VIM AND FDM FOR SOLVING BOLTZMANN EQUATION OF SELF-GRAVITATING GAS-DUST SYSTEM

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Specially dedicated to my beloved family

and those people who have given consistent support and guide me.

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ABSTRACT

In this research, Boltzmann equation of Self-Gravitating Gas-Dust System is introduced. The model consists of dust component equations, gas dynamics equations and also Poisson equation. The mathematical model is used in order to visualize the movement of the gas and dust in environment. This research only focuses on sequential algorithm and solving the model in 2 Dimension (2-D). Numerical solutions obtained by using the Variational Iteration Method (VIM) and Finite Difference Method (FDM). The results are computed by using Borland C++ Builder, MATLAB 2008a and MAPLE 13. Then, the graphs are visualized by using Microsoft excel. The result is based on numerical analysis. Numerical analysis under consideration is pattern of the graph, gradient of the graph, time execution and computational complexity. The movement of the gas-dust showed differently with the different method. At the end, the results show that the VIM and FDM are successfully in visualizing the movement of the gas-dust in reality. Based on theoretically, VIM is in solving gas-dust problem compared to FDM.

ABSTRAK

Dalam kajian ini, persamaan Boltzmann telah diperkenalkan. Model ini terdiri daripada persamaan habuk, persamaan gas dan juga persamaan Poisson. Model matematik ini digunakan untuk melihat pergerakan habuk dan gas dalam persekitaran. Kajian ini hanya memberi tumpuan pada algoritma berjujuran dalam 2 dimensi (2-D). Penyelesaian berangka telah diperolehi melalui kaedah lelaran variasi dan kaedah beza terhingga. Penyelesaian ini telah dilakukan dengan menggunakan perisian seperti Borland C++ Builder, MATLAB 2008a dan MAPLE 13 Manakala, graf dilakarkan menggunakan microsof excel. Keputusan adalah berdasarkan analisis berangka. Analisis berangka yang di kaji adalah corak pergerak graf, kecerungan graf, masa pelaksanaan dan pengiraan komplek. Pergerakan habuk dan gas menunjukkan perbezaan apabila menggunakan kaedah penyelesaian yang berbeza. Hasil kajian menunjukkan kaedah lelaran variasi dan kaedah beza terhingga telah berjaya dalam mengambarkan pergerakan-pergerakan habuk-gas di alam realiti. Berdasarkan teori, kaedah lelaran variasi adalah keadah yang berkesan untuk menyelesaikan masalah habuk dan gas berbanding kaedah beza terhingga.

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LIST OF SYMBOLS

a	Acceleration
E	Energy
r	Coordinates
u	Velocities
v	Gas velocity
p	Pressure
Φ	Gravitational potential
Φ_1	Outer potential
Φ_2	Self-consistent potential
ρ_p	Dust density
ρ_g	Gas density
k_{fr}	Coefficient of friction

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Nowadays, many difference numerical methods have been introduced in solving several of linear and nonlinear partial differential equation (PDE). The methods commonly used are Adomian Decomposition Method (ADM), Homotopy Perturbation Method (HPM), Homotopy Analysis Method (HAM) and Variational Iteration Method (VIM) (Jafani, 2009). On the other hand, Finite Difference Method (FDM) is also one of the most popular methods in solving linear and nonlinear PDE (Popov & Fryazinor, 2009). It is because the method is easy to apply directly to the problems.

Since the research on dust movement continues to be a big interest, the building of mathematical, analytical and numerical approaches are developing as well. In this research, we applied VIM and FDM method in solving numerical model which included of dust component, gas components and gravitational interaction (Kireev, 2009). Normally, the model was solved by other famous methods. Dust components usually solved by particle-in-cell method (PIC) and gas components solved by fluid-in-cell method (FLIC) (Kuksheva et. al, 2003). Poisson equation for gravitational interaction is solved on grid in cylindrical coordinate system (Kukshere et. al, 2009), "particle-particle" method (Kuksheva, 2003) and by Fourier transforms method (Kireev, 2009).

