EV) gets the attention and interest of scientists due to its advantages of zero green house gaseous emissions and higher efficiency. Battery pack is utilised as energy storage element in EV. Strict handling on battery pack is important to ensure battery pack performs in safe and consistent manner under various load demand and driving state. Therefore, an efficient Battery Management System (BMS) which can perform State Of Charge (SOC) estimation, cell equalisation and temperature control, should be put as the primary concern. In this aspect, an accurate battery model is required to give high quality SOC estimation and battery management. Equivalent circuit model is widely used as the battery model since it can be easily connected to external circuit in a simulation platform. However, the existing battery models are generally built for low capacity battery and do not take into account on nonlinear capacity effect. In this thesis, equivalent circuit model for 18 Ah Lithium Ferro Phosphates (LiFePO_4) battery is developed. LiFePO_4 battery is a good energy storage element for EV since it has good thermal and chemical stabilities. The thesis studies the existing battery modelling technique and investigates the dynamic characteristics of 18 Ah LiFePO_4 battery. A new battery modelling approach with consideration of nonlinear capacity effect has also been proposed for high capacity LiFePO_4 battery. Moreover, a simplified methodology for battery modelling is proposed to improve existing battery model. Parameter extraction is discussed and the proposed battery model is validated from the experiment data. The comparison between experiment and simulation results shows that the proposed model is capable of predicting dynamic behaviours of the battery with minimum error.

ABSTRAK

Kenderaan elektrik (EV) menarik perhatian dan minat daripada saintis kerana ia tidak mempunyai masalah pelepasan gas rumah hijau dan ia mempunyai kecekapan yang tinggi. Pek bateri diguna sebagai elemen simpanan tenaga dalam EV. Pengendalian bateri pek adalah sangat penting untuk memastikan bateri pek berfungsi secara selamat dan konsisten dalam pelbagai permintaan beban dan keadaan memandu. Maka, sistem pengurusan bateri (BMS) yang cekap dalam penganggaran status caj (SOC), pengimbangan sel dan pengawalan suhu perlu dijadikan sebagai fokus utama. Dalam aspek ini, model bateri yang tepat amat diperlukan dalam penganggaran SOC dan pengurusan bateri. Model litar setara banyak digunakan sebagai model bateri kerana ia mudah disambungkan kepada litar luar dalam platform simulasi. Walau bagaimanapun, model bateri yang wujud biasanya dibina untuk bateri yang berkapasiti rendah atau tidak mengambil kira kesan kapasiti tak linear. Dalam tesis ini, model litar setara yang sesuai bagi 18 Ah bateri Litium Ferro Fosfat (LiFePO_4) dibangunkan. Bateri LiFePO_4 merupakan alat penyimpanan tenaga yang sesuai untuk EV kerana ia mempunyai ciri-ciri yang stabil dari segi kimia dan terma. Tesis ini mengkaji teknik pemodelan bateri yang sedia ada serta mengkaji ciri-ciri dinamik pada 18 Ah LiFePO_4 bateri. Pemodelan model bateri yang baru untuk LiFePO_4 berkapasiti tinggi juga dicadangkan. Model baru ini mengambil kira kesan kapasiti tak linear. Selain itu, metodologi pemodelan bateri juga dipermudahkan dan dicadangkan dalam tesis. Langkah-langkah pengenalan parameter telah dibincangkan dan ketepatan model baru yang dicadangkan ini juga disahkan melalui data uji kaji. Perbandingan antara keputusan uji kaji dan simulasi menunjukkan bahawa model yang dicadangkan dapat mewakili ciri-ciri dinamik bateri dengan ralat yang minimum.

