

# Numerical Analysis of Disc Brake Squeal Considering Temperature Dependent Friction Coefficient

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**Abstract:** Passenger cars have become one of the main transportations for people travelling from one place to another. Indeed, vehicle quietness and passenger comfort issues are a major concern. One of vehicle components that occasionally generate unwanted vibration and unpleasant noise is the brake system. Brake squeal noise is the most troublesome and irritant one to both car passenger and the environment, and is expensive to brakes and carmakers in terms of warranty costs. It is well accepted that brake squeal is due to a friction-induced dynamic instability and it frequently occurs at frequency above 1 kHz and is described as sound pressure level above 78 dB. Brake squeal has been studied over 20 years ago through experimental, analytical and numerical methods in an attempt to understand, to predict and to prevent squeal occurrence. In recent years, the finite element (FE) method has become the preferred method due to inadequacy of experimental methods in predicting squeal at early stage in the design process. However, the drawbacks of the FE method are over-predictions and missing unstable modes in the squeal frequency range. This paper attempts to improve the drawbacks by considering temperature dependent friction coefficient ( $T$ - $\mu$  dependency), which is typically neglected by many previous investigators. Prediction of disc brake squeal is performed using complex eigenvalue analysis that available in ABAQUS V6.4. In doing so, a validated and detailed 3-D finite element model of a real disc brake is used. Predicted results are then compared to those obtained in the experimental results with and without  $T$ - $\mu$  dependency.

**Keywords:** Squeal, Disc Brake, Complex Eigenvalue, Temperature, Friction Coefficient

## 1. Introduction

Passenger cars have become one of the main transportations for people travelling from one place to another. Indeed, vehicle quietness and passenger comfort issues are a major concern. One of vehicle components that occasionally generate unwanted vibration and unpleasant noise is the brake system. As a result, carmakers, brake and friction material suppliers face challenging tasks to reduce high warranty payouts. Akay [1] stated that the warranty claims due to the noise, vibration and harshness (NVH) issues including brake squeal in North America alone were up to one billion US dollars a year. Similarly, Abendroth and Wernitz [2] noted that many friction material suppliers had to spend up to 50 percent of their engineering budgets on the NVH issues.

In a recent review, Kinkaid *et. al.* [3] listed a wide array of brakes noise and vibration phenomenon described by its own terminology. Squeal, creep-groan, moan, chatter, judder, hum, and squeak are among the names that can be found in the open literature. Of these noises, squeal is the most troublesome and irritant one to both car passengers and the environment, and is expensive to the brakes and car manufacturers in terms of warranty costs [4]. It is well accepted that brakes squeal is due to friction - induced vibration or self-excited vibration via a rotating disc. Brake squeal frequently occurs at frequency above 1 kHz [5] and is described as sound pressure level above 78 dB [6].

Brake squeal has been studied since 1930's by many investigators through experimental, analytical and numerical methods in an attempt to understand, to predict and to prevent squeal occurrence. In recent years, the finite element (FE) method, particularly using complex eigenvalue analysis, has become the preferred method in studying brake squeal. The popularity of finite element analysis (FEA) is due to the inadequacy of experimental methods in predicting squeal at early stage in the design process. Moreover, FEA can potentially simulate any changes made on the disc brake components much faster and easier than experimental methods. A recent review [7] stated that experimental methods are expensive due to hardware costs and long turnaround time for design iterations. In addition, discoveries made on a particular type of brake are not always transferable to other types of brake and quite often product developments are based on trial-and-error basis. Furthermore, a stability margin is frequently not found experimentally.

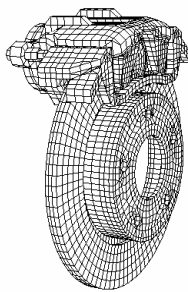
However, the crux now lies in how the FE method can be a predictive tool rather than a diagnosis tool. Kung *et. al.* [8] commented that although the complex eigenvalue analysis was successfully used in brake squeal problems, the shortcomings of this method were over-predictions and sometimes missing, unstable modes in the squeal frequency range. In a similar paper, they suggested that in order to improve the complex eigenvalue predictability, user experience and engineering judgement were essential to obtain reliable results. In addition to this, they also stated that realistic friction coupling between pad and disc interface, consideration of friction-induced (positive and/or negative) damping and lining wear could also play important role for improving the predictability. AbuBakar [9] in his thesis found that

combination of realistic surface topography of brake pads with friction-induced negative damping could produce better prediction in the complex eigenvalue analysis.

