NUMERICAL SOLUTION OF MASS TRANSFER TO MICROPOLAR FLUID FLOW PAST A STENOSED ARTERY

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Specially dedicated to my beloved family

and those people who have given consistent support and guide me.

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ABSTRACT

A study about blood flow behaviour numerically and mathematically becomes invaluable tool in interpreting and analyzing the circulatory system. In this study, a mathematical model of fluid flow and mass transfer past through a stenoses artery is developed. The blood behaves as non-Newtonian fluid. The blood flow is considered to be unsteady, nonlinear, axisymmetric, two-dimensional and fully developed which is described by micropolar fluid. Meanwhile, the arterial wall is considered to be rigid. The geometry of stenosis is given by cosine-shaped plotted using MATLAB programme based on existing coding. The governing equations of the problem consist of continuity equation, momentum equation and convection-diffusion equation that govern mass transport to flow are formulated in cylindrical coordinate system. Then, all of the governing equations are written in dimensionless form by using dimensionless quantities in order to avoid difficulties. A radial coordinate transformation of the governing equation together with the set of initial and boundary are also needed in order to avoid interpolation error during discretization. A numerical technique has been performed in this study. Finite difference method in staggered grid known as Marker and Cell (MAC) method aid by existing MATLAB programme has been selected to solve all the governing equations involced. Successive over relaxive (SOR) method is also proposed in handling the poisson equation for pressure. Finally, two stability restriction which optimized the time step size at each calculation play an important role to limit the numerical computational. As a result, the value of wall shear stress and separation zone for micropolar fluid are lower than Newtonian fluid due to the presence of rotational viscosity. The values of wall shear stress and velocity were highest at around of critical stenotic region. While, the mass concentration were decrease and converge to zero at the arterial wall as the both wall shear stress and axial velocity decrease.

ABSTRAK

Kajian mengenai aliran darah secara berangka dan matematik dianggap sebagai satu alat yang sangat berharga bagi mentafsir dan menganalisis system peredaran darah. Dalam kajian ini, model pengaliran darah dan perpindahan jisim melalui stenosis arteri dibentuk. Darah bercirikan sebagai bendalir tak Newtonan iaitu bendalir mikropolar. Aliran darah dianggap tak mantap, tak mendatar, berpaksi simetri, dua dimensi dan terbentuk sepenuhnya. Manakala, dinding arteri dianggap keras. Geometri stenosis adalah dibentuk daripada lengkung kos dan sin yang diplot menggunakan pengatucaraan MATLAB berdasarkan kod yang sedia ada. Persamaan penaklukan dalam masalah ini yang melibatkan persamaan keselanjaran, persamaan momentum dan persamaan perolakan-resapan yang mengawal pengangkutan jisim untuk mengalir dalam system polar koordinat silinder diformulakan. Kemudian, persamaan penaklukan ditulis dalam bentuk tak dimensi untuk mengelakkan kesukaran. Jelmaan jejari koordinat bersama-sama syarat sempadanya juga diperlukan supaya kesilapan boleh dielakkan ketika pengembangan. Penyelesaian berangka telah dilaksanakan dalam kajian ini. Kaedah beza terhingga iaitu kaedah MAC dibantu oleh pengatucaraan MATLAB berdasarkan kod yang sedia ada telah dipilih untuk menyelesaikan semua persamaan yang terlibat. Kaedah pengenduran berlebihan berturutan (SOR) digunakan untuk menyelesaikan persamaan poison tekanan. Dua sekatan kestabilan yang berfungsi untuk memastikan kesesuaian saiz langkah masa pada setiap perhitungan memainkan peranan yang penting untuk mengawal pengkomputeran berangka. Keputusannya, nilai ketegangan permukaan dinding dan rantau pemisah untuk bendalir mikropolar adalah lebih rendah berbanding dengan bendalir Newtonan disebabkan oleh wujudnya putaran tebal dan melekat. Nilai tertinggi ketegangan permukaan dinding dan halaju berlaku disekitar kawasan stenosis. Jisim konsentrasi berkurang dan menuju ke kosong disebabkan oleh penurunan ketegangan permukaan dinding dan halaju.

