SIMULATED ANNEALING APPROACH IN SOLVING THE MINIMAX PROBLEM WITH FIXED LINE BARRIER

TUAN MARIAM BINTI TUAN MAHMUD

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Specially dedicated to my beloved mother Che Engku Lijah Binti T. Ali, to my supporting family members and my loyal friends that are always there for me.

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In the name of Allah, the Beneficent, the Merciful.

All praise is due to Allah, the Lord of the Worlds. Thee do we serve and Thee do we beseech for help. Without His help, I do not think that this report can be done successfully. There are so many conjectures in finishing this study. Thank you to Allah, He makes all the difficulties became easier to solve.

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ABSTRACT

Minimax location model is a class of location problems in which customers need the facility especially in emergency situation. The objective of this problem is to minimize the maximum distance between facility and the existing customers. The facility can be hospital, fire station and military service. This study involves fixed line barrier where the customers need to go through the passage on the barrier in order to move from one point to another point if necessary. Examples of line barrier are rivers, lakes and mountains. The single-facility problem is solved exactly by solving the MINLP problem using LINGO. Simulated Annealing approach is used in order to solve the multi-facility problem, coded using C++ programming. The procedure of SA algorithm is provided. The results for single facility and multi-facility problems are provided.

ABSTRAK

Model lokasi minimaks merupakan salah satu jenis masalah penempatan sesuatu kemudahan. Kemudahan bagi jenis model ini boleh digunakan dalam situasi kecemasan. Objektif bagi masalah ini ialah ingin meminimumkan jarak yang paling maksimum antara kemudahan dengan penduduk berdekatan. Contoh kemudahan kecemasan ialah hospital, balai bomba dan perkhidmatan ketenteraan. Kajian ini melibatkan halangan seperti sungai, tasik dan bukit bukau. Jika perlu, penduduk perlu melalui satu medium atau laluan agar dapat merentasi halangan tersebut. Selain itu, kajian ini menggunakan kaedah *Simulated Annealing (SA)* untuk menyelesaikan masalah tersebut. Prosedur mengenai SA juga dibincangkan dalam kajian ini. Penyelesaian masalah ini juga menggunakan bantuan perisian daripada LINGO dan program C++. Keputusan bagi masalah ini yang melibatkan satu dan dua kemudahan turut disediakan.

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LIST OF SYMBOLS

i	the number of iterations
n	the number of fixed points (or customer points)
$N(x)$	the number of neighbourhood points
w_j	demand or weight of customer, $j=1,2,\ldots,n$
a_j	location of customer, $j=1,2,\ldots,n$
$d(X_i, a_j)$	distance between facility i and cusotomer j
$F(x)$	objective function value for the current trial solution
$F(x')$	objective function value for the current candidate to be the next trial solution
T_k	temperature of iteration where $k = 0,1,2,\ldots$
$P(\delta)$	probability acceptance
α	rate of cooling

LIST OF ABBREVIATIONS

SA	Simulated Annealing
MINLP	Mix Integer Nonlinear Programming

CHAPTER 1

INTRODUCTION

1.1 Introduction to Minimax Location Problem

Uncapacitated facility location problems take a great variety of forms, depending on the nature of the objective function (minisum, minimax, problems with covering constraints), on the time horizon under consideration (static, dynamic), on the existence of hierarchical relationships between the facilities and on the inclusion or not of stochastic elements in their formulation (probabilistic, deterministic). When we consider the possible combinations of the categories above, numerous different types of the problem can be defined (Roberto, 2003). *Center* or *minimax* location problems are recently paid attention by many researchers of operational research. Potential applications of this problem are as follows: Warehouse location, public service centers, emergency service centers and military service (Amiri-Aref *et al.*, 2011).

In addition, facility location models can also differ in the distance metric applied, the number and size of the facilities to locate, and several other decision indices, depending on the specific application, inclusion and consideration of these various indices in the problem formulation will lead to very different location models (Farahani *et al.*, 2009).

One of the factors that are widely considered in the studies of facility location problem is distance. Euclidean distance assumes that travelling is possible to any orientations at any points. However, this assumption does not usually hold for facility location in urban areas. In this case, the block norm which is assumed that it can be traveled to a given several allowable orientations of movement with weights at any points is more applicable. Rectilinear distance is regarded as one of the block norms such that there are two allowable orientations which cross at right angles with the same weights (Uno *et al.*, 2007).

Besides, location problem in which regions are excluded from siting new facilities, but travelling through is allowed are called *restricted location problems*. These problems have been solved for median and center objectives (Nandikonda *et al.*, (2003). In reality, facility location problems involve the consideration of restrictions imposed by barriers. Miyagawa (2012) defined barriers as regions where traveling as well as locating new facilities is prohibited. Examples of barriers include lakes, parks, and military areas. Barriers also take place because of the disasters and accidents that cause damage to road networks. The shape of barriers also varies, for instance there are convex polygonal, circular, line with passages and rectangular barriers.

