# On The Use of Non uniform inflow Model for Aerodynamics Analysis Helicopter Rotor Blade in the Forward Flight 

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#### Abstract

The aerodynamics analysis of rotor blade helicopter in forward flight by using a non uniform inflow models are presented. This work can be considered as a continuation from the previous work as described in the Ref.1. Here the linear inflow model would be used as the basic idea in solving the aerodynamic problems oppose with the Ref. 1 which used a uniform inflow model. Physical flow phenomena around a rotor blade helicopter had been recognized very complicated. Rotor blade behaves like a finite wing. The presence of lift upon a finite wing would be followed by wake vortex sheet this vortex sheet promotes induced velocity along span wise non uniformly. Only certain wing plan form namely elliptic wing plan form could generate uniform induced velocity along span wise. For an arbitrary wing plan form the induced velocities were normally non uniform. The shape of wake vortex sheet released from a finite wing relatively simple, the vortex sheet can be considered as a plane of an infinitesimal thickness starting from the trailing edge line goes down in parallel to the free stream flow. In the case of the rotor helicopter, the rotor blade becomes a rotating lifting surface. As result the shape of wake vortex sheet becomes more complex than a fixed finite wing. Hence the non uniformity of the induce velocity become apparent.


## Notation

| $\boldsymbol{C}_{\boldsymbol{T}}$ | Thrust Coefficient |
| :--- | :--- |
| $\boldsymbol{D}$ | Drag |
| $\boldsymbol{L}$ | Lift |
| $\boldsymbol{P}$ | Power |
| $\boldsymbol{Q}$ | Torque |
| $\boldsymbol{R}_{\boldsymbol{B}}$ | Blade Radius |
| $\boldsymbol{U}_{\boldsymbol{T}}$ | Velocity normal to blade leading edge |
| $\boldsymbol{U}_{\boldsymbol{P}}$ | line |
| $\boldsymbol{U}_{\boldsymbol{R}}$ | Out plane velocity |
| $\mathbf{U}_{\infty}$ | Radial Velocity |
| $\boldsymbol{\alpha}$ | Incoming flow velocity |
| $\boldsymbol{\alpha}_{\text {eff }}$ | Angle Of Attack |
| $\boldsymbol{\beta}$ | Effective Angle Of Attack |
| $\boldsymbol{\lambda}_{\text {ave }}$ | Coning Angle |
| $\boldsymbol{\mu}$ | Uniform inflow ratio |
| $\boldsymbol{\psi}$ | Advance ratio |
| $\boldsymbol{\chi}$ | Azimuth angle |

$\varepsilon_{\mathrm{ct}} \quad$ Prescribed value

## I. Introduction

The aerodynamics analysis of rotor blade helicopter in forward flight by using a non uniform inflow models are presented. This work can be considered as a continuation from the previous work as described in the Ref.1. The previous work used uniform inflow model as the basic idea in solving the aerodynamic problems of rotor blade. Using this approach, various rotors had been analyzed in order to identify the influence of rotor blade parameter geometry, flight condition as well as the required numerical parameter imposed by the blade element requirements. Physical flow phenomena around a rotor blade helicopter had been recognized to be very complicated. The rotor blade could be considered as a rotary wing. As result, as the blade rotates, it would be accompanied by the presence of wake vortex sheet. The shape of wake vortex sheet released from a finite wing is relatively a simple. This sheet can be considered as a plane of an infinitesimal thickness started from the trailing edge line goes down stream in the direction parallel to the free stream velocity. While the rotor blade helicopter as a rotary wing would generate the wake vortex sheet with the shape is more complex than its counter part. The wake shape would, at least, at helical form. This vortex sheet in return would promote induced velocities along span wise which it is expected to distribute in non uniform manner. It is, therefore, the use of uniform inflow model seems is not adequate to perform an accurate aerodynamics analysis of the rotor blade helicopter.
There are various model had been introduced to accommodate the variation induced velocity both in longitudinal ( radial direction ) as well as in lateral direction ( azimuth direction) such as presented in Ref. 2 and 3.

The present work introduces the use of non uniform inflow models to be incorporated into the combined Momentum Theory and The Blade Element Theory. For a comparison purposes two types of a rotor blade helicopter were analyzed. The first type relates to the rotor blade which it was assumed no twist and no coning angle and the blade plan form is simple a rectangular. The second type is similar to the first but with the coning angle which varying with respect to the azimuth positions.

The comparison result with a uniform flow model is clearly indicated that all those six type of inflow models provide the average thrust coefficients are relatively lower than the result using a uniform inflow model. If the local thrust coefficient is plotted against the blade azimuth positions, the non uniform flow models shift the locations blade azimuth position at which the thrust coefficient reach maximum or minimum value. However to establish which the most accurate inflow model, further work to compare with experimental result is needed. This represents the suggested for the future work.