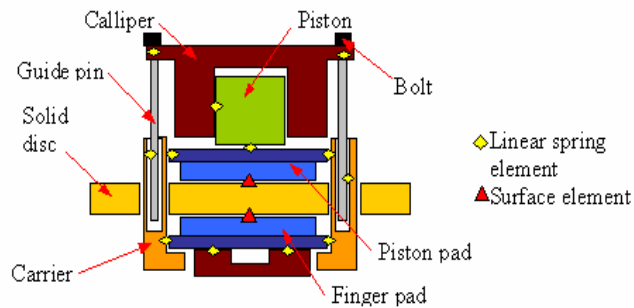
Sanders *et. al.* [10] in his work recommended that for up-front design and system modelling, it is desirable to describe the frictional behaviour of a brake lining as a function of contact pressure, sliding speed and temperature. Similarly, Mahajan *et. al.* [11] stated that asymmetric stiffness matrix that represented friction coupling between the pads and the disc interface due to friction coefficient which could be assumed as a function of contact pressure, sliding velocity and other factors such as temperature and humidity. However in their stability analysis, temperature and humidity effects were ignored. In the past, most of FE models [11-15] did account only for friction coefficient as a function of contact pressure and sliding velocity. Recently the authors found another way to improve the complex eigenvalue analysis predictability, i.e. by considering friction coefficient ( $\mu$ ) as a function of contact pressure, sliding velocity and temperature. In an attempt to do so, a validated and detailed FE model of a real brake assembly is used throughout this work. Complex eigenvalue analysis available in a commercial software package, ABAQUS v6.4 is utilised. Predicted results are then compared to those obtained in the experimental results with and without temperature- friction coefficient dependency ( $T-\mu$  dependency).

## 2. Finite Element Model

The finite element (FE) model, as shown in Fig. 1(a), consists of a disc (rotor), two pads, a caliper, a carrier, a piston and two guide pins. Damping shims are not present in the FE model. The model uses up to 8000 solid elements and approximately 37,200 degrees of freedom. Fig. 1(b) shows a schematic diagram of contact interaction that has been used in the disc brake assembly model. Surface based elements are used at the disc/pad interfaces while spring elements are used for other contact interactions between the disc brake components. A rigid boundary condition is imposed at the boltholes of the disc and of the carrier bracket, where all six degrees of freedom are rigidly constrained.



a) Finite element model



b) Contact interaction

Figure 1. A solid disc brake assembly

Since the contact between the disc and friction material surface is crucial, a realistic representation of those interfaces should be made. The friction material of pads has a rougher surface and is softer than the disc, which has quite a smooth and flat surface, and is less prone to wear. Therefore in this work, actual surfaces on macroscopic scale at the piston (inboard) and finger (outboard) pads are measured and considered. A Mitutoyo linear gauge LG-1030E and a digital scale indicator are used to measure and provide readings of the surface respectively, as shown in Fig. 2. Node mapping, as shown in Figure 2 is required so that surface measurement can be made at desirable positions, which are generated from the FE model. By doing this, information that is obtained in the measurement can be used to adjust the coordinates of the nodes of piston and finger pads in the brake pad interface model. There are about 227 nodes on the piston pad interface and 229 nodes on the finger pad interface. The FE model is then verified through three validation stages and has been described in [9].

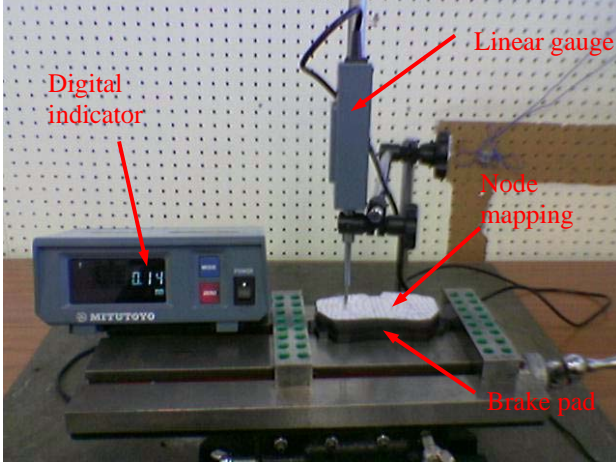


Figure 2. Measurements of the friction material interface

### 3. Complex Eigenvalue Analysis

The complex eigenvalue analysis made available in ABAQUS is utilized to determine disc brake assembly stability. The positive real parts of the complex eigenvalue indicate the degree of instability (unstable frequencies and unstable modes) of the disc brake assembly and are thought to imply the likelihood of squeal occurrence. The essence of this method lies in the asymmetric stiffness matrix that is derived from the contact stiffness and the friction coefficient at the disc/ pads interface [14]. In order to perform the complex eigenvalue analysis using ABAQUS, four main steps are required [15]. They are given as follows:

1. Nonlinear static analysis for applying brake-line pressure
2. Nonlinear static analysis to impose rotational speed on the disc
3. Normal mode analysis to extract natural frequency of undamped system
4. Complex eigenvalue analysis that incorporates the effect of friction coupling

In this analysis, the complex eigenvalues are solved using the subspace projection method. The eigenvalue problem can be given in the following form:

$$(\lambda^2 \mathbf{M} + \lambda \mathbf{C} + \mathbf{K}) \mathbf{y} = 0 \quad (1)$$

where  $\mathbf{M}$  is the mass matrix,  $\mathbf{C}$  is the damping matrix,  $\mathbf{K}$  is the unsymmetric (due to friction) stiffness matrix and  $\mathbf{y}$  is the eigenvector. This unsymmetrical stiffness matrix leads to complex eigenvalues and eigenvectors. In the third step stated above, the symmetric eigenvalue problem is first solved, by dropping damping matrix  $\mathbf{C}$  and the unsymmetric contributions to the symmetric stiffness matrix  $\mathbf{K}_s$ , to find the projection subspace. Therefore the eigenvalue,  $\lambda$  becomes a pure imaginary where  $\lambda = i\omega$ , and the eigenvalue problem now becomes:

$$(-\omega^2 \mathbf{M} + \mathbf{K}_s) \mathbf{z} = 0 \quad (2)$$

This symmetric eigenvalue problem then is solved using subspace eigensolver [16]. The next step is that the original matrices are projected in the subspace of real eigenvectors,  $\mathbf{z}$  and given as follows:

$$\mathbf{M}^* = [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n]^T \mathbf{M} [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n], \quad (3a)$$

$$\mathbf{C}^* = [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n]^T \mathbf{C} [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n], \quad (3b)$$

$$\mathbf{K}^* = [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n]^T \mathbf{K} [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n], \quad (3c)$$

Now the eigenvalue problem is expressed in the following form:

$$(\lambda^2 \mathbf{M}^* + \lambda \mathbf{C}^* + \mathbf{K}^*) \mathbf{y}^* = 0 \quad (4)$$

The reduced complex eigenvalues problem is then solved using the QZ method for a generalized nonsymmetrical eigenvalue problem. The eigenvectors of the original system are recovered by the following:

$$\mathbf{y}^k = [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n] \mathbf{y}^{*k} \quad (5)$$

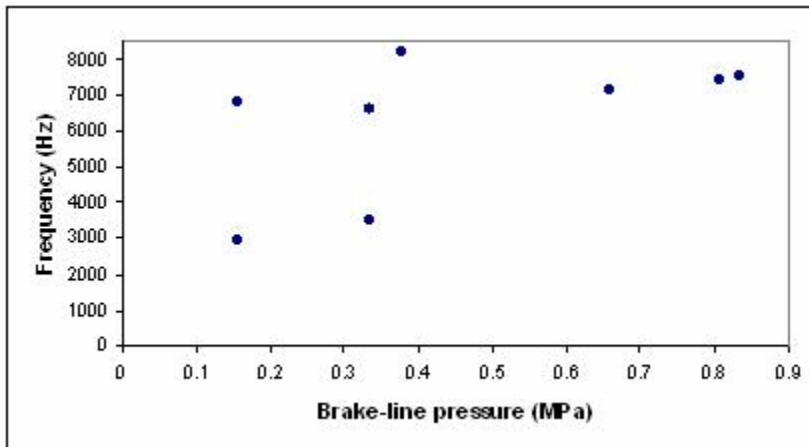
where  $\mathbf{y}^k$  is the approximation of the  $k$ -th eigenvector of the original system.

