# A STUDY ON THE CHARACTERISTICS OF RAINFALL DATA AND ITS PARAMETER ESTIMATES

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# A STUDY ON THE CHARACTERISTICS OF RAINFALL DATA AND ITS PARAMETER ESTIMATES

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Especially dedicated to my parents

Arumugam Munusamy

&

Sukla Arumugam,

and my sister, Zena

for their continuous support and encouragement

at all times,

also to

all who have been a source of guidance and inspiration to me

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# ABSTRACT

The modeling of the rainfall process has been of interest in simulation studies to assess its impact in the fields of agriculture, water management and others. The rainfall data series used has been obtained from the Malaysian Drainage and Irrigation Department for a 33 year period from 1975 to 2007. The purpose of this study is to investigate the Tweedie family of distributions and determine the appropriate distribution to model the rainfall data using the parameter estimated. Based on the estimated parameter, it was suggested that the Gamma distribution which is a special case of the Tweedie family of distributions is suitable to model the rainfall data instead of the Poisson-gamma distribution. Consequently, a combination of a first order Markov chain and gamma distribution function is identified to model the rainfall process of occurrence and amount separately on a monthly timescale for the ten selected rain gauge stations across Peninsular Malaysia. These model parameter estimates were obtained using the method of maximum likelihood. Conversely, during the estimation of these model parameters certain general characteristics were revealed. Firstly, the transitional probability of a wet day to a wet day was higher but parallel to the transition from a dry day to a wet day. This characteristic revealed the linear relationship between the transitional probabilities and the monthly fraction of wet days. Secondly, the $\beta$ parameter in the gamma distribution functions used to describe the amount of rainfall, is related to the monthly amount of rain per wet day. Therefore, a short method is proposed using the regression technique to estimate the model parameters from these monthly summaries. The relative error analysis revealed that there was no significant difference between the long and short method parameter estimates. Hence, this short method would be very useful in cases where there is a lack of detailed daily rainfall data available.

# ABSTRAK

Kajian simulasi amat menitikberatkan pemodelan proses hujan bagi menilai kesannya terhadap bidang pertanian, pengurusan air dan lain-lain. Rekod data curahan hujan sepanjang tempoh 33 tahun dari tahun 1975 hingga 2007 yang diperolehi daripada Jabatan Pengairan dan Saliran Malaysia digunakan dalam kajian ini. Tujuan kajian ini adalah untuk menyiasat taburan keluarga Tweedie bagi menentukan taburan yang paling sesuai bagi memodelkan data curahan hujan menerusi anggaran parameter taburan. Berdasarkan parameter yang dianggarkan, didapati bahawa taburan gamma yang merupakan kes khas taburan keluarga Tweedie merupakan taburan paling sesuai bagi memodelkan data curahan hujan berbanding taburan Poisson-gamma. Maka, gabungan rantai Markov order satu dan taburan gamma dikenal pasti bagi memodelkan proses hujan iaitu kejadian hujan serta jumlah hujan secara berasingan untuk setiap bulan bagi sepuluh stesen curahan hujan yang dipilih dari seluruh semenanjung Malaysia. Kaedah “maximum likelihood” digunakan bagi memperolehi anggaran parameter model. Proses anggaran parameter model ini telah mendedahkan beberapa ciri-ciri umum. Pertama sekali, didapati bahawa kebarangkalian peralihan dari hari hujan ke hari hujan lebih tinggi tetapi selari dengan kebarangkalian peralihan dari hari kering ke hari hujan. Daripada ciri ini telah dapat disimpulkan bahawa wujudnya hubungan linear diantara anggaran kebarangkalian peralihan dan pecahan hari hujan bulanan. Seterusnya, parameter $\beta$ daripada taburan gamma yang menggambarkan jumlah hujan, mempunyai hubung kait dengan min jumlah hujan pada hari-hari yang hujan. Maka, dengan mengaplikasikan teknik regression, satu kaedah pendek dicadangkan bagi menganggarkan parameter model dari statistik data curahan bulanan dan bukannya data curahan harian. Selepas itu, ralat relatif bagi anggaran parameter menggunakan kaedah panjang serta pendek dikira, membuktikan bahawa tidak terdapat perbezaan yang signifikan diantara kedua kaedah tersebut. Maka secara umumnya, kaedah pendek ini amat berguna bagi kes dimana data harian curahan didapati tidak lengkap.

# TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF FIGURES

## LIST OF ABBREVIATIONS

| | | |
|---|---|---|
| CV | - | Coefficient of Variation |
| AIC | - | Akaike's Information Criterion |
| BIC | - | Bayesian Information Criterion |
| CGF | - | Cumulant Distribution Function |
| NSRP | - | Neyman–Scott Rectangular Pulses |
| CERES | - | Crop Estimation through Resource and Environmental Synthesis |
| LAD | - | Least Absolute Deviation |
| SPI | - | Standardized Precipitation Index |
| NDVI | - | Normalized Difference Vegetation Index |
| EDM | - | Exponential Dispersion Models |
| GLM | - | Generalized Linear Models |
| MLE | - | Maximum Likelihood Estimate |
| pdf | - | Probability density function |

# LIST OF SYMBOLS

| | | |
|---|---|---|
| $\mu$ | – | Mean |
| $\sigma$ | – | Standard Deviation |
| $k$ | – | Kurtosis |
| $p$ | – | Index parameter |
| $\alpha$ | – | Shape parameter for gamma distribution |
| $\beta$ | – | Scale parameter for gamma distribution |
| $p_{00}$ | – | Transition probability from a dry day to a dry day |
| $P(W\|D)$, $p_{01}$ | – | Transition probability from a dry day to a wet day |
| $p_{10}$ | – | Transition probability from a wet day to a dry day |
| $P(W\|W)$ , $p_{11}$ | – | Transition probability from a wet day to a wet day |
| $a$ | – | Intercept parameter for simple linear regression method |
| $b$ | – | Slope parameter for simple linear regression method |

# LIST OF APPENDICES

# CHAPTER 1

# INTRODUCTION

## 1.1 Introduction

Malaysia is a Southeast Asian country straddling the South China Sea. Due to Malaysia's proximity to the equator, it has a tropical rainforest climate. Thus, Peninsular Malaysia experiences summer all year round including two monsoon seasons which largely contribute to the amount of rainfall observed in Peninsular Malaysia. The country experiences the Northeast monsoon during November to February and the Southwest monsoon in the months from May till August. The Northeast monsoon brings with it heavy rain spells along the east coast of Peninsular Malaysia while the Southwest monsoon leaves most parts of the Peninsula dry.

Due to Malaysia's tropical rainforest climate there isn't an extremely dry season and every month has a mean precipitation amount of at least 60 millimeters. It is suggested that rainfall is frequent in this region. Therefore, it is necessary to study the rainfall process by modeling the rainfall data. Rainfall modeling has been widely studied in various disciplines such as climatology, meteorology and hydrology towards facilitating a better understanding of rainfall pattern and its characteristics. The models obtained have been useful in various fields. For example, water resource planning, water management and, agricultural planning. It has also been studied to aid in drought and flood prediction, rainfall runoff modeling, crop growth studies, impact of climate change studies and become beneficial to numerous other fields of study. Thus, rainfall modeling plays a crucial role in many fields that are either directly or indirectly impacted by the rainfall process.

The rainfall data is a unique data series since it consist of zeroes and non-zero values. Zero values indicate non-rainy days while the non-zero values represent the rainy days. Due to the characteristic of the rainfall data, two different models have been used in modeling rainfall data separately. The first model is denoted as rainfall occurrences which are used to model the sequence of wet and dry days while the rainfall amount is used to describe the amount of rainfall observed during a wet day. Recently it was discovered that the Tweedie family distribution which contains the Poisson-gamma distribution with a fixed range of the index parameter value, could model the rainfall process of occurrence and amount simultaneously. Hence this study will investigate the Tweedie family distributions to identify the best distribution that can be used to model the monthly rainfall data series at a few selected rain gauge stations in Peninsular Malaysia.