## II. Methodology

### 2.1 Theoretical Background

The Momentum Theory and The Blade Element Theory represent two independent methods. Both can be used as a tool for aerodynamics analysis of the rotor blade helicopter. However on the use to those methods independently need additional information. The momentum theory which is not required the detail of rotor blade geometry only can be used if the thrust coefficient is known then a uniform induced velocity cross the rotor disk plane can be deduced. In other way if induced velocity is known then the thrust coefficient can be estimated. The blade element Theory required the detail information of the rotor blade geometry. It is means the method allows to study the influence of rotor blade parameter geometry affected to the aerodynamics performance of the rotor blade helicopter. Those two methods combine together would eliminate the lack of information. The induced velocity would be provided from the Momentum theory which then would be used in The blade element theory in order to obtain the thrust coefficient $\mathrm{C}_{\mathrm{T}}$. An iteration process was required, since the thrust coefficient from the Blade Element Theory would not the same value used by the Momentum Theory. Finally the iterative calculation The Momentum Theory and The Blade Element theory would reach certain condition where the difference value between two successive iteration would not exceed a prescribed value $\varepsilon_{\mathrm{ct}}$. Here one choose $\varepsilon_{\mathrm{ct}}$ as small as possible. However in used of $\varepsilon_{\mathrm{ct}}$ is equal to 0.005 is adequate which that the difference result between two iteration value would not exceed than $2 \%$. The detailed derivation of those two methods can be found in Ref. 2,3 and 4.

For a given a thrust coefficient $C_{T}$, the incoming velocity $U_{\infty}$ and the disk plane angle $\alpha_{\text {TPP }}$ accordingly to the Momentum theory gives the uniform inflow ratio $\lambda_{\text {ave }}$ would $\mathrm{be}^{4}$ :

$$
\lambda_{\text {ave }}=\mu_{x} \operatorname{tg} \alpha+\frac{\mathrm{C}_{\mathrm{T}}}{\sqrt{\mu_{x}^{2}+\lambda_{\text {ave }}^{2}}}
$$

In above equation $\mu_{\mathrm{x}}$ represent the advance ratio parallel to the disk plane. For a given the rotor rotational speed and the rotor blade radius are denoted by $\Omega$ and $\mathrm{R}_{\mathrm{B}}$ respectively. The advance ratio $\mu_{\mathrm{x}}$ is defined as :

$$
\mu_{\mathrm{x}}=\frac{\mathrm{U}_{\infty} \cos \alpha_{\mathrm{TPP}}}{\Omega \mathrm{R}_{\mathrm{B}}}
$$

While the advance ratio perpendicular with respect to the disk plane is denoted by $\mu_{z}$ defined as:

$$
\mu_{\mathrm{z}}=\frac{\mathrm{U}_{\infty} \sin \alpha_{\mathrm{TPP}}}{\Omega \mathrm{R}_{\mathrm{B}}}
$$

The term $\Omega R_{B}$ is called as the blade tip speed $U_{\text {tip }}$.
The equation 1-1 represents the non linear equation in term of the inflow ratio $\lambda_{\text {ave }}$. Hence the solution for the $\lambda$ needs to be done iteratively. Using a Newton Raphson iterative method, the iterated value of $\lambda_{\text {ave }}^{n}$ at the nth iteration would be ${ }^{4}$ :

$$
\lambda_{\text {ave }}^{n}=\lambda_{\text {ave }}^{n-1}-\left|\frac{\mathrm{f}\left(\lambda_{\text {ave }}^{n-1}\right)}{\mathrm{f}^{\prime}\left(\lambda_{\text {ave }}^{n-1}\right)}\right|
$$

Where:

$$
\begin{array}{lr}
\mathrm{f}\left(\lambda_{\text {ave }}\right)=\lambda_{\text {ave }}-\mu_{x} \operatorname{tg}\left(\alpha_{\text {TPP }}\right)-\frac{\mathrm{C}_{\mathrm{T}}}{\sqrt{\mu_{x}{ }^{2}+\lambda_{\text {ave }}{ }^{2}}} & 1-4 \mathrm{a} \\
\mathrm{f}\left(\lambda_{\text {ave }}\right)=1-\lambda_{\text {ave }} \frac{\mathrm{C}_{\mathrm{T}}}{\left(\mu_{x}{ }^{2}+\lambda_{\text {ave }}{ }^{2}\right) \sqrt{\mu_{x}{ }^{2}+\lambda_{\text {ave }}{ }^{2}}} & 1-4 \mathrm{~b}
\end{array}
$$

The initial value of the inflow ratio $\lambda_{0}$ for starting the iteration process is given by:

$$
\lambda_{\text {ave }}^{0}=\sqrt{\frac{\mathrm{C}_{\mathrm{T}}}{2}}
$$

Here one can implement the criteria for finishing the iteration process by the following equation :

$$
\left|\frac{\lambda_{\text {ave }}^{\mathrm{n}}-\lambda_{\text {ave }}^{\mathrm{n}-1}}{\lambda_{\text {ave }}^{\mathrm{n}}}\right| \leq \varepsilon_{\lambda}
$$

The $\varepsilon_{\lambda}$ represents a prescribed value which can be chosen arbitrary. It could be, normally, below 0.005 . If the chosen value $\varepsilon \lambda$ is set equal to 0.005 , then the iteration process would be terminated at the difference value between two successive iteration results would not exceeded more than $0.5 \%$.