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LIST OF SYMBOLS/NOTATIONS

Roman Letters

C	-	mass concentration of the solute
Cs	-	reference mass concentration at the inlet
D	-	coefficient of diffusion
I_0	-	modified Bessel function of zeroth
I_1	-	modified bessel function of first order
j	-	microinertia constant
L	-	the finite difference of vessel or arterial segment
M	-	hartmann number
n	-	power law index, time direction
p	-	pressure
Pt	-	pressure at the right of the top position
Pb	-	pressure at the right of the bottom position
Q	-	flow rate
(r,θ,z)	-	coordinates of a material point in cylindrical polar coordinates system
R_0	-	unconstructed radius of the stenosed artery
$R(z)$	-	radius of arterial segment
Re	-	reynold number
ReG	-	generalized Reynold number
Sc	-	schmidt number
t	-	parameter for time
u	-	axial velocity
U	-	cross-sectional average velocity

v	-	radial velocity
z_0	-	half length of stenosed artery
z_1	-	centre of the stenosed

Greek Letters

ρ	-	density of blood
κ	-	rotation viscosity
μ	-	viscosity of blood
ω	-	angular velocity of micropolar fluid
ζ	-	material constant
τ_u	-	wall shear stress
Λ	-	resistance to flow
δ	-	maximum width of stenosis
δt	-	time increament
δz	-	width of $(i.\ j)^{th}$ control volume
δx	-	length of $(i.\ j)^{th}$ control volume
β	-	combination factor
ϕ	-	momentum flux
ω_o	-	over-relaxion parameter
ω_1	-	under-relaxion parameter
$\frac{\partial p}{\partial z}$	-	pressure gradient in z-direction

CHAPTER 1

INTRODUCTION

1.1 Background of Study

Good health is a dream of everyone in whole country. Currently, a lot of healthy products like Herbalife, HPA, ECPI, supplement and others are introduce and act as a precaution by some of them in order to attain enough nutrients, vitamins, irons and calciums. Other than that, some of them will spend their time for exercise to make sure they are prevented from any sickness. Unfortunately, there are some people who still did not take serious about their health. They still looked at foods that contain a lot fats, cholesterols and carbohydrates which are exposing themselves to a disease known as stenosis.

Stenosis can be varying. One of them is atherosclerosis. Atherosclerosis is defined as arteriosclerotic vascular disease (ASVD). (ASVD) is a condition in which an artery wall thickens as a result of the accumulation of fatty materials such as cholesterol. The artery wall is complex and it contains 3 different layers. The layers are intima, media, and adventitia which all of them play a role in establishing the overall mechanics of the wall. An accumulation of plaque occurs on the inner layer of artery or intima media. Atherosclerosis can affect any artery in the body, including arteries in the heart, brain, arms, legs, pelvis, and kidneys. As a result, different diseases may develop based on which arteries are affected. One of them is Coronary heart disease (CHD) which is the first killer of both men and women in the United States. CHD occurs if plaque builds up in the coronary arteries. These arteries supply oxygen-rich blood to the heart. Plaque which is made up of fat, cholesterols, calcium and other substances found in the blood can narrows the coronary arteries and reduces blood flow to your heart muscle. Plaque build up also makes it more likely that blood clots will form in your arteries. Blood clots can partially or completely block blood flow. The narrowing or blockage does not occur suddenly but builds up over several years where fatty materials, like cholesterol, fat, smooth muscle cell, line the blood vessels. As the result, the arteries become constricted, the elasticity of arterial wall and blood volume that travels through the arteries are reduce. This phenomenon will lead to lack of nutrients and transportation to the organ.