1.2 Background of the Research

Numerical model of the components consist of several equations. The dynamics of dust components is described by the Vlasov-Liouville kinetic equation (Collisionless Boltzmann equation) (Kireev, 2009). Besides, the equations of gas dynamics are employed to consider gas movement. It is commonly known that the equations of gas dynamics are the mathematical expressions of conservation laws. It consist combination of mass, impulse, momentum and energy equation (Kuksheva, 2003). The gravitational potential is satisfying by Poisson equation. The movement takes places based on outer potential and self-consistent potential. All the equations (dust, gas dynamics and gravitational potential) directly involved in collision between the particles and gas molecules which called as Brownian motion.

Recently, VIM widely used in research world. This method is very famous among researches in order to solve or to find the exact solution of the problems. This method was proposed by Ji Huan He which had found wide application for the solution of nonlinear ordinary and PDE. In 2005, VIM successfully used in Schrodinger-KdV, generalized KdV and shallow water equation (Soliman & Abdou, 2005) and Burger's and coupled Burger's (Soliman & Abdou, 2005). Followed by solving nonlinear RDEs, Hirota Sotsuma Coupled KdV system and Drinefel's Sokolov-Wilson equation (Soliman & Abdou, 2006). VIM also is a powerful device to solve the boundary value problem (BVP) and initial value problem (IVP) such as the third order nonlinear ODE (Blasius equation) which arises in boundary layer problem in the fluid dynamics (Liu & Kurra, 2011) and Brati equation (Aslam Noor & Syed Tauseef, 2011). In 2012, VIM was proven in finding exact solution of gas dynamics equations (Matinfar et. al ,2012). Modified VIM was successfully solve Schrodiger and Laplace problems (Abeer, 2012) and in deriving analytical solution of seventh order deferential equation (Nikker & Mighani, 2012).

On the other hand, FDM is also one of the methods for solving gas dynamics equations. Two-dimensional difference scheme was applied to simulate gas dynamics equations. The result shows that the scheme is homogeneous and provides an automatic shock capturing (Popov & Fryazinov, 2008).

1.3 Statement of the Problem

In this research, we will investigate the accuracy of VIM and FD for solving the mathematical model by Kireev (2009). The other investigation is based on the performance of numerical analysis of both methods. The performance evaluations under consideration are pattern and gradient of the graphs, computational complexity, and time execution.

1.4 Objective of the Research

The main objectives of this research are:

a) To use the VIM for solving the conventional gas-dust Boltzmann equation.
b) To produce the numerical analysis of finite difference method of gas dynamics equations based on Jacobi Method.
c) To visualize the result from VIM and finite difference method.

1.5 Scope of the Research

This research is only focused on six mathematical models (Kireev, 2009). The equations will be solved 2nd order of forward finite difference method and variational iteration method. The initial and boundary conditions will be fixed and the specific parameters will be used. We will use Borland C++ Builder, MAPLE 13, MATLAB 2008a and Microsoft Excel in visualizing the result. Figure 1.1 indicates the scope of research involving the mathematical modeling of the self-gravitating dust-gas system, research methodology under consideration and computational platform.

1.6 Significant of the Research

The significant of this research is to capture the gas and dust movement of dynamic model based on changing parameters such as time and space (Reeves, 2009). MABLAB 2008a, MAPLE 13 and Microsoft Excel software are used to visual the difference directions of dust and gas movement by changing the independent parameters.

The visualization is based on two methods; Variational iteration method (VIM) and Finite Difference Method (FDM). Numerical performance measurements will use to analyze the comparison of two methods. The measurements are run time, convergence rate, accuracy and relative error. This research proves that VIM is the alternative method to capture the dynamic movement of gas and dust.