To accommodate the variation of inflow ratio in both longitudinal and lateral one can use the variation of inflow model suggested by Glauert ${ }^{4}$ as:
$\lambda(r, \Psi)=\lambda_{\text {ave }}\left(1 \dashv \mathrm{k}_{\mathrm{x}} \mathrm{r} \cos \Psi \dashv \mathrm{k}_{\mathrm{z}} \mathrm{r} \sin \Psi\right)$
In above equation $\mathrm{k}_{\mathrm{x}}$ and $\mathrm{k}_{\mathrm{y}}$ can be viewed as weighting factors and represent the deviations of the inflow from the uniform value predicted by the Momentum theory Eq. 1.1.

There are various attempts had been made to provide the weighting factor $k_{x}$ and $k_{z}$. Table 1.1 bellows shows a summarized of the suggested value of those two weighting factors adopted from Ref. 4.

| Author | $\mathrm{k}_{\mathrm{x}}$ factor | $\mathrm{K}_{\mathrm{z}}$ factor |
| :---: | :---: | :---: |
| Colement et al (1945) | $\tan (\mathrm{X} / 2)$ | 0 |
| $\begin{aligned} & \hline \text { Drees } \\ & (1949) \end{aligned}$ | $\frac{4}{3} \frac{\left(1-\cos X-1.8 \mu_{x}\right)}{\sin X}$ | $-2 \mu_{\mathrm{z}}$ |
| $\begin{aligned} & \text { Payne } \\ & \text { (1959) } \end{aligned}$ | $\frac{4}{3}\left(\frac{\mu_{x}}{\lambda_{\text {ave }}}\right) \frac{1}{\left(1.2+\frac{\left.\mu_{x} / \lambda_{\text {ave }}\right)}{}\right)}$ | 0 |
| White and Blake <br> (1979) | $\sqrt{2} \sin \mathrm{X}$ | 0 |
| Pitt $\&$ <br> Peters  <br> $(1981)$  | $\left(\frac{15}{23} \pi\right) \tan (\mathrm{X} / 2)$ | 0 |
| Howlett (1981) | $\sin ^{2}(\mathrm{X})$ | 0 |
|  |  |  |

Table 2.1 Weighting factor kx and $\mathrm{k}_{\mathrm{z}}$ for non uniform inflow model Eq.1.7 Adopted from Ref. 4.
The variable $\chi$ in table 2.1 above is called as the wake skew angle and is given by :
$\mathrm{X}=\operatorname{arctg}\left(\frac{\mu_{\mathrm{x}}}{\mu_{\mathrm{z}}+\lambda_{\text {ave }}}\right)$
1-8

The inflow ratio $\lambda(r, \Psi)$ was obtained by using Eq. 1.7 , would be as an input for the Blade element Theory. General speaking, The Blade Element Theory is similar to the strip theory commonly use a finite wing aerodynamics. Here the blade was considered as composed of a number of aerodynamically independent cross-sections, whose characteristics are the same as a blade at a proper angle of attack. The lift and drag are estimated at the strip by using 2-D airfoil characteristics accordingly to the local flow velocity. It is, therefore, necessary to determine the magnitude as well as the direction of the airflow in the immediate vicinity of the blade element under consideration.

Suppose that the motion of the rotor blade under consideration has a variable coning angle $\beta$ as function of blade azimuth position $\Psi$ can be written as $^{4}$ :
$\beta(\Psi)=\beta_{0}+\mathrm{A} \cos \beta+\mathrm{B} \sin \beta$
The coefficient $\beta_{0}$, $A$ and $B$ above equation are specified. Different helicopter might have different value coefficients.
The incoming flow velocity $U_{\infty}$ to the disk plane in forward flight, can be resolved into two component velocities, namely the component velocity parallel to the disk plane $\mathrm{U} \infty_{/ /}$, and the component velocity $\mathrm{U} \infty \perp$ which is perpendicular to it. This incoming velocity would superimposed with the angular velocity of the blade $\Omega$, induced velocity $\mathrm{v}_{\mathrm{i}}$, and the velocity which generated by the change of coning angle $\beta(\Psi)$ as the blade rotates to produce the effective resultant velocity sees by the blade section. This resultant velocity can be split into three component velocity. They are namely: the component velocity normal to blade leading edge line $U_{T}$, the radial velocity $U_{R}$ and the out plane velocity $U_{P}$.

Those three component velocities, of course, would be function of blade azimuth position $\Psi$ and distance of blade section $r$ to the axis of rotation. Figure 2.1 show the schematic diagram of velocities work on the rotor blade.