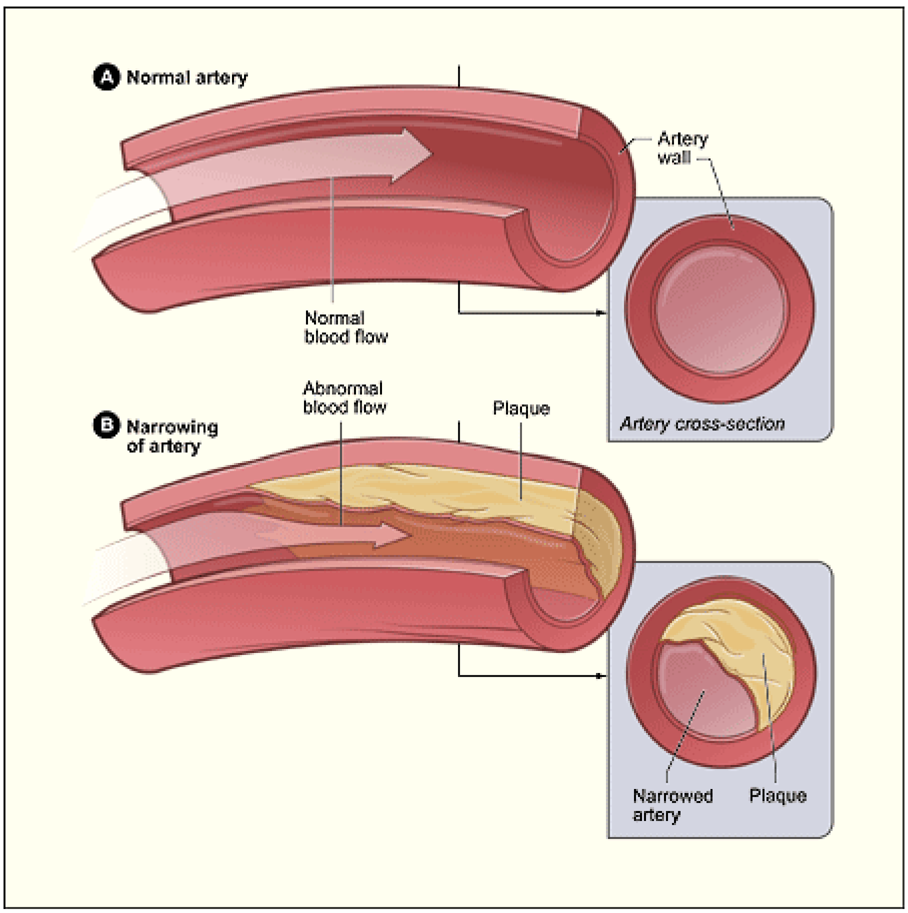

Figure 1.1 The difference between normal and diseased artery

Blood exhibit mainly of blood plasma with red blood cells (erythrocyctes), white blood cells (leucocytes) and blood platelets (thrombocytes). The transport of oxygen and other solutes, and the regulation of the body's PH and temperature are its main function. The erythrocyctes contain about 45% of the blood volume, dominate the rheological behavior of the blood. Overall, the blood flow characteristics were affected by many factors where mass transport is one of them.

Mass transport in arteries can be defined as a movement of atherogenic molecules which refers to blood bone components such as oxygen and low density lipoproteins (LDLs) from flowing blood into the arterial wall or vise versa. Atherosclerosis may occur based on shear-dependent mass transfer mechanism of cholesterol between blood and arterial wall as proved by several of previous researchers

Liu *et al*. (2004) stated that the initiation and localization of atherosclerosis is refered to local hemodynamic factors such as wall shear stress. Although there remains some uncertainty about the exact hemodynamic factors responsible for the initiation of atherosclerosis, it has been noticed that the atherosclerosis development is strongly associated to the blood flow characteristics in the arteries.

The theory of microfluids existing microscopic effects arising from the local structure and micro-motion of the fluid elements. Such fluids support stress and body moments admitting the rotary inertia. A Micropolar fluid is a subclass of microfluids which support couple stress, body couples, microrotational effects, and microrotational inertia. Liquid crystals, suspension and animal blood are examples of micropolar fluid which consists of randomly oriented bar-like elements or dumbbell molecules and each volume element has microrotation about its centroid, explained in average sense by skew-symmetric gyration tensor.

Most of the study has been done on the aspect of computational fluid dynamics (CFD). CFD is a branch of fluid mechanics that take into account numerical methods and algorithms in solving the problem which consist of fluid flow. Numerical simulations of fluid flow are carried out in CFD. These simulations result obtains from numerical solutions of differential equations derived from physical conservation laws for flows. Moreover, CFD is able to provide insight into the type of the flow and could provide some solutions to the problems related with blood flow.

In fluid dynamics, several of experimental and computational methods extensively been used as research tools. Numerous experiments have been conducted to study the geometry and the micro rotational fluid effects consist of the mass transfer on the condition. These studies therefore have provided valuable data. But, there are limitations. However, these experimental techniques need to be investigated further by using other methods. Recently, computational techniques have demonstrated a strong ability in modeling flow behavior within stenosed arteries of varying geometry and conditions that can specified based on the realistic conditions. This method is quite fast, economical and present extensive results that allow for deeply understanding of the disease.

