OPTICAL SOLITON GENERATION IN FIBER OPTICS: FREE & FORCED NONLINEAR SCHRÖDINGER EQUATION

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My utmost dedication to mum and dad.

Thank you always being there for me.

I love you.

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ABSTRACT

Future telecommunication will depend on effective data transmission, high quality video encoding, and efficient networking using optical solitons in fiber optic. At this moment, we encounter slow data transmission, fiber loss and chromatic dispersion that can be considered as forcing terms. Fiber optics is a man-made tool that has changing refractive index and this cause fiber loss. Nonlinear Schrödinger (NLS) equation which is a nonlinear partial differential equation can models the non-forced system effectively that combines the effect of nonlinearity and dispersion. In this research, a numerical method that consists of semi-implicit Pseudo-Spectral method scheme will be implemented to solve NLS equation. Comparing the results from analytical solution between numerical solution of NLS equation to determine an accurate and stable code so that it can be used to solve forced nonlinear Schrödinger (fNLS) equation that models forcing system. MATLAB computer programming which is user friendly will be used to implement the numerical scheme that produces various graphical outputs to simulate the propagation of solitons.

ABSTRAK

Telekomunikasi masa depan akan bergantung kepada keberkesanan penghantaran, pengekodan video yang berkualiti tinggi dan rangkaian cekap yang menggunakan soliton optik dalam gentian optik. Pada masa ini, kita dapati penghantaran data yang lambat, kehilangan gentian dan serakan kromatik yang boleh dianggap sebagai rintangan. Gentian optik adalah alat buatan manusia yang mempunyai indeks biasan yang berubah-ubah dan menyebabkan kehilangan gentian. Persamaan tak linear Schrödinger (NLS) merupakan satu persamaan pembezaan separa tak linear yang boleh memodelkan sistem bebas rintangan dengan berkesan yang memggabungkan kesan ketaklinearan dan penyebaran. Dalam kajian ini, kaedah berangka (pseudo-Spectral) dikemukaan untuk menyelesaikan persamaan NLS. Hasil daripada penyelesaian analitikal antara berangka untuk persamaan NLS yang bebas dibandingkan untuk menentukan kod yang tepat dan stabil supaya is boleh digunakan bagi menyelesaikan sistem paksaan (fNLS). MATLAB adalah satu pengaturacaraan yang mesra pengguna dan dapat menjana pelbagai grafik yang menunjukkan interaksi solitons.

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LIST OF SYMBOLS

2D		2-Dimensional
u	-	Envelope function
t	-	Time propagation
x	-	Distance
α	-	Nonlinearity coefficient
β	-	Dispersion coefficient
γ	-	Fiber attenuation coefficient
i	-	Imaginary unit
P	-	Power in the fiber
M		Number of loops
N		Number of discrete points

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CHAPTER 1

INTRODUCTION

1.1 Introduction

We now live in a world where computers, television, cell phones and other tools are a necessity. This advancement has been made possible via the progression of information technology. Fiber optics technology has played a major role in contributing to this rapid advancement of technology. Without us realizing it, fiber optics is an essential part of our everyday lives and is becoming an increasingly common replacement for traditional standard copper wire. Both materials are used to transmit signals from one location to another via fiber optic cables. Transmission of information includes data transmission, image transmission, and energy transmission from one source to another. However, a fiber-optic line has significant advantages compared to copper wire, including the ability to carry a larger amount of bandwidth over a greater distance at faster speeds, all for a lower maintenance cost and with increased resistance to electromagnetic interference from objects, such as radios and other cables. Optical fiber is also the safest way in transmitting data as it does not leak data or information due to the fact that the transmissions signals are guided through the optic fiber and not through copper wires like it's done in a cable.

1.2 Background of Study

There are two different materials used to fabricate optics which operate as a guide for the light waves to travel within the cables which carry information sent from one end to another. Polymeric fibers are commonly used for short distance transfer and installations in rough surroundings, whereas glass fibers are used for high quality and long distance data transfer. The light transmitted through the fiber is confined due to total internal reflection within the material. Optical communication use “wavelength division multiplexing with different wavelengths to carry different signals in the same fiber”. In other words, information is encoded into different wavelengths of light to allow information to travel in different directions without interference, therefore, making it possible for high speeds of data transmission within one small strand of fiber optic cable. The information contained by fiber these cables travel with the light which reflects off the inner walls of the cable and is guided throughout a fiber optic circuit.

Fiber optics can transmit information in the same way that copper wire can transmit electricity. However, copper transmits only a few million electrical pulses per second, compared to an optical fiber that carries up to 20 billion light pulses per second. This means telephone, cable and computer companies can handle huge amounts of data transfers at once, compared to the limited capabilities of conventional wires. Fiber optic cable was developed due to the massive surge in the quantity of data over the past 20 years. Without fiber optics cable, the modern Internet and World Wide Web would not be possible. If you were to make a phone call to Europe, traditionally the signal would go up to a satellite and then back down to Europe. With fiber optics, if the call is transmitted through a transatlantic fiber optic cable, there is a direct connection.