Fig.2.1 Diagram Velocities over the rotor blade helicopter (Leishman, 2000)
The three component velocities as mentioned above can be written, respectively as:

$$
\begin{array}{lr}
\mathrm{U}_{\mathrm{p}}=\left(\mathrm{U}_{\infty} \sin \alpha_{\text {TTP }}+\mathrm{v}_{\mathrm{i}}\right)+\mathrm{r} \frac{\partial \beta}{\partial \mathrm{t}}+\mu_{\mathrm{x}} \Omega \mathrm{R}_{\mathrm{B}} \beta(\Psi) \cos (\Psi) & 1-8 \mathrm{a} \\
\mathrm{U}_{\mathrm{T}}=\Omega \mathrm{r}+\mathrm{U}_{\alpha} \cos \alpha_{\text {TPP }} \operatorname{Cos}(\Psi) & 1-8 \mathrm{~b} \\
\mathrm{U}_{\mathrm{R}}=\mathrm{U}_{\alpha} \cos \alpha_{\mathrm{TTP}} \operatorname{Sin}(\Psi) & 1-8 \mathrm{c}
\end{array}
$$

Let Eq. 1.8 is divided by the tip speed ratio $\Omega R_{B}$ in order to form in non dimensional term as:
$\begin{array}{lr}\frac{\mathrm{U}_{\mathrm{p}}}{\Omega \mathrm{R}_{\mathrm{B}}}=\left(\mu_{\mathrm{z}}+\lambda(\mathrm{r}, \Psi)\right)+\frac{\mathrm{r}}{\Omega \mathrm{R}_{\mathrm{B}}} \frac{\partial \beta}{\partial \mathrm{t}}+\mu_{x} \beta(\Psi) \cos (\Psi) & 1-9 \mathrm{a} \\ \frac{\mathrm{U}_{\mathrm{T}}}{\Omega \mathrm{R}_{\mathrm{B}}}=\frac{\mathrm{r}}{\mathrm{R}_{\mathrm{B}}}+\mu_{\mathrm{x}} \operatorname{Sin}(\Psi) & \\ \frac{\mathrm{U}_{\mathrm{R}}}{\Omega \mathrm{R}_{\mathrm{B}}}=\mu_{\mathrm{x}} \cos (\Psi) & 1-9 \mathrm{~b} \\ \end{array}$
Let consider a typical element or strip shown in the Figure 2.3.


Figure 2.3: Velocity Diagram on the blade section (Leishman, 2000).
This element has a pitch angle equal to $\theta$, which represents the angle between the plane of rotation and the line of zero lift. Many rotor blades are, normally, twisted, so the pitch angle $\theta$ might varies with r. Hence it is necessary to note as $\theta(r)$. If the blade section just sees an in-plane velocity $U_{T}$ only, the pitch angle would be the section angle of attack at that section. The component out plane velocity $U \infty \perp$, and induced inflow velocity vi, would change the flow direction by amounts $\Phi$, as shown in the Figure 2.3 above, namely,
$\Phi=\operatorname{arctg}\left(\frac{\mathrm{U}_{\mathrm{p}}}{\mathrm{U}_{\mathrm{T}}}\right)$
Considering velocity components equation $1-8 \mathrm{a}$ and $1-8 \mathrm{~b}$ for $\mathrm{U}_{\mathrm{p}}$ and UT, both varying in longitudinal (radial direction) as well as in the azimuth direction. As result that the inflow angle $\Phi$ need to be viewed as $\Phi(\mathrm{r}, \Psi)$.
The effective angle of attack $\boldsymbol{a}_{\text {eff }}$, then, can be defined as :
$\alpha_{\text {eff }}(\mathrm{r}, \Psi)=\theta(\mathrm{r})-\Phi(\mathrm{r}, \Psi)$

The airfoil lift and drag coefficients $C_{1}\left(\alpha_{\text {eff }}\right)$ and $C_{d}\left(\alpha_{\text {eff }}\right)$ at this effective angle of attack $\alpha_{\text {eff }}$ may be obtained from look up table of airfoil data. The lift and drag forces will be perpendicular to, and along the apparent stream direction. The effective velocity works on the differential blade element $\Delta \mathrm{r}$, creating a differential lift $\Delta \mathrm{L}(\mathrm{r}, \Psi)$ and differential drag $\Delta \mathrm{D}(\mathrm{r}, \Psi)$ as :

$$
\begin{align*}
& \left.\Delta \mathrm{L}(\mathrm{r}, \Psi)=\frac{1}{2} \rho\left(\mathrm{U}_{\mathrm{p}}^{2}+\mathrm{U}_{\mathrm{T}}^{2}\right)^{2} \mathrm{c}(\mathrm{r})\right) \mathrm{C}_{\downarrow}\left(\alpha_{\mathrm{eff}}\right) \Delta \mathrm{r} \\
& \left.\Delta \mathrm{D}(\mathrm{r}, \Psi)=\frac{1}{2} \rho\left(\mathrm{U}_{\mathrm{p}}^{2}+\mathrm{U}_{\mathrm{T}}^{2}\right)^{2} \mathrm{c}(\mathrm{r})\right) \mathrm{C}_{d}\left(\alpha_{\text {eff }}\right) \Delta \mathrm{r}
\end{align*}
$$