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LIST OF SYMBOLS

$a' \text{ to } e'$	-	Parameters of curve fitting
$A \text{ to } F$	-	Parameters for unloaded conditions in simplified model
α	-	Battery capacity
α^C	-	Consumed capacity
α^F	-	Full charged capacity
α^U	-	Usable capacity
β	-	Peukert's constant in Peukert's law
BMS	-	Battery management system
C-rate	-	Rate of current
CCF	-	Correction factor for capacity fading
CDT	-	Constant discharge test
CO_2	-	Carbon dioxides
C_{CAP}	-	Capacitance in VRC model which used to represent battery capacity
C_{DL}	-	Double layer capacitor in energetical model
C_{dl}	-	Double layer capacitance
C_S	-	Storage capacitor in energetical model
C_r	-	Capacitance in RC network in energetical model
C_1	-	Capacitance in first RC parallel network
C_2	-	Capacitance in second RC parallel network
C_3	-	Capacitance in third RC parallel network
C'_1	-	Capacitance in first RC parallel network for unloaded conditions
C'_2	-	Capacitance in second RC parallel network for unloaded conditions
C'_3	-	Capacitance in third RC parallel network for unloaded condition

c	-	Fraction of total capacity for available capacity in KiBaM
DAQ	-	Data Acquisition
EIS	-	Electrochemical Impedance Spectroscopy
EKF	-	Extended Kalman Filer
EMS	-	Energy management system
EV	-	Electric vehicle
ε	-	Faded capacity
f	-	Discharge phase
f_c	-	Capacity factor which related to capacity fading
f_T	-	Temperature factor
f_{max}	-	Frequency of the peak of semicircle
G	-	Voltage raise in the unloaded condition
GHGs	-	Green house gaseous
h_1	-	Height of available charge well
h_2	-	Height of bound charge well
I-V	-	Current-Voltage
ICE	-	Internal combustion engine
IEA	-	International Energy Agency
I_L	-	Battery current
KF	-	Kalman Filter
KiBaM	-	Kinetic battery model
K	-	Polarization constant in Sheperd equation
k	-	Fixed conductance
k'	-	Conductance
κ	-	Coefficient for cycling effect
L	-	Total operating time
Li-ion	-	Lithium-ion
Li-P	-	Lithium Polymer
LiCoO ₂	-	Lithium Cobalt Oxide
LiFePO ₄	-	Lithium Ferro Phosphate
M_I	-	Gain correction for R_I

M_2	-	Gain correction for R_2
M_3	-	Gain correction for R_3
m	-	Computational factor
N	-	Initial capacity or nominal capacity
NiCd	-	Nickel-Cadmium
NiFe	-	Nickel-Iron
NiMH	-	Nickel-Metal Hydride
NiZn	-	Nickel-Zinc
N_1	-	Time constant gain for first time constant in time constant correction
N_2	-	Time constant gain for second time constant in time constant correction
N_3	-	Time constant gain for third time constant in time constant correction
n	-	Number of cycle
n_L	-	Estimated number of usable cycle
OCV	-	Open Circuit Voltage
OEC D	-	The Organisation for Economic Co-operation and Development
PDT	-	Pulse discharge test
PHEV	-	Plug-in hybrid vehicle
PNGV	-	Partnership for new generation of vehicle
p	-	Probability of recover one charge unit in one time slot,
q_i	-	Probability of required i charge units in one time slot
RC	-	Resistor-Capacitor
RMSE	-	Root mean square error
R_Q	-	Ohmic resistance
R_F	-	Internal resistance for fully charged condition
R_S	-	Series resistance
R_{S0}	-	Initial of series resistance
R_T	-	Total resistance in simplified model
R_{ct}	-	Charge-transfer resistance

R_{cycle}	-	Increment of internal resistance caused by the cycling effect
R_{ex}	-	Resistive load for model development
R_{int}	-	Internal resistance
R_1	-	Resistance in first RC parallel network
R_2	-	Resistance in second RC parallel network
R_3	-	Resistance in third RC parallel network
R'_1	-	Resistance in first RC parallel network for unloaded conditions
R'_2	-	Resistance in second RC parallel network for unloaded conditions
R'_3	-	Resistance in third RC parallel network for unloaded conditions
r	-	Probability of staying in the same SOC
r_{HF}	-	Ohmic resistance in energetical model
r_r	-	Resistance in RC network in energetical model
SOC	-	State-of-charge
SOH	-	State-of-health
SPKF	-	Sigma-point Kalman Filter
S_1	-	Exponential zone amplitude in Sheperd equation
S_2	-	Exponential zone time constant in Sheperd equation
T	-	Measured temperature
$T_{AMBIENT}$	-	Ambient temperature
$T_{DEFINED}$	-	Defined temperature
T_{ref}	-	Reference temperature
t	-	Battery runtime
t_E	-	Ending time of relaxation
t_R	-	Ending time of loaded condition or starting time of unloaded condition
t_S	-	Starting time of loaded condition
UPS	-	Uninterruptible power supply
VRC	-	Voltage-Resistor-Capacitor
VTF	-	Vogel-Tammann-Fulcher
V_O	-	Battery constant voltage in Sheperd equation
V_{SOC}	-	Voltage across C_{CAP} in VRC model