1.2 Problem Statement

It is long familiar fact that severe stenosis may easily cause critical flow condition related to artery collapse, plaque cap rupture which leads directly to stroke and heart attack. It also can lead to reducing the blood flow because of the blockage by emboli. Some people did not notice about the disease early and too late for them to attain the best treatment. Unfortunately, they were get sudden death due to this phenomenon. Based on this situation, numerical solution of mass transfer to micropolar fluid flow past a stenosed artery is investigated. Formulation of blood flow equation (Eringen’s micropolar fluid) and mass transfer to the blood (convection-diffusion equation) need to be done. Besides that, we want to distinguish how to solve governing equations of motion accompanied by the appropriate choice of boundary conditions by using Marker and Cell (MAC) method. By this study, prediction on the best planning for them who have the arterial disease can be done.

1.3 Objective of Study

The objectives of this study are:

i. To formulate the equation of micropolar blood flow and mass transport equation.
ii. To solve all governing equations by using MAC method.
iii. To detail on discretization expression for each of equation.
iv. To analyze the blood flow characteristic under the response of mass transfer to micropolar blood flow through cosine-shape stenosis in the unsteady state aid by MATLAB software.

1.4 Scope of Study

This study takes into account the non-Newtonian flow. The blood flow is considered to be micropolar fluid, nonlinear, unsteady, axisymmetric, two-dimensional, and fully developed. Besides, the artery wall is treated as rigid having cosine-shaped stenosis. The unsteady governing equations of motion coupled to the mass transport equations together with the appropriate initial and boundary conditions are solved numerically by MAC method aid by existing coding of MATLAB software.

1.5 Significance of Study

The significance of study is valuable in order to exhibit the blood flow characteristic and mass concentration profile over the entire arterial segment. Moreover, the outcomes accomplished from the non-Newtonian models give more approximate results as in the real situation. In addition, computational models of blood flow in the cardiovascular system provide insight into normal and diseased conditions in blood vessels and have applications in areas such as surgical planning and medical device design. These models also involve a low computational cost. Definitely, it will increase the real life example in application of MAC method.

1.6 Outline of the Study

This study is included of six chapters starting from introduction and end with conclusion and recommendation. The first chapter is divided into five sections that are background of study, problem statement, objective of study, scope of study and significance of study. In this study, a mathematical model of unsteady non-newtonian blood flow together with the mass transfer through stenosis artery is developed. The problem is solved numerically by using finite difference method in staggered grid known as marker and cell (MAC) method.

Chapter 2 consists of literature review on stenotic artery, micropolar fluid, mass transfer in fluid and method of solution. This chapter will discuss about the rheology of the blood. The explanation on Newtonian and non-newtonian also included. A lot of experimental and numerical method has been introduced in this chapter. Subsequently, the details are discussed in depth regarding to previous research.

Chapter 3 discuss about the geometry of stenosis that plotted using MATLAB. Other than that, the formulation of all governing equation involved also stated. All of the governing equations are reduce into non-dimensional form in order to avoid difficulties. Then, radial coordinate transformation is applied so that it can avoid interpolation error while discretizing the governing equations.

Next, chapter 4 briefly explains the numerical method of solution involved in this research. All the governing equations together with the set of initial and boundary condition are solved using finite difference method. Control volume-based finite difference discretization of all equations is carried out in staggered grid known as MAC method. Primarilly, some basic knowledge of MAC method will be introduced. Discretization of continuity equation, momentum equation and convection-diffusion equation are derived in this chapter. The convective term in momentum equations are restricted by stability limitations. Since the pressure does not appear in the continuity equation, we have to couple the equation of continuity, axial momentum and radial momentum equation in order to construct poison equation. The poison equation for pressure is solved by successive over relaxive (SOR) method. The pressure and velocity correction also be mentioned. Moreover, the determination of blood flow characteristic is defined clearly as the understanding of the term is important for this study. This chapter is ended with the flowchart of numerical algorithm.

Numerical results and discussion will be presented in chapter 5. The results are obtained by using MAC method and SOR method aided by MATLAB programming with existing coding. The obtain results are focus on discussing about distribution of wall shear stress, streamlines pattern, velocity profile and mass concentration profile.

Hence, conclusion of this study will be preview in last chapter that is chapter 6. This chapter consists of summary of the study and recommendation for the future study.

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