#### 4. Prediction of Disc Brake Squeal

In this work, only part of the experimental results obtained in [17] are utilised for comparison. Table 1 shows operating conditions for six squeal tests and their respective squeal frequencies captured in the experiments. From the table, it is shown that test numbers T01 and T19 have two squeal frequencies at 3.6 and 6.6 kHz, and 2.9 and 6.8 kHz respectively. For the other test numbers (T06, T27, T45 and T47) there is only one squeal frequency captured as given in Table 1. All those squeal frequencies are characterised by either 4, 5, 6 or 7 nodal diameters (ND) of the disc. For illustration, Fig. 3 shows distributions of squeal frequencies against brake-line pressure. It shows that there are two squeal frequencies at brake-line pressure range of 0.1~0.2 MPa, three squeal frequencies at range of 0.3~0.4 MPa, one squeal frequency between 0.6~0.7 MPa and two squeal frequencies at brake-line pressure range of 0.8~0.9 MPa. This figure will be used later to compare with prediction results from the FE method.

**Table 1. Operating conditions of squeal tests [17]**

Test No.	Average Temperature, $T$ (°C)	Pressure, $p$ (MPa)	Speed, $\Omega$ (rad/s)	Kinetic Friction Coefficient, $\mu$	Squeal Frequency (Hz)
T01	33.0	0.33	3.22	0.46	3516, 6599
T06	83.7	0.83	6.32	0.50	7540
T19	255.8	0.16	25.57	0.48	2944, 6797
T27	131.7	0.66	3.23	0.55	7112
T45	76.2	0.81	3.22	0.57	7420
T47	232.2	0.38	3.278	0.50	8201



**Figure 3. Experimental results of disc brake squeal noise**

Firstly, complex eigenvalue analysis is simulated without  $T$ - $\mu$  relationship. From the analysis, it is found that there are 48 unstable frequencies predicted compared to 8 squeal frequencies captured in the experiments as shown in Fig. 4. It is

suggested that there are over-predictions in the simulation results even though those 8 unstable frequencies are reasonably matched with the experimental data. From Fig. 4, there are about 8 unstable frequencies predicted for brake-line pressure range of 0.1~0.2 MPa whereas only 2 squeal frequencies captured in this range. For brake-line pressure between 0.3~0.4 MPa, complex eigenvalue analysis predicts about 19 unstable frequencies whilst in the experiments there are only 3 squeal frequencies captured and thus it indicates that there are 15 unstable frequencies over predicted. At brake-line pressure range of 0.6~0.7 MPa, there are about 7 unstable frequencies obtained in the complex eigenvalue analysis compared to only one squeal frequency appears in the squeal tests. Finally, for brake-line pressure between 0.8~0.9 MPa, 14 unstable frequencies are predicted compared to 2 squeal frequencies in the experiments. This brings to over-predictions of 12 unstable frequencies. By looking at those over-predictions for the whole range of brake-line pressure, it is suggested that improvement in the predictability particularly using complex eigenvalue analysis should be made. Thus, this leads to the authors to find a way to improve predictability in complex eigenvalue analysis. In this paper, the authors attempt to include temperature dependent friction coefficient ( $T$ - $\mu$  dependency) and it is interesting to see the predicted results.

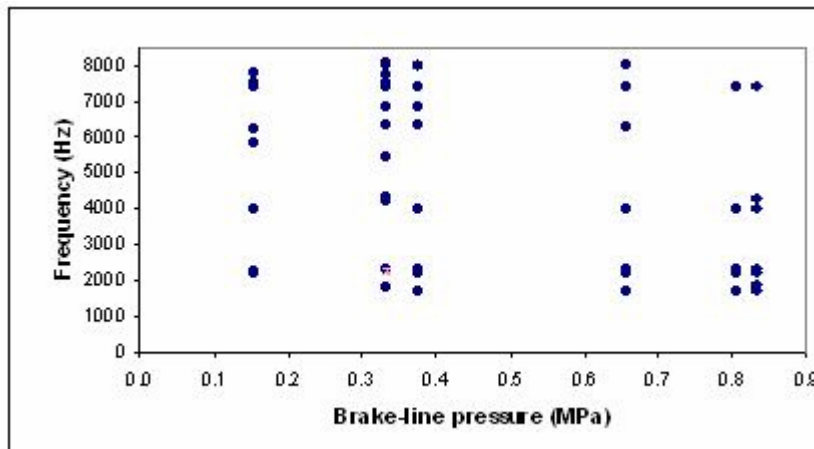


Figure 4. Predicted unstable frequencies without consideration of  $T$ - $\mu$  dependency