## 1.2 Background of Study

The modeling of monthly rainfall data can be performed via two known approaches firstly the modeling of rainfall occurrence and rainfall amount simultaneously. Secondly the modeling of rainfall occurrence and rainfall amount using two separate models for each process. Modeling rainfall occurrence and amount can be carried out on numerous time scales ranging from hourly to daily to monthly or even seasonally based on the rainfall data in hand. In this study, the approach shall be to model rainfall occurrence and amount on the monthly timescale at selected rain gauge stations in Peninsular Malaysia.

Commonly, there have been numerous studies to model rainfall occurrence (Gabriel and Neumann, 1962; Stern and Coe, 1984; Deni et al., 2009) and there have also been various studies on modeling rainfall amount (Aksoy, 2000; Srikanthan and Pegram, 2009; Hussain et al., 2010). Then there have been research on the modeling of both rainfall occurrence and also amount using separate models. And in the recent years studies have suggested the modeling the rainfall process of occurrence and amount simultaneously using the Poisson-gamma distribution which belongs to the Tweedie family distributions (Dunn, 2004).

Accordingly the Poisson-gamma distribution that belongs to the Tweedie family distributions has been capable of modeling rainfall occurrence and amount simultaneously when the index parameter estimate for the rainfall data analyzed fell between (1,2) with a 95 percent confidence interval. Exploring the Tweedie family of distributions in studies has been due to its unique characteristics of assigning the appropriate distribution based on the value of its index parameter estimate. This special characteristic makes the Tweedie family of distributions interesting to be studied in order to obtain the parameter estimate and determine the appropriate distribution suitable to model the rainfall data. The Tweedie distribution contains the Poisson-gamma (Tweedie) distribution and also a few distributions that are known as its special cases.

The special cases of the Tweedie family of distributions are dependent on the value of the index parameter estimate of the data series. Hence these special cases include the Normal distribution (index parameter equals to 0), Poisson distribution (index parameter equals to 1), Gamma distribution (index parameter equals or greater than 2), the inverse Gaussian distribution (index parameter equals to 3) and the Tweedie distribution better known as the Poisson-gamma distribution (index parameter values between 1 and 2). Meanwhile there isn't any distribution that exists for index parameter values between 0 and 1 (Jorgensen, 1987).

Consequently, the concept of this study is to investigate the rainfall data through the estimation of the index parameter for the Tweedie family distributions. Followed by the fitting the rainfall data to an appropriate distribution within the Tweedie family of distributions based on the index parameter estimates obtained. Thus in the nutshell, this study shall model the rainfall process of occurrence and amount for the rainfall data. So the study will investigate the characteristic of the rainfall data series in order to determine the type of distribution from the Tweedie family distributions which is suitable to model the rainfall data based on the index parameter estimate obtained.

Conversely, during the evaluation of the model parameters certain general characteristics were revealed. Firstly the transitional probability of a wet day to a wet day is higher but parallel to the transition from a dry day to a wet day. This lead to a linear relationship discovered between the transitional probabilities and the monthly fraction of wet days. Secondly, the $\beta$ parameter in the gamma distribution functions used to describe the amount of rainfall, is related to the monthly amount of rain per wet day. Therefore, a short method is proposed using the regression technique to estimate the model parameters from these monthly summaries.