1.2 Background of the Problem

According to Amiri-Aref *et al*. (2011), the main purpose of the minimax location problem is to minimize the maximum distance from the facility to the demand points. The planar minimax location problem is first introduced by Sylvester in 1887. After that, Elzinga and Hearn efficiently solved the Euclidean center location problems with equal weight in 1972. But most of the real problems involve unequal weight. So, in 1982, Charalambous, Hearn and Vijay researched on the minimax problem with unequal weight distance, separately (Amiri-Aref*et al.*, 2011). Weights can be defined as the demand of the corresponding existing facilities (Biscoff *et al.,* 2007).

In reality, barrier is a very important constraint in solving the facility location problem. So, there are many researchers that researched on the minimax problems involving barriers. Aupperle and Keil (1989) proposed polynomial time algorithm for the Euclidean *p*-minimax problem when the demand points are restricted to lie on a fixed number of parallel lines. Then, Frieß *et al.,* (2005), solve the minimax problems in the presence of polyhedral barriers with Euclidean distance using propagation of circular wavefronts approach. By using the same type of barriers, Bischoff *et al.,* (2009) presented the Euclidean multifacility location-allocation problem and proposed two heuristics to solve the problem.

Most real problems have interaction with rectilinear or block norm because the distance is not always linear or on straight line. Consequently, many researchers approach to the minimax problem in the presence of barriers are based on rectilinear or block distance or block norm distances which allows for problem decompositions and discretizations. According to Amiri-Aref*et al.* (2011), Chakrabarty and Chaudhuri considered a constrained rectilinear distance minimax location problem and presented a geometric solution approach in 1990 and 1992. After that, there is study on the restricted center location problem under polyhedral gauges by Nickel in 1998. Then, Dearing *et al.* in 2002 came with a new type of barrier. He considered the rectilinear distances center facility location problem with polyhedral barriers and derived a finite dominating set result for the problem. There are researchers who extended similar ideas to a more general class of location problems. Segars Jr. (2000) and Dearing and Segars Jr. (2002a, b) developed a decomposition approach on which the objective function of a location problem with barriers is convex and optimized the problem using convex optimization methods. Then, by using the same problem, Dearing *et al.* (2005) used block norm distances in place of the rectilinear distances. The researchers presented new barriers which is arbitrary shaped barriers. They considered a single finite-size facility location problems with Manhattan (i.e., rectilinear) distance metric.

Based on the work of Savas *et al.* in 2002, Kelachankuttu *et al.* (2007) introduced a new facility location problem by applying a contour line. Then, extending the work of Savas *et al.* in 2002, Sarkar *et al.* (2007) addressed the problem involving finite facility location problem with only user-facility interactions. Nandikonda *et al.* (2003) changed the objective function of the problem, they considered the rectilinear distance in center problem with the presence of arbitrary shaped barriers. After that, Canbolat and Wesolowsky (2010) proposed a solution approach for the rectilinear Weber problem with a probabilistic line barrier. Then, in 2011, Amiri-Aref *et al.* extended the study by Canbolat and Wesolowsky (2010). The study concentrated on the center problem instead of Weber problem but it is still in the presence of the same type of barrier which is probabilistic line barrier.

However, based on the works discussed earlier, it can be seen that most of the problems are solved by using exact method, in many applications, the exact solution for the facility location problem is not feasible because of its complexity due to large number of variables, inadequate knowledge of how the variable interact, long computations times, and high noise environments that mask system functionality, among other factors. Therefore, heuristic method is applied. Figure 1.1 presents the scenario leading to the research problem considered in this study.

