THE STABILITY OF A CROSS ROLLS IN A RAYLEIGH-BENARD CONVECTION IN A POROUS MEDIA OF AN INFINITE EXTENT.

NUR HAMIZAH BINTI NORIZAN

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Universiti Teknologi Malaysia

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To my beloved parents,

Norizan bin Md Amin and Nona Asiah bt Daud,

My sisters,brothers and friends,

Thanks for your support.

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ABSTRACT

The process of flow through porous media is of interest to a wide range of engineers and scientists. The convective flow in porous media is one of the main topics of heat transfer which has been investigated in the last several decades. Porous medium is a material that contains the pores. It also has the skeletal portion of the material which is called the “matrix” or “frame”. The instability of a fluid layer which is confined between two thermally conducting plates, and is heated from below to produce a fixed temperature difference is called Rayleigh Bĕnard convection. The aim of this study is to analyze small perturbation effects due to the leading convection term. The method of weakly nonlinear analysis is used to determine the convection threshold. The amplitude shows that the bifurcation is the stable branches and bifurcation of amplitude clearly shows similarity to linear distribution. The method of Runge-Kutta is used to determine the bifurcation of the cross rolls. The rolls with the higher amplitude will prevail.

ABSTRAK

Proses aliran dalam sesuatu medium yang mempunyai liang adalah sangat penting kepada jurutera dan ahli saintis. Aliran pemanasan dalam medium yang mempunyai liang adalah topik utama pemindahan haba yang telah diteliti sejak beberapa dekat dahulu. Sesuatu medium yang berliang adalah bahan yang mempunyai bahagian yang dipanggil "matriks" atau "bingkai". Suatu sistem yang mempunyai lapisan bendalir yang terkurung di antara dua plat haba dipanaskan dari bawah adalah untuk menggalakan perolakan yg bakal berlaku. Tujuan utama kajian ini adalah untuk menganalisis kesan yang berlaku terhadap sistem apabila terdapat perubahan ϕ. Kaedah *weakly nonlinear analysis* digunakan untuk menentukan perolakan yang berlaku terhadap sistem. Kaedah *Runge-Kutta* juga digunakan untuk melihat perubahan yang berlaku diantara dua sistem. Sistem yang mempunyai amplitud yang lebih tinggi akan digunakan.

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LIST OF SYMBOLS

R_a	Rayleigh Number
g	acceleration due to gravity
α	thermal expansion coefficient
β	coefficient of thermal expansion
T_0	temperature of horizontal lower bound
T_1	temperature of horizontal upper bound
h	depth of the layer of the fluid
κ	thermal diffusivity
ν	kinematic viscosity
ϕ	porosity of porous medium
μ	dynamic viscosity of fluid
K	permeability
P	pressure
T_s	temperature of solid
T_f	temperature of fluid
q'''	rate of internal heat per unit volume
ρ	fluid density
c	specific heat of incompressible substance
c_p	specific heat at constant pressure
ε	amplitude
A_p	pore volume

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CHAPTER 1

INTRODUCTION

1.1 Background of the Study

A porous media is a material that contains the pores. It also has the skeletal portion of the material which is called the “matrix” or “frame”. The pores are typically filled with a fluid (liquid, gas or both) and the skeletal material is usually a solid, but the structures like for example, foams are often usefully analyzed by using the concept of porous media. So, the concept of porous media is encountered in many engineering disciplines. For examples, the civil engineering deals with the flow of water in aquifers, the movement of moisture through and under engineering structures, transport of pollutants in aquifers and the propagation of stresses under foundations of structures. Furthermore, agricultural engineering deals, for example, with the movement of water and solutes in the root zone in the soil. In all these examples, one or more extensive quantities are transported through the solid or the fluid phases that together occupy a porous medium domain. To solve a problem of transport in such a domain means to determine the spatial and temporal distributions of state variables which is velocity, mass density and pressure of a fluid phase, concentration of a solute and stress in the solid skeleton, that have been selected to describe the state of the material system occupying that domain.

The measurement for porous media is called porosity, denoted by φ, is defined as the fraction of the total volume of the total volume of the medium that is occupied by void space. Thus $1 - \varphi$is the fraction that is occupied by solid. For an isotropic medium the "surface porosity" which is, the fraction of void area to total area of a typical cross section will normally be equal to φ. In defining φ in this way we are assuming that all the void space is connected. If in fact one has to deal with a medium in which some of the pore space is disconnected from the remainder, then one has to introduce an "effective porosity," defined as the ratio of connected void to total volume.

In this study, the heat transfer is one of the most important problems. Heat transfer is a process by which internal energy from one substance transfer to another substance. Under the kinetic theory, the internal energy of a substance is generated from the motion of the molecules. Then, the heat energy is the form of energy which transfers this energy from one system to another. This heat transfer can take place in three ways which are conduction is when heat flows through a heated solid, convection is when heated particles transfer heat to another substance, such as coking something in boiling water and the radiation is when the heat is transferred through electromagnetic waves.

