

MODELLING AND FORECASTING VOLATILE DATA BY USING ARIMA AND
GARCH MODELS

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*To my beloved father, Miswan bin Bibet, mother, Hamidah binti Karimin,
all my siblings, Mohamad Nizam, Nur Azlin, Muhammad Faizal and him,
Muhammad Sayyidi Afiq bin Awang.*

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ABSTRACT

Modelling and forecasting of volatile data have become the area of interest in financial time series. Volatility refers to a condition where the conditional variance changes between extremely high and extremely low values. In the current study, modelling and forecasting will be carried out using two sets of real data namely crude oil prices and kijing emas prices. The models investigated are Box-Jenkins Autoregressive Integrated Moving Average (ARIMA) model and Generalized Autoregressive Conditionally Heteroscedasticity (GARCH) model. In estimating the parameters for the Box-Jenkins ARIMA model, two estimation methods are used. These are Maximum Likelihood Estimation (MLE) and Ordinary Least Squares Estimation (OLS). The capabilities of these two methods in estimating the ARIMA models are evaluated by using Mean Absolute Percentage Error (MAPE). The modelling performances of ARIMA and GARCH models will be evaluated by using Akaike's Information Criterion (AIC) while the forecasting performances of both models will be evaluated by using Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). The processes of modelling and forecasting will be done by using R and Eviews statistical softwares. As a result of the study, it can be concluded that in terms of parameters estimation of ARIMA models, MLE gives more precise forecast for crude oil prices data while OLS gives more precise forecast for kijing emas prices data. In terms of forecasting performances between ARIMA and GARCH models, it can be concluded that GARCH is a better model for kijing emas prices data while ARIMA is a better model for crude oil prices data.

ABSTRAK

Permodelan dan ramalan data tidak menentu (atau data turun-naik) telah menjadi bidang penting dalam ekonomi dan kewangan. Turun-naik merujuk kepada keadaan di mana perubahan varians bersyarat berlaku antara nilai yang sangat tinggi dan sangat rendah. Dalam kajian ini, pemodelan dan ramalan akan dijalankan dengan menggunakan dua set data sebenar iaitu harga minyak mentah dan harga kijang emas. Model yang dikaji adalah model Purata Bergerak Bersepadu Autoregresi Box-Jenkins (ARIMA) dan model Heteroscedastik Bersyarat Autoregresi Teritlak (GARCH). Dalam menganggar parameter bagi model ARIMA, dua kaedah anggaran digunakan. Kaedah tersebut adalah Anggaran Kemungkinan Maksima (MLE) dan Anggaran Kuasa Dua Terkecil Biasa (OLS). Keupayaan kedua-dua kaedah ini dalam menganggar model ARIMA dinilai dengan menggunakan Min Ralat Peratus Mutlak (MAPE). Keupayaan kedua-dua model dalam permodelan data turun-naik akan dinilai dengan menggunakan Kriteria Maklumat Akaike (AIC) manakala keupayaan dalam ramalan data akan dinilai dengan menggunakan Min Ralat Mutlak (MAE) dan Min Ralat Peratus Mutlak (MAPE). Proses permodelan dan ramalan akan dilakukan dengan menggunakan sofwer R dan Eviews. Hasil kajian mendapati dari segi menganggar parameter bagi model ARIMA, MLE memberi ramalan yang lebih tepat bagi data harga minyak mentah manakala OLS memberi ramalan yang tepat bagi data harga kijang emas. Dari segi permodelan dan ramalan, di antara model ARIMA dan GARCH, dapatlah disimpulkan bahawa GARCH adalah model yang lebih baik bagi data harga kijang emas manakala ARIMA adalah model yang lebih baik bagi data harga minyak mentah.

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CHAPTER 1

INTRODUCTION

1.0 Introduction

Time series refers to a collection of observations that are made sequentially at regular time intervals. Applications of time series cover all areas of statistics but some of the most important areas are economic and financial time series as well as many areas of ecological and environmental data. Examples of time series data are daily rainfall, daily exchange rate, monthly data for unemployment and share prices.