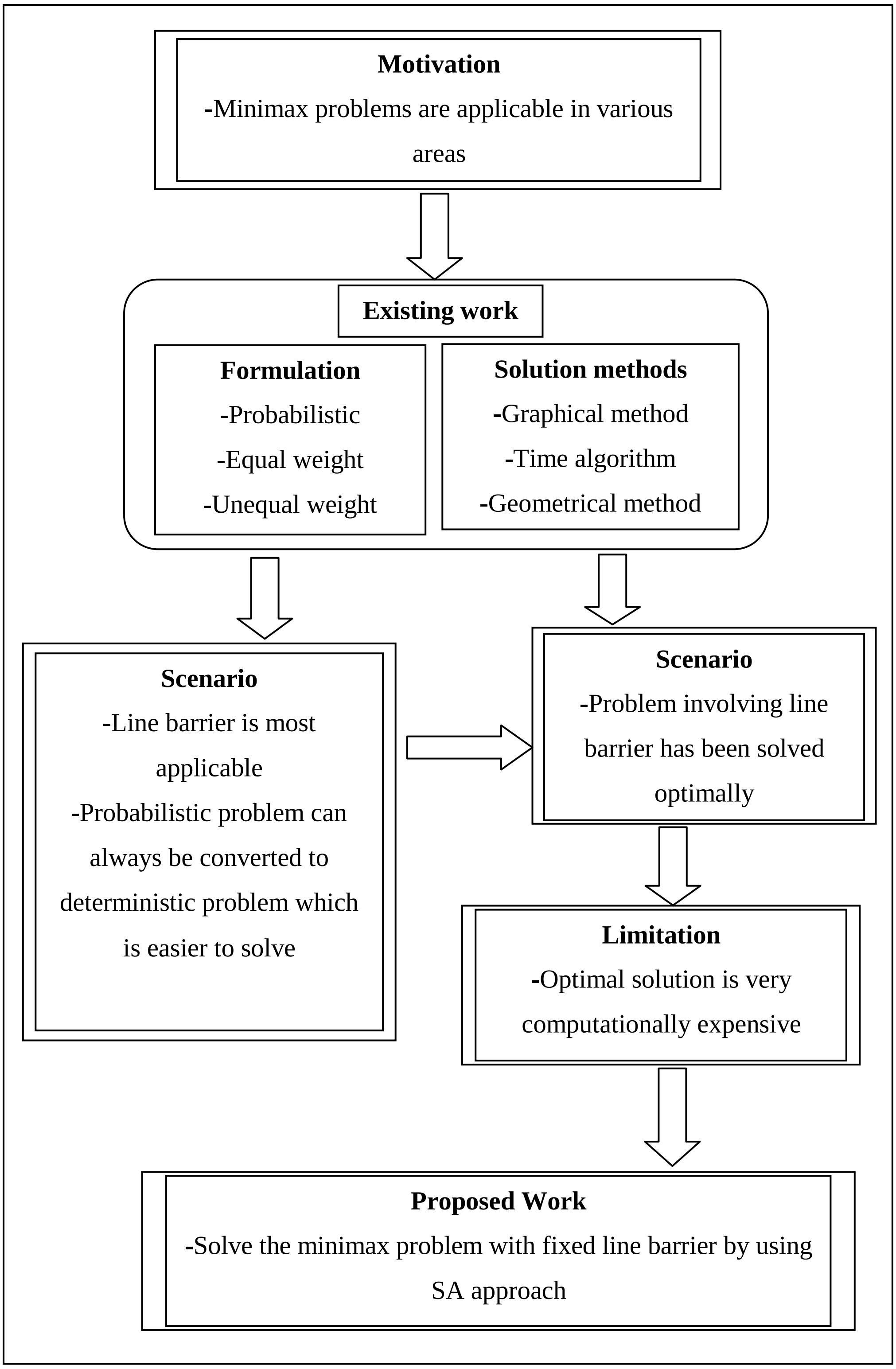

Figure 1.1: Scenario Leading to the Research Problem

1.3 Statement of the Problem

In this study, minimax problem involving fixed line barrier is considered. This problem was solved optimally but in many applications, solution for the problem is not feasible because of its complexity. Therefore, this study proposed a heuristic approach namely Simulated Annealing (SA) in solving the problem.

1.4 Objective of the Study

The objectives of the study are:

a) To solve Mix Integer Nonlinear Programming (MINLP) model for single-facility problem using LINGO.
b) To develop Simulated Annealing (SA) algorithm for solving single and multi-facility problem.
c) To implement the developed SA procedure using C++ programming.
d) To investigate the performance of various parameter settings in the algorithms.

1.5 Scope of the Study

The problem involving random 50 fixed points are considered. Generated data is used for experimental purpose. Deterministic is algorithm whose resulting behavior is entirely determined by its initial solution and it always arrives at the same final solution through the same sequence of solutions. Continuous location problems involves an infinite set of possible locations for a new facility

1.6 Significance of the Study

This study focuses in developing the Simulated Annealing algorithm for solving minimax problem with fixed line barrier. The proposed technique will be able to solve large size problem with less computational effort. The main contributions of the research are summarized as follows:

i) Development of Simulated Annealing algorithm for solving minimax problem with fixed line barrier.
ii) Evaluation of the performance of the variants of the proposed algorithm for different temperature decrement rule and stopping criteria.
iii) As a reference for solving real minimax problem involving fixed line barrier.

1.7 Outline of the Thesis

This thesis contains five chapters. Chapter 1 is the introduction of this research. Chapter 2 provides the literature review on the minimax location problems without barriers and also problems with barriers. This chapter also shares information about the method used in solving the problems. Then, Chapter 3 presents the research methodology adopted in carrying out the work. The chapter explains the basic algorithm of Simulated Annealing (SA) technique and the factors that affect the efficiency of annealing process. Chapter 4 presents the framework of solving the minimax problem, the implementation of LINGO and discussion on the result obtained from LINGO and C++ programming. Finally, the last chapter gives the conclusion and the recommendation for future work.

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