Phenomenon of fluid dynamics which occurs when there is a temperature difference within a liquid or gas is called convection cell. Fluids are materials which exhibits the property of flow. When a volume of fluid is heated, it expands and becomes less dense and thus become more buoyant than the surrounding fluid. The cooler, denser fluid settles underneath and this forces the warmer and less dense fluid to rise. Such a movement is called convection, and the moving body of liquid is referred to as a convection cell. Rayleigh-Běnard convection is the instability of a fluid layer which is confined between two thermally conducting plates, and is heated from below to produce a fixed temperature difference. Since liquids typically have positive thermal expansion coefficient, the hot liquid at the bottom of the cell expands and produces an unstable density gradient in the fluid layer. If the density gradient is sufficiently strong against viscosity, the hot fluid will rise, causing a convective flow which result in enhanced transport of heat between the two plates. Since there is a density gradient between the top and the bottom plate, gravity acts by trying to pull the cooler, denser liquid from the top to the bottom. This gravitational force is opposed by the viscous damping force in the fluid. The balance of these two forces is expressed by a non-dimensional parameter called the Rayleigh number. The Rayleigh Number is defined as:

$$R = \frac{g\alpha\beta}{\kappa\nu} h^4 \quad , \qquad (1.1)$$

where, g is the acceleration due to gravity,

α the coefficient of thermal expansion of the fluid,

$\beta = \left|\frac{dT}{dz}\right| = (T_0 - T_1)/h$ the vertical temperature gradient with T_0 the temperature on the lower surface and T_1 on the upper surface,

h the depth of the layer of the fluid,

κ the thermal diffusivity,

v the kinematic viscosity.

The statement of the Rayleigh-Bènard problem is based on the set of the hydrodynamic equations in the Boussinesq approximations. In fluid dynamics, the Boussinesq approximation is used in the field of buoyancy-driven flow, also known as natural convection. It states that the density differences are sufficiently small to be neglected, except where they appear in terms multiplied by g, the acceleration due to gravity.

In some basic principles, the heat transfer known as the laws of thermodynamics, which defines how heat transfer relates to work done by a system and place some limitations on what is possible for a system to achieve. In the porous media it will use the first law of thermodynamics. It is an expression of the principle of conservation of energy. The law states that the energy can be transformed which it is changed from one form to another, but cannot be created or destroyed. It is usually formulated by stating that the change in the internal energy of a system is equal to the amount of heat supplied to the system, minus the amount of work performed by the system on its surroundings.

1.2 Problem Statement

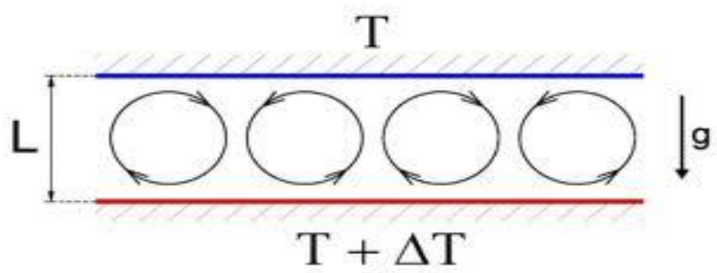

Figure 1.1: A layer of liquid between two impenetrable parallel plates

Figure 1.1 show a problem which is we set-up uses a layer of liquid between two impenetrable parallel plates as top and bottom boundaries which now considered in the form of infinite extend. The height of the layer is small compared to the horizontal dimension. The temperature of the bottom plane is set to be slightly higher than the top plane and to create a possible variation in temperature. At the onset of convection, the liquid will experience changes. When the temperature difference between lower and upper plate is positive and large enough, buoyancy forces lead to the destabilization of the quiescent state and a distinct convective pattern appears in the fluid layer. Its means that the fluid has a reaction towards each other due to the buoyancy and the viscosity of the liquid interacting with each other. At the stage of in equilibrium, the movement commences the pattern of convection which forms the second stage of stability. In this case, we have the parallel rolls aligned in such a way. Ahlers suggested that three-dimensional convection normally occurs at the onset. A cross roll is a case to considered and the variation of the thermal forcing is taken. We want to see the stability of a cross rolls arrange in various angle settings in

Rayleigh-Bĕnard convection in a porous media of an infinite extends. In this project, we will use the MATLAB to solve the equations that arise.

1.3 Objectives of the Study

The objectives of this research are summarized as:

I. To use weakly non-linear method to determine the convection threshold.

II. To analyze small perturbation effects due to the leading convection term.

1.4 Scope of Study

The Rayleigh–Bĕnard system is a standard setup to study pattern formation outside of equilibrium. It consists of a fluid layer confined by two impenetrable parallel plates perpendicular to the direction of gravity that are kept at a constant temperature. When the temperature difference between lower and upper plate is positive and large enough, buoyancy forces lead to the destabilization of the quiescent state and a convection pattern appears in the fluid layer.The statement of the Rayleigh-Bĕnard problem is based on the set of the hydrodynamic equations in the Boussinesq approximations. In fluid dynamics, the Boussinesq approximation is used in the field of buoyancy-driven flow, also known as natural convection. It states that the density differences are sufficiently small to be neglected, except where they appear in terms multiplied by g, the acceleration due to gravity.

By using the weakly nonlinear analysis, we resolve to get the representation of convection in terms of heat transfer. The next step is to create the possibility of cross roll in the analysis and the different angle of interaction of rolls, ϕ is considered. In this dissertation, we would focus on the cross-roll convection of Rayleigh-Bènard convection in a porous media. We will analyze the solution given by the weakly nonlinear expansion of the convection under the influence of an imperfect heating with the characteristic of periodic thermal forcing. The natural approach is to model it without the periodic thermal forcing.

1.5 Significance of the Study

Heat transfer and transport phenomena problem are the main focus of this analysis. The findings of this research would give us the characteristics or criteria of convection in porous media. Through this research, we sought to find the analytical solution to the convection problem stated. The study will give us a better understanding of how convection flows occurs and the kind of competition exist in the process. It adds to the literature of this discipline. A comparative study of stability against parallel rolls will be giving a good insight of the result of competitions between the two structures in a system.

1.6 Organisation

In Chapter 2, we will discuss about the literature review of the porous media, Rayleigh-Benard convection, stability of the convection, Mechanics of fluid flow through a porous media, heat transfer in porous media, and the conservation equation. While, in Chapter 3, we will discuss about the weakly nonlinear analysis. Then, we will discuss about the stability of rolls. Lastly, in Chapter 5 we have the conclusion and recommendations.

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