There are two main goals of time series analysis. Firstly to describe and summarise the time series data, and secondly to make prediction of the future values of time series variables. Both of these goals need the pattern of observed time series data. Once the pattern of time series data is identified, interpretation of the data can be made. As an example, an increasing pattern of the data can lead to the increasing forecast value for the future.

Most time series patterns can be classified into two basic components, which are trend and seasonality. For trend analysis, it is normally referred to as a long-term movement, either consistently increasing or decreasing. For example, the price of gold is considered to have an increase in trend. For seasonality analysis, the variation of time series is dependent on the time of the year. For example, the sales volumes during Chinese New Year are increased compared to other days.

1.1 Background of the Study

Modelling and forecasting of volatile data have become the area of interest in financial time series. Volatility refers to a condition where the conditional variance changes between extremely high and low value. In finance, measuring volatility by the conditional variance of return is often adopted as a crude measure of the total risk of the asset. Many values at risk (VaR) models used for measuring the risk of market require the forecast of the volatility coefficients.

In the current study, modelling and forecasting will be carried out using two sets of real data. These are crude oil prices and kijang emas prices. These data are chosen because apart from being volatile as that is the area of focus for the current study, these two series are of great importance to mankind. Crude oil is claimed as one of the world's treasures. It is a natural resource of earth and has many valuable uses. It is a flammable liquid that consists of a complex mixture of organic compounds and hydrocarbon. Crude oil is discovered mostly through oil drilling and is refined and separated by boiling point. Its appearance varies depending on its composition. Pure crude oil are black or dark brown in colour, but sometimes it may be reddish, yellowish and greenish.

Crude oil prices are volatile time series. The prices just like any other volatile commodity have huge price swings in periods of oversupply or shortage. The crude oil prices cycle may last over several years responding to demand changes. Crude oil prices give impact to the cost of gasoline, manufacturing, home heating oil and electric power generation. The increase of oil prices will lead to the increase in cost of everything especially food and daily needs. This is because our daily necessities depend on transportation. This high oil prices will finally cause or increase inflation.

Crude oil prices affect many related sectors that depend heavily on the usage of crude oil. The inconsistency of crude oil prices make the modelling and forecasting of crude oil prices an important area of research. Apart from providing the information

about the future oil prices to the public, crude oil forecasting is also crucial in determining the world's economic movement.

Another volatile community data in the financial market under investigation in the current study is Kijang emas prices. Kijang Emas is an officially Malaysian gold and is minted by the Royal Mint of Malaysia. It was issued on 17th July 2001 and the gold comes in 1 oz, ½ oz and ¼ oz of bullion coin in weights. Since kijang emas is a type of gold, its prices movement is just like other prices of gold in the world. The forecasting of kijang emas prices is important for investment purposes in Malaysia.

Gold is a valuable metal and it is found in nature as a free metal. Gold is always yellow in colour and it is an electropositive element. The chemical name for gold is Aurum and it is symbolised as Au, from the word Aurora which means dawn. Gold is a very soft metal, ductile, which means it can be shaped and stretched easily and also malleable. Pure 24 carat gold is always yellow and because of its softness, it has to be alloyed. This means that gold is rarely used pure.

Gold bullion coin or gold jewellery is made from alloying gold with other metals. The addition of other metals to gold will tend to bleach its original colour. Strong bleachers of gold are nickel and palladium while moderate bleachers of gold are silver and zinc. Alloying to other metals produce weak to moderate effects. For example, alloying pure gold with nickel and palladium will produce white gold, while the gold remains yellow in colour when alloying pure gold with copper and silver.

In the case of volatile market conditions, people start to replace their money investments with gold either by buying jewellery or gold bullion coin. This is to ensure that the investors do not lose their purchasing power later. The gold demand is increasing and this condition drives the prices of gold to go up. Just like other yellow gold prices, kijang emas prices does not depend on economic changes, political effects and environmental conditions.

Crude oil prices and kjang emas prices will be used as case studies in the current research. Suitable time series models will be determined so as to obtain models that will be precise enough for forecasting volatile data.