V_S	-	Voltage relative to R_{S0}
V_{S2}	-	Voltage relative to ΔR_S
V_{k1}	-	Voltage across first RC parallel network
V_{k2}	-	Voltage across second RC parallel network
V_{k3}	-	Voltage across third RC parallel network
V_t	-	Battery voltage
v_1 and v_2	-	Parameters of VTF equation
W	-	Warburg impedance
ω	-	Frequency
y_1	-	Amount of available charge
y_2	-	Amount of bound charge
$y_{1,0}$	-	Initial available charge
$y_{2,0}$	-	Initial bound charge
γ	-	State of charge
γ_D	-	Discharged SOC
γ_0	-	Initial SOC of battery
Z	-	Impedance of battery
Z_{eq}	-	Equivalent impedance of battery model
Z_p	-	Faradic impedance in energetical model for characterising electrode porosity
Z_t	-	Faradic impedance in energetical model for characterising concentration impedance of electrolyte
σ	-	Constant related to diffusion rate
θ	-	Coefficient of discharge rate
σ	-	Warburg coefficient
λ	-	Time constant of SOC increment
ϕ	-	Increment factor of capacity recovery
$\Delta E(T)$	-	OCV correction value for temperature effect
$\Delta\gamma$	-	SOC increment
ΔR_S	-	Series resistance increment

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CHAPTER 1

INTRODUCTION

1.1 Background

World energy supply is highly dependent on unsustainable resources, such as oil (33.2 %), coal (27 %) and natural gas (21.1 %) [1]. Resources of these fossil fuels are limited and expected could be exhausted within 40 years [2]. The consumption of these fossil fuels also produces green house gaseous (GHGs). For instance, fossil fuels are responsible for 85 % of anthropogenic carbon dioxides (CO₂) emissions [3]. International Energy Agency (IEA) has conducted a BLUE Map scenario by 2008, which describes the transformation on energy technology by 2050, in order to reduce the annual emissions of carbon dioxides (CO₂) [4].

According to the statistics provided by the Organisation for Economic Co-operation and Development (OECD) [1], there are approximately 27.3 % of the energy supplies consumed by the transportation sector. Moreover, it was stated that 61.4 % of the world oil is consumed by transportation while the price of oil is rising due to the depletion of oil resources. Therefore, the transformation of energy in this sector may greatly reduce the overall GHGs emissions and oil demands. These situations encourage the research and development activities to build up higher efficiency and cleaner transportation. Electric vehicle (EV) gets the attention and interest of scientists due to its advantages of zero GHGs emissions and higher efficiency. It has been proposed to replace internal combustion engine (ICE) as the mainstream vehicle in the near future. The renewable energy source can be used to

generate the energy sources of EV and thus help to reduce the reliance on fossil fuels and reduce the GHGs [5]. A technology roadmap for Electrical and Plug-in Hybrid Vehicle (EV/PHEV) is also carried out by IEA. The roadmap outlines the strategies to popularise the adoption of EV and PHEV worldwide and provide a significant reduction in light-duty vehicle CO₂ emissions by 2050 [4].