Those two differential forces must be rotated in directions normal to, and tangential to the rotor disk, respectively, and producing the differential thrust $\Delta \mathrm{T}(\mathrm{r}, \Psi)$ and the differential axial force $\Delta \mathrm{F}_{\mathrm{x}}(\mathrm{r}, \Psi)$ as given below :

$$
\begin{align*}
& \Delta \mathrm{T}(\mathrm{r}, \Psi)=(\Delta \mathrm{L} \cos (\Phi)-\Delta \mathrm{D} \sin (\Phi))= \\
& \frac{1}{2} \rho\left(\mathrm{U}_{\mathrm{T}}^{2}+\mathrm{U}_{\mathrm{P}}^{2}\right) \mathrm{c}\left(\mathrm{C}_{1} \cos (\Phi)-\mathrm{C}_{\mathrm{d}} \sin (\Phi)\right) \Delta \mathrm{r} \\
& \Delta \mathrm{~F}_{\mathrm{x}}(\mathrm{r}, \Psi)=(\Delta \mathrm{D} \cos (\Phi)+\Delta \mathrm{L} \sin (\Phi))= \\
& \frac{1}{2} \rho\left(\mathrm{U}_{\mathrm{T}}^{2}+\mathrm{U}_{\mathrm{P}}^{2}\right) \mathrm{c}\left(\mathrm{C}_{\mathrm{d}} \cos (\Phi)+\mathrm{C}_{1} \sin (\Phi)\right) \Delta \mathrm{r}
\end{align*}
$$

The differential torque $\Delta \mathrm{Q}(\mathrm{r}, \Psi)$ and differential power $\Delta \mathrm{P}(\mathrm{r}, \Psi)$ can be obtained, respectively, as :
$\Delta \mathrm{Q}(\mathrm{r}, \Psi)=\mathrm{r} \Delta \mathrm{F}_{\mathrm{x}}(\mathrm{r}, \Psi) \quad 1-14 \mathrm{a}$
and
$\Delta \mathrm{P}(\mathrm{r}, \Psi)=\Omega \mathrm{r} \Delta \mathrm{F}_{\mathrm{x}}(\mathrm{r}, \Psi)=\Omega \Delta \mathrm{Q}(\mathrm{r}, \Psi)$
Finally, the thrust $\mathrm{T}(\Psi)$, torque $\mathrm{Q}(\Psi)$ and power $\mathrm{P}(\Psi)$ may be found by integrating $\Delta \mathrm{T}(\mathrm{r}, \Psi), \Delta \mathrm{Q}(\mathrm{r}, \Psi)$ and $\Delta \mathrm{P}(\mathrm{r}, \Psi)$ above from root to tip ( $\mathrm{r}=0$ to $\mathrm{r}=\mathrm{R}_{\mathrm{B}}$ ), and multiplying the results by the total number of blades $\mathrm{N}_{\mathrm{b}}$ for a given certain blade azimuth position $\Psi$. They are namely: $T(\Psi)=\int_{\mathrm{r}=\mathrm{R}_{0}}^{\mathrm{r}=\mathrm{R}_{\mathrm{B}}} \frac{1}{2} \rho\left(\mathrm{U}_{\mathrm{T}}^{2}+\mathrm{U}_{\mathrm{P}}^{2}\right) \mathrm{c}\left(\mathrm{C}_{1} \cos (\Phi)-\mathrm{C}_{\mathrm{d}} \sin (\Phi)\right) \mathrm{dr}$

$$
\begin{aligned}
& 1-15 \mathrm{a} \\
& \mathrm{Q}(\Psi)=\int_{\mathrm{r}=\mathrm{R}_{0}}^{\mathrm{r}=\mathrm{R}_{\mathrm{B}}} \Delta \mathrm{~F}_{\mathrm{x}}(\mathrm{r}, \Psi) \mathrm{r}= \\
& \int_{\mathrm{r}=0}^{\mathrm{r}=\mathrm{R}_{\mathrm{B}}} \frac{1}{2} \rho\left(\mathrm{U}_{\mathrm{T}}^{2}+\mathrm{U}_{\mathrm{P}}^{2}\right) \mathrm{c}\left(\mathrm{C}_{\mathrm{d}} \cos (\Phi)+\mathrm{C}_{\mathrm{l}} \sin (\Phi)\right) \mathrm{rdr} \\
& \mathrm{P}(\Psi)=\int_{\mathrm{r}=\mathrm{R}_{0}}^{\mathrm{r}=\mathrm{R}_{\mathrm{B}}} \Delta \mathrm{~F}_{\mathrm{x}}(\mathrm{r}, \Psi) \Omega \mathrm{r}= \\
& \int_{\mathrm{r}=0}^{\mathrm{r}=\mathrm{R}_{\mathrm{B}}} \frac{1}{2} \rho\left(\mathrm{U}_{\mathrm{T}}^{2}+\mathrm{U}_{\mathrm{P}}^{2}\right) \mathrm{c}\left(\mathrm{C}_{\mathrm{d}} \cos (\Phi)+\mathrm{C}_{1} \sin (\Phi)\right) \Omega \mathrm{rdr} \\
& 1-15 \mathrm{c}
\end{aligned}
$$