1.7 Organization of Dissertation

Chapter 1 contains general introduction of the mathematical model and numerical methods of gas and dust movement. The background, objectives, statement and the significance of the research are also stated.

Chapter 2 defines the Boltzmann equation and Vlasov equation of dynamic method. Then followed by review some literature involve two method that under consideration. Numerical analysis and computational platform system are also discussed. At the end of this chapter, the survey of references and the summary of literature review under consideration are organized in the table.

Chapter 3 begins with discretization of the conventional gas-dust Boltzmann equation by using finite difference method (FDM). The entire steps taken from the beginning to the end have shown clearly. The sequential algorithm for step is showed.

While in Chapter 4, the numerical model of gas-dust Boltzmann equation is solved using Variational Iteration Method (VIM). The selected initial conditions are used in computing the solution. This chapter emphasizes Poisson equation in order to get the exact solution.

Chapter 5 discusses the results obtained the numerical solution of FDM and VIM as in chapter 3 and chapter 4. A few tables and graphs are used to discuss the comparison of FDM and VIM. Some performance results will be drawn to investigate the differences between two methods.

Finally, this report will end with chapter 6 where the conclusion will be made and some recommendations for future research are provided. Then it is followed by references and appendices.

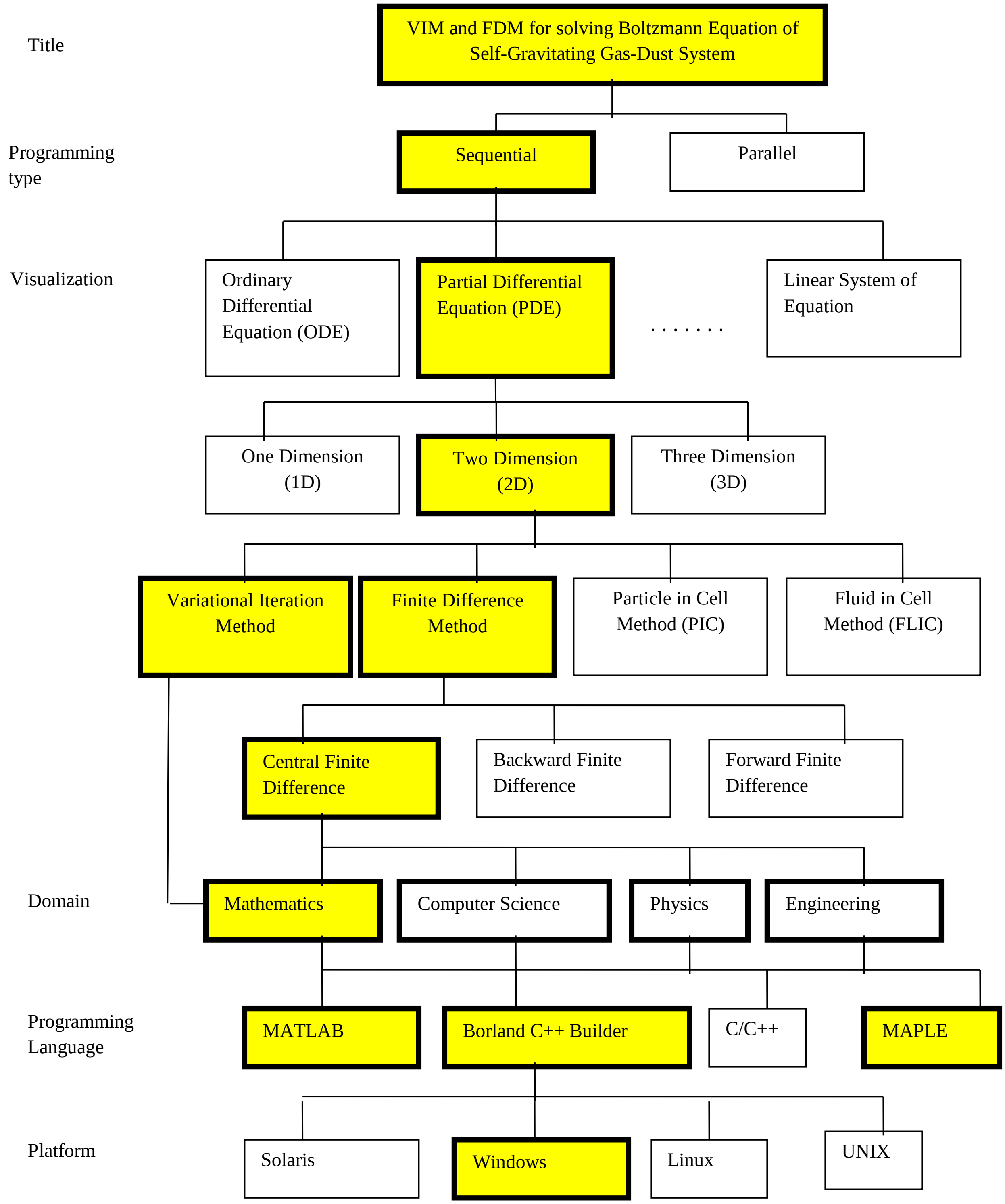

Figure 1.0: Research scope of the mathematical modeling of the self-gravitating dust-gas system

REFERENCES

Kireev, S. A Parallel 3D Code for Simulation of Self-gravitating Gas-Dust Systems. *Springer-Verlag Berlin Heidelberg.* 2009. LNCS 5689, pp.406 – 413.