1.1.1 Electric Vehicle

The technology and idea of EV is not entirely new, but has existed for around three centuries. The world's first electric vehicle (EV) was made in 1830s and it used non-rechargeable batteries as the energy storage devices. The EV with rechargeable batteries was then released and it reached the first peak at the end of 19th century [2]. EVs were mass-produced and widely adopted as cars, taxis, buses and delivery vehicles. Even though the first internal combustion engine (ICE) vehicle was made in 1886, the ICE vehicle is not popularly implemented due to their frowziness and inconvenience of manual start [2, 6]. These limitations of ICE vehicle made EVs seem to be a more attractive choice. However, after the self starter of ICE was invented in 1911 and the cheap oil was broadly available, ICE vehicles became more attractive than EVs [6]. The poor EV's performances, such as the higher price of the battery compared to petrol, the long charging time of battery, and the short travel distance of EV, have cause the decline of the EVs' market after 1910s [2, 6]. The world crisis of fuel that happened by the mid of 20th century gave the second chance of EV development. However, the crisis was solved when the Middle East countries provided cheap fuel to the market [2]. The development of EV was aroused again at the beginning of 21st century due to the shortage of fuel resources and the issue of environmental pollution [2, 5].

Advantages of EV include zero emissions of GHG and air pollutants, very low noise, very high efficiency, and relatively low cost of electric motor [4]. For EV, electric motor is used for electric propulsion system while battery is used as the energy storage device. The battery is recharged from grid electricity, regenerative

braking or photovoltaic panels [4]. However, the battery has lower specific energy and specific power than ICE fuel [4]. Therefore, the performance of battery becomes the key for EV development. New battery technology and battery management system are two important aspects that enhance the performance of battery [2].

1.1.2 Battery Technologies

Rechargeable battery is an electrochemical device which converts electrical energy to chemical energy during charging and converts chemical energy to electrical energy during discharging [7]. Although there are several devices, such as ultra-capacitor, super-capacitor and ultra-high speed flywheels that are potentially applied as the energy storage element in EV [7], battery is still preferable due to its high specific energy and its capability of recharged for regenerative braking practice [8]. Apart from EV, batteries are broadly used as an energy storage element for portable electronic device, uninterruptible power supply (UPS), distributed generation, and avionics system.

In the aspect of battery technology, the battery is improved from Lead Acid battery to Nickel-based battery and from Nickel-based battery to Lithium-based battery [2]. Even though the battery technology has significantly improved, the battery technology is still unable to keep up with the pace of the current technology [9-11]. Specific energy, specific power, efficiency, maintenance requirement, cost, management, environment friendliness and safety are the requirements for EV's energy storage element. In this aspect, specific energy is the most important consideration for EV since the rate of specific energy determines the travel range of EV. A heavier battery is required if the battery with lower specific energy is applied. Ragone plot of the energy storage element is shown in Figure 1.1. By referring to Figure 1.1, Lithium ion (Li-ion) battery has the highest specific power and specific energy compared to others [4, 12].