The average thrust $\mathrm{T}_{\text {ave }}$, torque $\mathrm{Q}_{\text {ave }}$ and power $\mathrm{P}_{\text {ave }}$ can be found by integrating $\mathrm{T}(\Psi), \mathrm{Q}(\Psi)$ and $\Delta \mathrm{P}(\Psi)$ from $\Psi=0^{0}$ to $\Psi=360^{\circ}$ as:

$$
\begin{align*}
& \mathrm{T}_{\text {ave }}(\Psi)=\frac{1}{2 \pi} \int_{0}^{2 \pi}\left(\int_{\mathrm{r}=\mathrm{R}_{0}}^{\mathrm{r}=\mathrm{R}_{\mathrm{B}}} \frac{1}{2} \rho\left(\mathrm{U}_{\mathrm{T}}^{2}+\mathrm{U}_{\mathrm{P}}^{2}\right) \mathrm{c}\left(\mathrm{C}_{\mathrm{l}} \cos (\Phi)-\mathrm{C}_{\mathrm{d}} \sin (\Phi)\right) \mathrm{dr}\right) \mathrm{d} \Psi \\
& \mathrm{Q}_{\text {ave }}(\Psi)=\frac{1}{2 \pi} \int_{0}^{2 \pi}\left(\int_{\mathrm{r}=0}^{\mathrm{r}=\mathrm{R}_{\mathrm{B}}} \frac{1}{2} \rho\left(\mathrm{U}_{\mathrm{T}}^{2}+\mathrm{U}_{\mathrm{P}}^{2}\right) \mathrm{c}\left(\mathrm{C}_{\mathrm{d}} \cos (\Phi)+\mathrm{C}_{1} \sin (\Phi)\right) \mathrm{rdr}\right) \mathrm{d} \Psi
\end{align*}
$$

and

$$
\mathrm{P}_{\text {ave }}(\Psi)=\frac{1}{2 \pi} \int_{0}^{2 \pi}\left(\int_{\mathrm{r}=0}^{\mathrm{r} \mathrm{R}_{\mathrm{B}}} \frac{1}{2} \rho\left(\mathrm{U}_{\mathrm{T}}^{2}+\mathrm{U}_{\mathrm{P}}^{2}\right) \mathrm{c}\left(\mathrm{C}_{\mathrm{d}} \cos (\Phi)+\mathrm{C}_{\mathrm{l}} \sin (\Phi)\right) \Omega \mathrm{rdr}\right) \mathrm{d} \Psi
$$

The above integration can, in general, be only numerically done since the chord c , the sectional lift and drag coefficients may vary along the span wise as well as in the azimuth direction. The inflow velocity $\mathrm{v}_{\mathrm{i}}$ depends on T . Thus, an iterative process will be needed to find the quantity $v_{i}$. The iteration process would be accomplished, if the difference value between two successive iteration for the average thrust coefficient $\mathrm{C}_{\mathrm{T}}^{\mathrm{i}+1}$ and $\mathrm{C}_{\mathrm{T}}^{\mathrm{i}}$ defined as:
$\left|\frac{\mathrm{C}_{\mathrm{T}}^{\mathrm{i}+1}-\mathrm{C}_{\mathrm{T}}^{\mathrm{i}}}{\mathrm{C}_{\mathrm{T}}^{\mathrm{i}+1}}\right| \leq \varepsilon_{\mathrm{ct}}$
In the right hand side above equation $\varepsilon_{\mathrm{ct}}$ represents an arbitrary prescribed value which here may one can chose to be equal 0.005.

### 2.2 Numerical Procedure

For the purpose of aerodynamic rotor blade helicopter, this approach would require data for the rotor blade parameter geometry and the flight condition. The first data would cover: chord distribution $\mathrm{c}(\mathrm{r})$, twist distribution $\theta(\mathrm{r})$, aerodynamic airfoil data for the blade section $\left(\mathrm{c}_{+}(a) \quad\right.$ and $\mathrm{c}_{d}(a)$, coning angle as function of blade azimuth position $\beta(\Psi)$, blade radius $\mathrm{R}_{\mathrm{B}}$ and blade number $\mathrm{N}_{\mathrm{B}}$. While the second data involved the incoming flow velocity $\mathrm{U}_{\infty}$, the angle of attack with respect to the disk plane $a_{\text {TPP }}$, and the rotational speed rotor blade $\mathrm{N}_{\text {RPM }}$. Additional data as numerical parameter calculation need to be supplied, namely : the number of blade division for each blade $\mathrm{N}_{\mathrm{BE}}$, the number of azimuth division to complete on rotational calculation $N_{R}$ and two others prescribed value for checking the convergence $\varepsilon_{\lambda}$ and $\varepsilon_{\mathrm{ct}}$. The prescribed value $\varepsilon_{\lambda}$ is the value to establish that the iteration process for determining the average inflow ratio $\lambda_{\text {ave }}$ was completed. While the prescribed value $\varepsilon_{\mathrm{ct}}$ in conjunction with iteration process for the thrust coefficient.