1.2 Statement of Problem

Financial markets always show high level of volatility as a result of non-constant variance, unexpected events and uncertainty in price. That is why in recent years, volatility in time series has become an important aspect in many financial decisions. The three main purposes of volatility modelling and forecasting are asset allocation, risk management and prediction on future volatility.

In this study, two sets of volatile community data are modelled and forecast by using time series models. The models proposed in this study are Box-Jenkins Autoregressive Integrated Moving Average (ARIMA) model and Generalized Autoregressive Conditionally Heteroscedasticity (GARCH) model. Box-Jenkins ARIMA model have been used widely in many areas of forecasting time series while GARCH models have been used widely in financial time series analysis.

In developing the models, parameter estimation is one of the crucial steps. Common methods of estimation include method of moment (MME), Ordinary Least Square Estimation (OLS) and Maximum Likelihood Estimation (MLE). According to William in 2006, MME is rarely used in time series analyses because it produces poor estimates. Although it is easy, MME is not an efficient estimation method for ARIMA model because it works for only Autoregressive models of large sizes.

OLS on the other hand is the best when there is a linear trend in the data (Hutcheson, 2011). Researchers have found that MLE works well in any linear and non-linear models (Myung, 2003). Engle, the developer of ARCH and Bollerslev, the

developer of GARCH have proven that MLE was the best estimation method for these models.

In this study, we want to compare the performances of two models. The following question will be explored in the current study:

Between Box-Jenkins Autoregressive Integrated Moving Average (ARIMA) model and Generalized Autoregressive Conditionally Heteroscedasticity (GARCH) model, which model is more accurate in forecasting volatile data? For ARIMA models, between MLE and OLS, which method gives better estimates?

1.3 Objectives of the Study

Objectives of this study are

- a) To explore the volatility in time series.
- b) To develop Box-Jenkins ARIMA and GARCH models in modelling volatile data.
- c) To forecast volatile data by using Box-Jenkins ARIMA and GARCH models.
- d) To compare the estimates of the ARIMA models using MLE and OLS.
- e) To compare the performances of Box-Jenkins ARIMA and GARCH models in forecasting volatile data.

1.4 Scope of the Study

This study focuses on the performances of time series models in modelling and forecasting volatile data by applying statistical tools incorporated in the Eviews and R

softwares in analysing the time series models involved in this study. There are two time series models considered, namely Box-Jenkins ARIMA model and GARCH model. The modelling performances of both models will be evaluated by using Akaike's Information Criterion (AIC) while the forecasting performances of both models will be evaluated by using Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE).

1.5 Significance of the Study

Using the basic concepts of time series, we can apply it to real life data. This study will apply time series modelling which are Box-Jenkins ARIMA and GARCH models in predicting the future values of volatile data. The process of modelling and forecasting will be done by using related statistical software. In this study, R and Eviews software will be used. The best model will be chosen to predict the values of the volatile data for the future.

Application of statistical tools in financial areas will strengthen the multidisplinary relationship between statisticians and economists. Better predictions based on statistical tools can be obtained and would benefit both parties.

1.6 Organisation of the Report

This study explores the potential of Box-Jenkins ARIMA and GARCH models in handling volatile data. For this purpose, two data will be used namely crude oil prices and kjang emas prices data. This report consists of five chapters. Chapter 1 discusses the research framework. It starts with the introduction of time series and followed by statement of problem, objectives of the study, scope of the study and significance of the study.

Chapter 2 reviews some past studies of the data forecasting and time series models related to the current study. It consists of the explanation and example of volatility in time series, reviews on crude oil forecasting, reviews on gold forecasting, reviews on ARIMA model and reviews on GARCH model. Chapter 3 describes the research methodology for this study. It consists of stationarity testing, Box-Jenkins ARIMA methodology and GARCH methodology.

Chapter 4 presents the analyses of the data. The best model in modelling the time series data will be chosen based on AIC while the best model in forecasting the time series data will be chosen based on MAPE and MAE. Discussion of the results will be presented in the last section of this chapter. Chapter 5 summarises and concludes the whole study. Some suggestions for future study are also presented.

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