Geng, Q. Airbone Dust Monitoring during Grain Handling. *2nd International Conference on Bioinformatics and Biomedical Engineering.* May 16-18, 2008. pp. 3745 – 3748.

Cheng, X., Cao, M., and Collier, M. An On-Line Detection System for Coal Mine Dust. *Proceeding of the 7th World Congress on Intelligent Control and Automation.* June 25-27. Chongqing, China. 2008. pp. 4166 – 4171.

Murnane, S.N., Barnes, R.N., Woodhead, S.R. ed. Electrostatic Modeling and Measurement of Airbone Particle Concentration. *IEEE Transaction On Instrumentation and Measurement.* April, 1996. Vol 45, no 2.

Oda, Y.,Nakata, T., Yamamoto, T., ed. Development of Dust Removal System for Fusion Reactor. *Journal of Fusion Enegy.* 1997. Vol 16, no 3.

Chen, J.X., Wegman, E.J., Fu, X., ed. Near Real-Time Simulation of Particle Systems. *Proceeding DIS-RT '99 Proceedings of the 3rd International Workshop on Distributed Interactive Simulation and Real-Time Applications.* IEEE Computer Society Washington, DC, USA. 1999. pp 33.

Tongnoi, N., and Parnklang, J. Portable Dust Monitoring Unit Using QCM. *International Conference on Control, Automation and System 2007.* Oct 17-20. COEX, Seoul Korea. 2007. pp 1374-1377.

Yamoto, F. and Sekiya, M. Two Evolutionary Paths of an Axisymmetric Gravitational Instability in The Dust Layer of A Protoplanetary Disk. *The Astrophysical Journal.* 2006. Vol 646, pp 155 – 158.

Wakita, S. and Sekiya, M. Numerical Simulations of The Gravitational Instability in The Dust Layer of A Protoplanetary Disk Using A Thin Disk Model. *The Astrophysical Journal.* 2008. Vol 675, pp 1559 – 1575.

National Institute. Hazard Prevention and Control in the Work Environment: Airborne Dust. Arbetslivinstitutet, Sweden. 2005.

Meister, K. Road dust: an overlooked urban pollutant. *Science for Environment Policy*. European Commission. March 15, 2012.

Nikkar, A. and Mighani, M.. Application of He's Variational Method for solving Seventh-Order Differential Equations. *American Journal of Computational and Applied Mathematics* 2(1). (2012). pp 37-40.