The implementation of the combined Momentum theory and the blade Element theory for the aerodynamics rotor blade helicopter analysis can be described as shown by the flow diagram as depicted in the Figure 2.4.

Given necessary input as described above, introduce the initial value of the thrust coefficient. Use this value to calculate the average inflow ratio $\lambda_{\text {ave }}$ by using the momentum theory Eq. 1.1 through an iteration process. As the average inflow ratio $\lambda_{\text {ave }}$ available calculate the local inflow ratio $\lambda(r, \Psi)$ as defining by Eq. 1.7 at each predetermining blade section $r$ and the blade azimuth position $\Psi$. The required wake skew angle $\chi$ can be obtained by using one of skew angle model as presented in the table 1.1. The resultant velocity and the effective angle of attack, then, can be obtained through out solving Eq. 1-9 to Eq. 1-11. Knowing the effective angle of attack $\alpha_{\text {eff }}(\mathrm{r}, \Psi)$, then, by using look up table airfoil data, the local lift coefficient and drag coefficient can be obtained. Finally implement the numerical integration along blade span, the thrust coefficient, torque coefficient as well as power coefficient at particular each blade azimuth can be found. Similar calculation need to be carried out for different blade azimuth position until a prescribed number of blade azimuth position $N_{R}$ are completed. Sum up those obtained thrust coefficient for those $N_{R}$ positions. The summation result, then, divided by $\mathrm{N}_{\mathrm{R}}$ would give the average thrust coefficient. The average torque and power coefficient could be done in similar way as in determining the average thrust coefficient. This average thrust coefficient, of course, would not have the same value as it used in the beginning calculation to determine the average inflow ratio according to the momentum theory. Recalculation is, therefore, needed. This would be repeated until two successive iteration value of the average thrust equation as defined by Eq. 1-17 is satisfied.

## III. Discussion and Result

For the purpose of comparison result between a uniform inflow model and non uniform inflow model, the rotor blade parameter geometry, flight condition as well as the required numerical parameters had been selected to have data as follows:

1. Blade number $\mathrm{N}_{\mathrm{B}}: 2$
2. Blade radius $\mathrm{R}_{\mathrm{B}}: 6$ meters
3. Uniform blade chord $\mathrm{c}(\mathrm{r}): 0.4$ meter
4. Rotational speed of the rotor blade : 400 Rpm
5. Angle of attack with respect to the rotor disk plane $\alpha_{\text {TPP }}=8^{0}$
6. Number of blade element $\mathrm{N}_{\mathrm{BE}}: 40$
7. Number of division of blade azimuth position $\mathrm{N}_{\mathrm{R}}: 60$
8. Inner blade radius for starting the Blade Element theory applied $\mathrm{R}_{0}=0.1$ meter
9. The Prescribed convergence value of the average inflow ratio $\varepsilon_{\lambda}=0.005$
10. The prescribed convergence value of the thrust coefficient $\varepsilon_{\mathrm{ct}}=0.005$.

Other required data would be added as the discussions proceed.

## Case the untwisted blade

Here the rotor blade was assumed to have a uniform pitch angle $\theta(\mathrm{r})=8^{0}$ and there is no conning angle $\left(\beta(\Psi)=0^{0}\right)$. The aerodynamics characteristic of the airfoil section defined to follow as example given in Ref. 4 as :

$$
\begin{aligned}
c_{+}(a) & =2 \pi a \\
c_{d}(\alpha) & =0.1+0.025 \alpha+0.65 \alpha^{2}
\end{aligned}
$$

$$
1-17 a \text { in radian. }
$$

This simple rotor blade model was chosen in order to give a better view in considering the difference result might be appeared between a uniform and non uniform inflow models. Instead of that, this suggested simple blade model was ,also, designed for the purpose of comparison result among various model of non uniform inflow model as presented in the table 1.1.