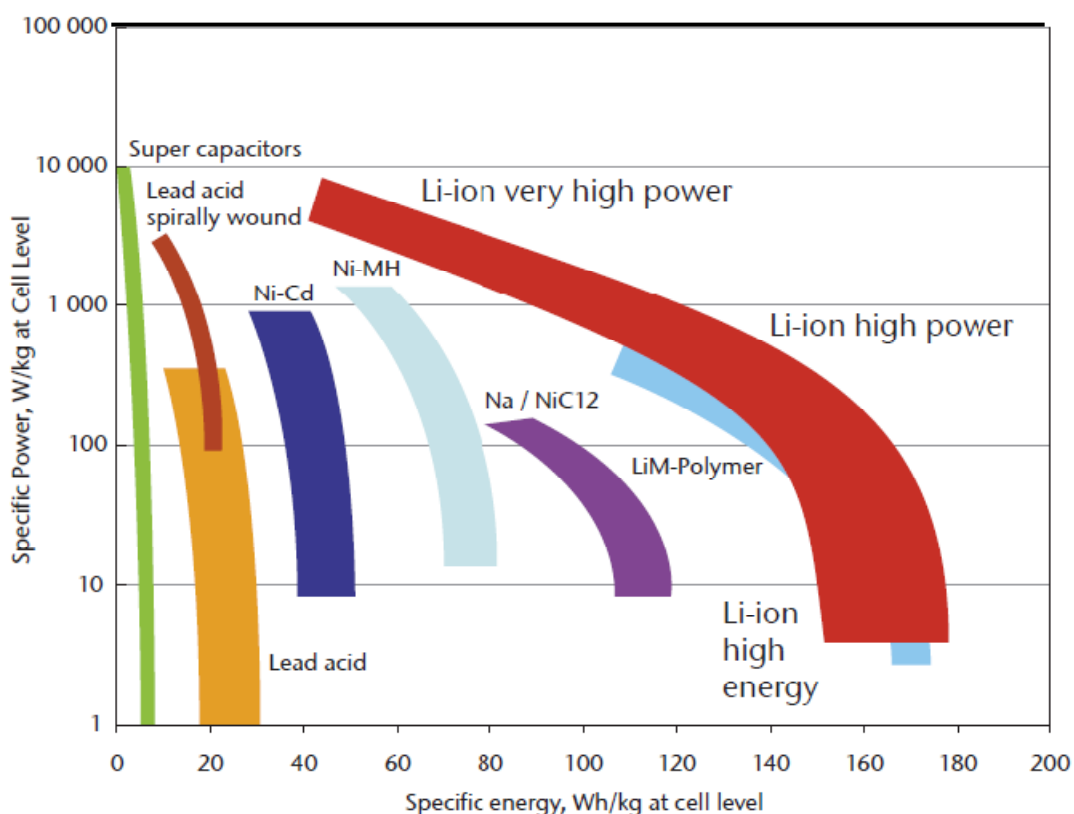


Figure 1.1 Ragone plot of energy storage element [4]

Lead Acid battery is constructed with lead, lead oxide and sulphuric acid. It is widely implemented in transportation sector due to low cost and maturity in technology. However, high molecular weight of lead has caused Lead Acid battery suffers from low specific energy. Additionally, Lead Acid battery also possesses poor temperature characteristic, especially at the low temperature (below 10 °C) [7]. Furthermore, the release of flammable hydrogen gas during self-discharging and the attendance of corrosive sulphuric acid may become safety threats to the vehicle [7].

Nickel-based battery can be categorized into several types, such as Nickel-Iron (NiFe), Nickel-Zinc (NiZn), Nickel-Cadmium (NiCd) and Nickel-Metal Hydride (NiMH). Nickel-based batteries possess higher specific energy and specific power than Lead Acid battery since nickel has lower molecular weight than lead. NiMH battery has highest rate of specific power and specific energy among all the Nickel-based battery. In addition to that, NiMH battery also has long cycle life, good temperature characteristics, low self-discharge rate, flat discharge profile and

negligible corrosion. However, NiMH battery has a high initial cost. Moreover, Nickel-based battery has suffers from memory effect [7]. In this aspect, the capacity of Nickel-based battery would be reduced because of it is not fully discharged before it is recharged [13]. NiMH battery has lesser extent of memory effect compared to NiCd battery [14].

Lithium Polymer (Li-P) and Li-ion are the two major technologies of Lithium-based battery. At the beginning of development, Li-ion battery suffered from safety issue due to the use of metal Lithium as negative electrode. This safety problem is then solved by using carbon material with Lithium insertion as the negative electrode material [12].

Lithium is the lightest metal and allows very high thermodynamic voltage. Therefore, Lithium-based battery has higher terminal voltage, higher specific energy and higher specific power compared to the other rechargeable batteries [12]. It is considered as the most promising battery in the future [7]. By referring to Ragone plot in Figure 1.1, the specific energy and specific power of Li-ion battery is the highest among all type of batteries. The specific energy of Li-ion battery is up to 150 Whkg^{-1} whereas the specific power of Li-ion battery can reach above 5 kWkg^{-1} [12]. The desired specific energy and specific power can be achieved by varying the thickness of electrodes [12]. In the field of portable electronic, the majority market of electronic devices is occupied by Lithium-based battery [12].

Lithium Ferro Phosphate (LiFePO_4) is one of the Lithium based battery which uses the phosphates as the cathode material. The theoretical capacity of LiFePO_4 battery is up to 170 mAh/g , which is the highest among lithium based batteries [15]. Moreover, the materials used in LiFePO_4 are widely available, economical and environmental friendly [15]. The handling of LiFePO_4 battery is safe and easy due to its excellent thermal stability in the fully charged condition and good humidity resistant [15]. The stability of LiFePO_4 effectively reduces the risk of explosion when the battery is accidentally overcharged and thus promises the safety of usage. Table 1.1 provides a comparison of Lithium-based battery according to their chemistry. For EV application, a good safety of the battery also ensures the

safety of usage. Hence, LiFePO_4 battery is more suitable to be applied as energy source in EV compared to another Lithium-based battery.