## 1.3 Statement of Problem

The Tweedie family of distributions contains the Poisson-gamma distribution that is able to model rainfall occurrence and amount simultaneously. In recent years, the Poisson-gamma or Tweedie distribution has been used to model the rainfall process successfully in various studies (Dunn, 2004). However the suitability of the application of the Poisson-gamma distribution to the Malaysia rainfall data series is yet to be studied in detail. Thus, the purpose of this study is to investigate the Tweedie family of distributions to determine if the rainfall data being studied can be modeled using the Poisson-gamma (Tweedie) distribution contained within. Hence, the estimation of the index parameter will allow for the identification of the suitable distribution from the

Tweedie family of distributions that can be used to model the monthly rainfall data. So we can determine based on the index parameter estimate whether the Poisson-gamma (Tweedie) distribution can fit the rainfall data.

Since one of the loopholes on modeling rainfall data is the unavailability of long series of historical daily rainfall data thus a method to estimate the parameter estimates using the monthly averages is very much beneficial to research on rainfall process modeling. Hence, based on certain general characteristics that appear to exist between model parameters and the monthly statistics of the rainfall data reveals a simple method for estimating model parameters. This proposed simple method will be known as the short method. The short method will be beneficial to those areas with restricted historical rainfall data series.

## 1.4 Research Objectives

The objectives of this study are :

1. To determine the characteristics of rainfall occurrence and rainfall amount.
2. To model the rainfall data series using the appropriate distribution from the Tweedie family of distributions
3. To propose an alternative method to obtain the model parameter of rainfall occurrence and amount using the technique of regression.

## 1.5 Scope of Study

This study will model the rainfall occurrence and amount for the rainfall data being studied. Thus the study will investigate the rainfall data to determine the index parameter estimate so as to determine the appropriate distribution that fits the data from the Tweedie family of distributions. Also the analysis will reveal whether the rainfall data can be modeled using the Poisson-gamma (Tweedie) distribution in the Tweedie family of distributions.

Monthly rainfall data from 10 selected rain gauge stations across Peninsular Malaysia over a period of 33 years (1975-2007) will be used in this study to obtain an appropriate monthly rainfall model at each station for rainfall occurrence and rainfall amount. A threshold value of 1mm shall be used in the study to categorize a dry or wet day as any amount of rainfall below the threshold value will be classified as a dry day with no rainfall and vice versa. This study shall identify a rainfall model, based on the Tweedie family of distribution that shall be used to model the monthly rainfall process to resemble the actual rainfall process as closely as possible.

## 1.6 Significance of the Study

In this study the rainfall process shall be modeled based on the appropriate distribution identified after the parameter estimate is obtained from the Tweedie family of distributions. Thus rainfall occurrence and rainfall amount shall be modeled simultaneously if the parameter estimate obtained belongs to the Poisson-gamma (Tweedie) distribution otherwise rainfall occurrence and amount shall be modeled separately using appropriate distributions. Hence through this study the appropriateness of modeling the rainfall data using the Tweedie distribution shall be investigated. Besides that through this study a comprehensive model for rainfall occurrence and amount shall be obtained.

The short method proposed using the linear relationship between the model parameters and the monthly summaries of the rainfall data is a method that can aid in the modeling of rainfall data where there are scarce resources of long series of daily rainfall data. Besides that, through this short method weather simulation models can be applied to those areas that were previously untouchable as there weren't enough rainfall data series to obtain the model parameter and consequently obtain the model and apply these results on simulation studies.

Water resources around the globe are being severely depleted in recent years due to climate change, population growth, pollution and so on. Thus being a form of freshwater, rainwater replenishes these depleting water resources around the globe that can be harvested for use in many ways. Hence rainfall modeling is a part of the water resource management initiative (Al-Qinna et al., 2011), can aid in water cycle planning and management (Wilks, 1999). Besides that, modeling has been known to have frequent inputs for agriculture (Semenov and Porter, 1995), hydrological models and is also essential to climate change studies (Wilks, 1999; Fowler et al., 2005) and it can also provide clues towards discerning climate change (Brissette et al., 2007) at the present moment and also for future events. Hence, this study will contribute to these fields in the future.

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