1.3 Statement of the Problem

The nonlinear Schrödinger (NLS) equation describes the phenomena that is very important nowadays; which is the propagation of waves in relation to the design of optical long-distance communications lines and all optical signal processing devices for reliable and high-bit-rate transmission of information. We will determine how the propagation of optical pulses in optical fiber can be modeled into NLS equation and also what are the main obstacles that affect the propagation of stable soliton pulses. Besides that, we will also solve the forced nonlinear Schrödinger (fNLS) equation to minimize fiber loss.

1.4 Objectives of the Study

The objectives of this research are:

a) obtaining mathematical modeling of optical soliton transmission in fiber optics.
b) solving nonlinear Schrödinger equation (NLS) analytically and numerically.
c) solving forced nonlinear Schrödinger (fNLS) equation numerically.

1.5 Scope of the Study

The main focus of this study is nonlinear Schrödinger (NLS) equation, a partial differential equation which has the nonlinearity and dispersion effect given by:

$$i\frac{\partial u}{\partial t} + \alpha|u|^2 u + \beta\frac{\partial^2 u}{\partial x^2} = 0 \tag{1.1}$$

Based on the physical model of fiber optics, the following assumptions are made:

a) There is no fiber loss along this fiber.

b) The shape will be maintained the whole time during propagation.

If the fiber optics is facing the fiber loss, the NLS equation with an added term representing the optical loss is observed. Forced nonlinear Schrödinger (fNLS) equation (forced system) is defined in the equation below:

$$i\frac{\partial u}{\partial t} + \alpha|u|^2U + \beta\frac{\partial^2 u}{\partial x^2} = \frac{1}{2}\gamma f(x) \tag{1.2}$$

where u, t and x are respectively the normalized parameters of the envelope function, the time propagation and the distance, meanwhile, α and β are real constant, γ is a magnitude of forcing, $f(x)$ is a function in terms of x and $i = \sqrt{-1}$ is an imaginary unit.

These equations focus on the physical problem of fiber optics.

1.6 Significance of the Study

The results of this research play an important role because sophisticated data transfer systems nowadays rely more and more on fiber optics for digital signal transmission. Central information superhighways can transmit up to 10 Gbits/seconds. High speed data transfer, highly reduced data loss, no electromagnetic problems, small dimensions and a low weight are the main features that characterize the use of fiber optic technology. There is a link between the physical phenomenon and mathematics, which results in us needing to understand the propagation of waves in the optical fiber. Forced nonlinear Schrödinger (fNLS) equation enables us to study in more depth about the existence of the fiber loss phenomenon in fiber optics. Through this equation, we are able to find ways to minimize the fiber loss.

1.7 Outline of the Study

This study focuses on optical solitons in fiber optics. It can be modeled into two mathematical equations, which are nonlinear Schrödinger (NLS) equation and forced nonlinear Schrödinger (fNLS) equation. The first chapter includes the background of study, the statement of problem, objectives of study, scope of studies and significance of study, which is explained in details.

In Chapter 2, literature review concerning the background of fiber optics will be discussed.

Chapter 3 will discuss on the nonlinear Schrödinger (NLS) equation. We will model the optical fiber using Maxwell equation. Later, we will focus on how to solve the NLS equation analytically. NLS equation will be solved analytically using the progressive wave solution method and the graphical outputs will be simulated. Meanwhile, the coefficient of α, β and θ is introduced to investigate the solitons generated.

Chapter 4 will introduce the numerical solution of NLS equation by using Pseudo-Spectral method so that the results can be compared with the analytical solution to find the percentage error of this numerical solution. Initialization block and forward scheme will be introduced and it will be used in developing the computational program for NLS equation. Simbiology Package will be used to simulate the graphical output based on the solutions obtained. Von Neumann stability analysis is also introduced to check the stability of finite difference schemes.

Chapter 5 will discuss on forced nonlinear Schrödinger (fNLS) equation. If the NLS equation is successfully solved using Pseudo-Spectral method, then the forced nonlinear Schrödinger (fNLS) equation can also be solved using this method. fNLS equation is generated due to the existing fiber loss term in optical fiber. fNLS will be solved numerically with a little change in the MATLAB programming that had been developed to solve NLS numerically. Various outputs of fNLS equation will be obtained through the results.

Chapter 6 is the final chapter of the thesis. The chapter is introduced to conclude the overall research. Some suggestions and recommendations for the future research in fiber optics will be discussed.

1.8 Conclusion

This chapter is an introduction chapter which gives a clear overview for the research will be going on the following chapter. The motivation of this research is clearly stated in the background of the problem with objectives. The scope of the research is discussed due to some limitations. The outline of the thesis is given in the end of this chapter to give the brief overview for what is going on every chapter.

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