Table 1.1 : Characteristics of Lithium-based batteries

Characteristics	Lithium cobalt oxide (LiCoO_2)	Lithium manganese oxide (LiMn_2O_2)	Lithium ferro phosphate (LiFePO_4)
Specific energy	Good	Average	Poor
Power	Good	Good	Average
Low temperature	Good	Good	Average
Calendar life	Average	Poor	Poor above 30°C
Cycle life	Average	Average	Average
Safety*	Poor	Average	Good
Cost/kWh	Higher	High	High
Maturity	High	High	Low

1.1.3 Battery Management System

Battery is the main energy storage device of EV. A handling of battery is necessary so that the battery performs as a safe, consistent and competent energy source under various load demand and driving state [16]. Besides, accurate battery information such as state-of-charge (SOC), state-of-health (SOH), current and voltage are vital for energy management system of EV [17]. Therefore, an efficient battery management system (BMS) which can perform SOC estimation, cell equalisation and temperature management should be put as the primary concern [17]. BMS gives battery protection, increases battery life and its performance. Figure 1.2 shows the block diagram of general BMS.

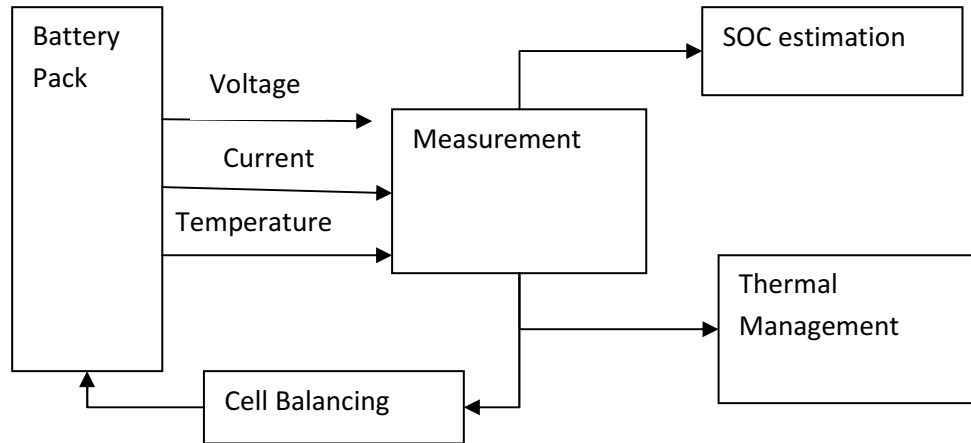


Figure 1.2 Battery management system (BMS)

SOC is the most important parameter to be realised since it demonstrates remaining capacity of the battery. The best performance of battery can be achieved by accurate assessment of SOC [18]. Unwanted harm of battery will occur if the value of SOC is extremely high (overcharged) or low (undercharged). The value of SOC is not readily measurable and thus estimation is required [16]. Several algorithms for SOC estimation have been proposed, such as Kalman Filter (KF) [19], Extended Kalman Filer (EKF) [20-21] and Sigma-point Kalman Filter (SPKF) [22-23]. High accurate performance of these SOC estimation algorithms is proven. These SOC estimation algorithms are model-based techniques which require a battery model that can provide current-voltage (I-V) information of the battery [17]. An accurate battery model is required since the accuracy of battery model would affect the quality of the SOC estimation [24]. Hence, accurate battery model should become the first issue in BMS design.

1.1.4 Battery Model

Battery model is important not only for SOC estimation, but also it is equally important for EMS controllers design, manage charge/discharge process, and lengthening the life of battery [5, 17]. By using accurate battery model, the

characteristics of battery under various charge/discharge conditions can be effectively forecasted and thus optimize the usage of battery [18].