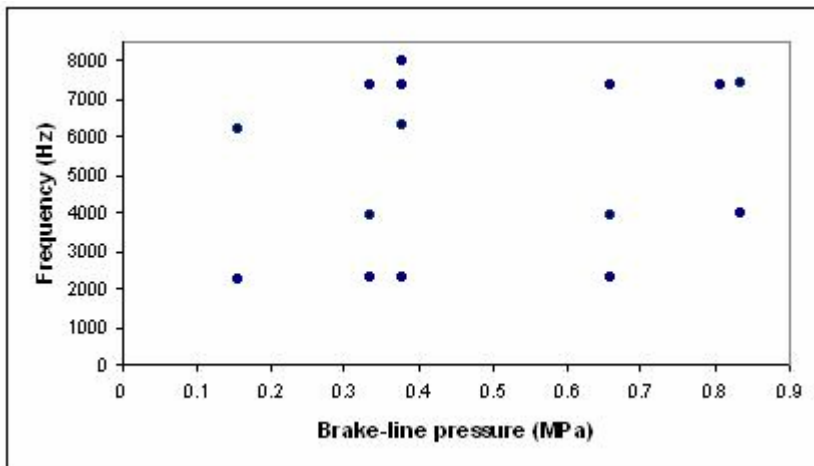


Figure 5. Predicted unstable frequencies with consideration of  $T$ - $\mu$  dependency

Using a similar approach i.e., complex eigenvalue analysis, six simulations are performed according to the operating conditions given in Table 1 and with the inclusion of  $T$ - $\mu$  dependency. From Fig. 5, it is shown that the numbers of unstable frequencies are far more less than those predicted without  $T$ - $\mu$  dependency. There are only 15 unstable frequencies appeared in complex eigenvalue analysis compared to 48 unstable frequencies (Fig. 4), which is about reduction of 69 percent. Now by looking at a certain range of brake-line pressure, Fig. 5 shows that at range of 0.1~0.2 MPa good agreement is achieved between predicted and measured data. There are 2 unstable frequencies predicted in complex eigenvalue analysis and these frequencies are almost the same as captured in the squeal tests. For brake-line pressure of 0.3~0.4 MPa, there are 7 unstable frequencies appeared in the analysis compared to 3 squeal frequencies

captured in the experiments which are 4 unstable frequencies over predicted. At brake-line pressure range of 0.6~0.7 MPa only 3 unstable frequencies are predicted which are two more than captured in the squeal tests. Finally for brake-line pressure between 0.8~0.9 MPa, complex eigenvalue analysis predicts 3 unstable frequencies compared to 2 squeal frequency in the experiments which is only one unstable frequency over predicted. In overall, with the inclusion of  $T-\mu$  dependency predictability in complex eigenvalue analysis can be improved and subsequently it increases reliability in the predicted results.

It is also important to note that correlations of the prediction results are not only based on the unstable frequencies alone but also in terms of their unstable mode shapes. For example, T19 has two squeal frequencies which are characterised by 4ND at 2944 Hz and 6ND at 6797 Hz. Similarly, in complex eigenvalue analysis, there are two unstable frequencies predicted at 2847 Hz and 6249 Hz which are characterised by 4ND and 6ND respectively as shown in Fig. 6. The results show that not only unstable frequencies are close enough to measured data but also their unstable modes match well with those mode shapes found in the experiments.

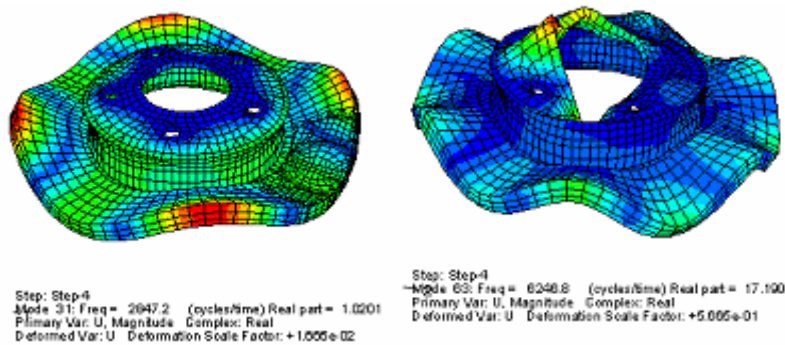


Figure 6. Unstable modes of the disc characterised by 4ND (left) and 6ND (right)