Ji-Huan H.. Variational iteration method—Some recent results and new interpretations. *Journal of Computational and Applied Mathematics* 207. (2007). pp 3 – 17.

Jassmin, A. M.. A modified variational iteration method for Schrodinger and Laplace problems. Int. J. Contemp. Math. Sciences. Vol. 7 no. 13.(2012). pp 615 – 624.

Matinfar, M., Saeidy, M. and et. al.. Variational iteration method for exact solution of gas dynamic equation using he's polynomials. Bulletin of Mathematical Analysis and Applications. ISSN:1821-1291 Volume 3 Issue 3. (2011). pp 50-55.

Abdou, M.A. and Soliman, A.A.. Numerical solution of nonlinear evolution equations using variational iteration method. Journal of Computational and Applied Mathematics 207. (2007).pp 111-120.

Abdou, M.A. and Soliman, A.A.. Variational iteration method for solving Burger's and coupled Burger's equations. Journal of Computational and Applied Mathematics 181 (2005). pp 245–251.

Abdou, M.A. and Soliman, A.A.. New application of variational iteration method. Physica D 211(2005). pp 1-8.

Ji-Huan H.. Approximate analytical solution for seepage flow with fractional derivatives in porous media. Comp. Methods Appl. Mech. Engrg. 167 (1998). pp 57-68.

Aslam Noor, M. and Tauseef Mohyud-Din, S.. Variational Iteration Method for Solving Initial and Boundary Value Problems of Bratu-type. World Applied Sciences Journal 13 (11). (2011). pp 2312-2322.

Ji-Huan, H. and Xu-Hong, W.. Variational iteration method: New development and applications. *Computers and Mathematics with Applications* 54. (2007). pp 881–894.

Aslam Noor, M. and Tauseef Mohyud-Din, S.. Variational Iteration Method for Solving Initial and Boundary Value Problems of Bratu-type. *Applications and Applied Mathematics: An International Journal (AAM).* Vol. 3, Issue 1. (June 2008). pp 89 – 99.

Guo-Zhong, Z., Xi-Jun, Yu. and et. al.. Variational iteration method forsolving compressible Euler equations. *Chin. Phys. B* Vol. 19 No. 7. (2010). pp 070203-01-070203-07.

Liu, Y. and N. Kurra, S.. Solution of Blasius Equation by Variational Iteration. *Applied Mathematics:*1(1). (2011).pp 24-27.

Eliasson, B., Shukla, P. K. and Avinash, K.. Dynamics of self-gravitating dust cloudsin astrophysical plasmas. *33rd EPS Conference on Plasma Phys. Rome 19 - 23 June 2006* ECA Vol.30I. (2006). pp 1-4 .

Avinash, K., Eliasson, B. and Shukla, P.K.. Dynamics of self-gravitating dust clouds and the formation of planetesimals. *Physics Letters A* 353. (2006). pp 105–108.

Chang, L. and Xiu-Lian, Z.. Analytical Study of Nonlinear Dust Acoustic Waves in Two Dimensional Dust Plasma with Dust Charge Variation. *Commun. Theor. Phys*. (Beijing, China) 44 . (2005) pp 247–251.

Batiha, K.. Numerical Solutions of the Multispecies Predator-Prey Model by Variational Iteration Method. *Journal of Computer Science* 3 (7) ISSN 1549-3636. (2007). pp 523-527.

Kumar Singh, K. and Gogoi, S.. A self-similar flow of a mixture of a non-ideal gas and small solid particles behind a spherical shock wave in the presence of a gravitational field. *International Journal of Physics and Mathematical Sciences* ISSN: 2277-2111 (Online). (2011). pp 9-21.

Wazwaz, A.M.. (2009). *Partial Differential Equations and Solitary Waves Theory*. Nonlinear Physical Science, Springer.