Numerous studies conducted on battery modelling techniques are published in various scientific journals. The battery models can be categorised into analytical, electrochemical and equivalent circuit models [25]. Electrochemical model is complex and involve time-varying spatial partial differential equations. It is also impossible to connect to the rest of the system directly [25]. On the other hand, analytical model is unable to give a good view of the internal electrochemical process of the battery [26] whereas equivalent circuit model has lower accuracy compared to electrochemical model [25]. However, equivalent circuit model is popularly used by circuit designers since the effective battery control is permitted by applying the mathematical equations that derived from equivalent circuit model.

1.2 Statement of Problem

UTM-PROTON Future Drive Laboratory, which was established in UTM Johor Bahru campus, is actively involved in the research and development of the EV's technologies. Research in areas such as battery management system, energy management system, machine controller and power converter are conducted.

Model of battery is vital as a guide for system designer to forecast the electrical characteristics of battery. By applying an accurate battery model, BMS can estimate the SOC and the runtime of the battery efficiently and optimise the performance of battery [27].

Generally, high capacity lithium based batteries pack is applied as energy sources of electric vehicle. LiFePO_4 is potentially to be implemented in electric vehicle since it promises safe usage. However, the research on battery model for LiFePO_4 is still limited. Moreover, most of the battery models in previous research are focus on low capacity battery and only suitable for certain type of battery.

Additionally, battery model that provided by MATLAB/Simulink does not able to accurately simulate the dynamic behaviours of the actual battery as presented in [28-29].

The lack of suitable and accurate battery model would lead to unreliable control of battery. Since battery model is the key of BMS design, it is important to develop an accurate battery model for high capacity LiFePO₄ battery in order to capture the nonlinearity of battery in term of I-V characteristic, SOC and runtime of high capacity LiFePO₄ battery.

1.3 Thesis Objectives and Contributions

The objectives of this study are:

1. to study the existing techniques used in battery modelling.
2. to investigate the dynamic characteristics of 3.2 V, 18 Ah LiFePO₄ battery.
3. to propose a new model for Lithium Ferro Phosphate battery.
4. to propose a simple yet significant method of improvement in existing battery modelling techniques.

While performing this study, the thesis makes the following contributions:

1. It develops modified model with nonlinear capacity effects consideration to improve the performance of the existing battery model. A novel method to capture nonlinear capacity effects is proposed.
2. It introduces a simple battery model which expresses parameters as a function of SOC and current. Parameters for loaded conditions are differentiated from parameters of unloaded conditions. This is because the characteristics of battery can be different in loaded and unloaded conditions as stated in [17], [30], and [31]. This will be discussed further in Chapter 6. Transient response correction is used to determine the

parameters for loaded conditions. The technique is simple, yet it improves the performance of battery model significantly.

1.4 Thesis Organisation

The rest of the thesis is organised as follows:

Chapter 2 discusses the dynamic behaviours and classifications of battery model. This chapter briefly discusses on the proposed battery model in previous researches, such as mathematical models, electrochemical models and equivalent circuit model. Model development for equivalent circuit model is reviewed.

Chapter 3 describes the experimental set-up used in the project. The procedures of battery tests, which are used to identify the parameters of the battery model are presented and described.

Chapter 4 discusses conventional battery model. In this chapter, conventional battery modelling technique is presented briefly. Simulation and the experimental results on the conventional battery model are also presented. The performance of conventional battery model is discussed.

Chapter 5 proposes a modified battery model with consideration of nonlinear capacity effect. In this model, a new approach of capturing nonlinear capacity effect is presented in detail. Simulation and the experimental results on the proposed battery model are also presented. The performance of modified battery model is discussed.

Chapter 6 proposes a simplified battery model. In this model, parameters are expressed as a function of SOC and current to eliminate the usage of look-up table in the model. Simulation and the experimental results on the simplified battery

model are also presented. The performance of simplified battery model is presented. The simplified model is further validated with random load tests.

Chapter 7 gives the conclusions of the thesis and possible directions of further research on this work.

1.5 Summary

As a prelude to the thesis, a brief background of electric vehicle that includes its history and its environmental benefits has been presented. Several issues that related to energy storage system, starting from battery technologies to battery management system, have been presented. Battery model which serves as the key of monitoring EV energy sources is briefly discussed. An equivalent circuit model has been chosen to use for this work due to its simplicity, accuracy and suitability for battery-powered system design.

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