## 5. Conclusions

This paper presents numerical analysis of disc brake squeal using the finite element (FE) method. Complex eigenvalue analysis that made available in a commercial software package called ABAQUS v6.4 is fully utilised to predict unstable frequencies and their modes. The FE model of a real disc brake assembly is simulated for two different cases i.e., with and without  $T-\mu$  dependency. It is found that with the inclusion of  $T-\mu$  dependency, predictability in the complex eigenvalue analysis can be improved where the results show that the numbers of unstable frequencies are reduced significantly than those predicted without  $T-\mu$  dependency. It is also found the predicted unstable frequencies are not only have reasonably good agreement with the experimental results but also their mode shapes are also matched well. This suggests that temperature dependent friction coefficient relationship cannot be easily ignored in predicting squeal using complex eigenvalue analysis as it can reduce over-prediction of unstable frequencies and thus produces more reliable and better correlation against experimental data.

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## References

- [1] Akay, A., 2002, Acoustics of friction, *J. Acoust. Soc. Am.*, 111(4):1525-1548.
- [2] Abendroth, H., and B. Wernitz, 2000, The integrated test concept dyno-vehicle; performance and noise, *S.A.E Technical Paper* 2000-01-2774.
- [3] Kinkaid, N. M., O. M. O'Reilly, and P. Papadopolous, 2003, Review of automotive disc brake squeal, *Journal of Sound and Vibration*, 267: 105-166.
- [4] Crolla, D. A., and A. M. Lang, 1991, Brake noise and vibration – The state of the art, *Vehicle Tribology Leeds-Lyon Tribology Series*, 18: 165-174.
- [5] Lang, A. M., and H. Smales, 1983, An approach to the solution of disc brake vibration problems, *Proc. IMechE*, C37/83.
- [6] Eriksson, M., 2000, Friction and contact phenomenon of disc brakes related to squeal, *PhD Thesis*, Faculty of Science and Technology, Uppsala University.



- [7] Ouyang, H., W. V. Nack, Y. Yuan, and F. Chen, 2005, Numerical analysis of automotive disc brake squeal: a review, *Int. J Vehicle Noise and Vibrations*, 1(3/4): 207-230.
- [8] Bajer, A., V. Belsky, and S. Kung, 2004, The influence of friction-induced damping and nonlinear effects on brake squeal analysis, *S.A.E. Technical Paper* 2004-01-2794.
- [9] AbuBakar, A. R., 2005. Modelling and simulation of disc brake contact analysis and squeal, *PhD Thesis*, Department of Mechanical Engineering, University of Liverpool.
- [10] Sanders, P. G., T. M. Dalka, and R. H. Basch, 2001, A reduced-scale brake dynamometer for friction characterisation, *J. Tribology International*, 34: 609-615.
- [11] Mahajan, S. K., Y. Hu, and K. Zhang, 1999, Vehicle disc brake squeal simulations and experiences, *S.A.E. Technical Paper* 1999-01-1738.
- [12] Goto, Y., T. Amago, K. Chiku, T. Matsushima, and T. Ishihara, 2004, Experimental identification method for interface contact stiffness of FE model for brake squeal, *Proc. IMechE International Conference- Braking 2004*, Leeds, pp. 143-155.
- [13] AbuBakar, A. R., H. Ouyang, L. Li, and J. E. Siegel, 2005, Brake pad surface topography Part II: Squeal generation and prevention, *SAE Technical Paper* 2005-01-3935.
- [14] Liles, G. D., Analysis of disc brake squeal using finite element methods, *S.A.E. Technical Paper* 891150.
- [15] Bajer, A., V. Belsky, and L. J. Zeng, 2003., Combining a nonlinear static analysis and complex eigenvalue extraction in brake squeal simulation, *S.A.E Technical Paper* 2003-01-3349.
- [16] ABAQUS Manual Version 6.4, 2003, Hibbit, Karlsson & Sorensen, Inc.
- [17] James, S., 2003, An experimental study of disc brake squeal, *PhD Thesis*, Department of Engineering, University of Liverpool.
- [18] Day, A. J., 1988, An analysis of speed, temperature, and performance characteristics of automotive drum brakes, *Journal of Tribology*, 110: 295-305.