The incoming velocity of forward flight was assumed $50 \mathrm{~m} / \mathrm{sec}$. With the rotor rotational speed at 400 RPM and the disk plane angle of attack $\alpha_{\text {TPP }}=8^{0}$. The problem in hand would correspond to the advance ratio at $\mu_{\mathrm{x}}=0.0197$. All calculations would be presented here used the same initial value for the thrust coefficient, $\mathrm{C}_{\mathrm{T} 0}=0.002$. The comparison result in term of the average thrust coefficient, the torque coefficient and the required number of iteration can be summarized as shown in the table 3.1 bellows:

| Inflow model | Thrust <br> Coef. <br> $\mathrm{C}_{\mathrm{T}}$ | Torque <br> Coef. <br> $\mathrm{C}_{\mathrm{Q}}$ | Iterati <br> on <br> numb <br> er |
| :--- | :---: | :---: | :---: |
| Uniform | 0.00684 | 0.00546 | 6 |
| Non uniform : <br> Colement et al <br> $(1945)$ | 0.00655 | 0.00526 | 6 |
| Non uniform : <br> Drees (1949) | 0.00666 | 0.00534 | 6 |
| Non uniform : <br> Payne (1959) | 0.00641 | 0.00516 | 6 |
| Non uniform : <br> White and <br> Blake (1979) | 0.00630 | 0.00508 | 6 |
| Non uniform : <br> Pitt \& Peters <br> $(1981)$ | 0.00626 | 0.00504 | 6 |
| Non uniform : <br> Howlett <br> $(1981)$ | 0.00647 | .00521 | 6 |

Table 3.1: Comparison result of thrust coefficient and torque coefficient between various inflow models.
Figure 3.1 showed the comparison result of thrust coefficient $\mathrm{C}_{\mathrm{T}}$ as function of blade azimuth position between a uniform flow model and various forms of non uniform inflow model as listed in the table 3.1 above. While Figure 3.2 showed the comparison in term of torque coefficient $\mathrm{C}_{\mathrm{Q}}$.

Those two figures show that the all non uniform flow models shift the peak of thrust coefficient at the blade azimuth position greater than a uniform flow model. The Coleman model give result closed to the uniform flow model, while the Pitt \& Peters model and White \& Blake model give the lowest curves. The Howlett model give $\mathrm{C}_{\mathrm{T}}$ curve in between among them.


Figure 3.1: Comparison result thrust coefficient $\mathrm{C}_{\mathrm{T}}$ as function blade azimuth position between a uniform and non uniform flow models


Figure 3.2: Comparison result torque coefficient $\mathrm{C}_{\mathrm{Q}}$ as function blade azimuth position between a uniform and non uniform flow models

## Case the Untwisted Blade with Coning Angle.

The following test case used a similar configuration of rotor blade in the previous test case. However the blade had been set to have certain a coning angle which is varying as function of blade azimuth position as given by:

$$
\beta(\Psi)=6^{0}-4^{0} \cos (\Psi)-4^{0} \sin (\Psi) \quad 1-18
$$

The blade coning angle as given by Eq. 1-18 was adopted from Ref. 4. The previous result showed that the Howlett model give a moderate result compared to another 6 inflow models. Hence this model had been selected as the model would be used in this test case.

Table 3.2 shows the comparison result of thrust coefficient between a uniform inflow model and non uniform inflow modeled by Howlett ${ }^{4}$ for the case with and without blade coning angle.

| Blade model | Inflow <br> model | Thrust coef. <br> $\mathrm{C}_{\mathrm{T}}$ |
| :--- | :--- | :--- |
| No conning angle | uniform | 0.00684 |
| No conning angle | Howlett <br> model | 0.00647 |
| Coning angle Eq. 1-18 | uniform | 0.00686 |
| Coning angle Eq. 1-18 | Howlett <br> model | 0.00650 |

Table 3.2 A comparison result of thrust coefficient between two inflow model
Considering above result as shown in The table 3.2 indicates that the coning angle as given by Eq. 1-18 do not give a significant effect to the thrust coefficient.

The Comparison result in term of thrust coefficient $\mathrm{C}_{\mathrm{T}}$ and torque coefficients presented as function of blade azimuth position for those two models as shown in the Figure 3.3 and Fig. 3.4 respectively. The result showed that the distribution of the local thrust coefficient as well as torque coefficient change significantly although in term of the average quantities as shown in the table 3.2 do not show so much different.


Figure 3.3 Comparison result of thrust coefficient $\mathrm{C}_{\mathrm{T}}$ as function of blade azimuth position between a uniform inflow


Figure 3.4 Comparison result of thrust coefficient $\mathrm{C}_{\mathrm{T}}$ as function of blade azimuth position between a uniform inflow model and Howlett inflow model.

## IV. Conclusion and Future work

The computer code for aerodynamics analysis of the rotor blade helicopter had been successfully developed. Six types of non uniform inflow models had been incorporated into the code. This would give an option for the user to select which inflow model would be used. In term of distribution thrust coefficient $\mathrm{C}_{\mathrm{T}}$ plotted against the blade azimuth position, the non uniform inflow model tends to produce a lower $\mathrm{C}_{\mathrm{T}}$ compared to the uniform flow model. Coning angle seems to give a significant effect to the performance of rotor blade helicopter. The shape of curve thrust coefficient $\mathrm{C}_{\mathrm{T}}$ completely differ with the result if there is no a coning angle.

A comparison with the experimental result to validate the present code is required. This would be useful in order to measure the degree of accuracy of the code and also to identify which inflow model offering the best fit to the experimental result. This represents the suggestion to the future work need to be carried out.

## V. References

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