MIXED CONVECTION BOUNDARY LAYER FLOW IN A VISCOELASTIC FLUID WITH INTERNAL HEAT GENERATION

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Specially dedicated to my beloved parents, family,

friends and those who have given consistent supports and guidance for me.

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ABSTRACT

The study in flows of viscoelastic fluid has generated many interests and become more important recently because of their wide applications in engineering and in several industrial processes. In this thesis, the mixed convection boundary layer of a viscoelastic fluid past a circular cylinder with constant heat flux with internal heat generation is discussed. The governing boundary layer equations are transformed into a system of non dimensional equations by using the non dimensional variables. The non dimensional governing equations are then solved numerically using the Keller-box method by augmenting an extra boundary condition at infinity. Numerical solutions are displayed graphically for the velocity and temperature profiles with the effects of the mixed convection parameter, λ, the viscoelastic parameter, K, the Prandtl number, Pr, and the internal heat generation parameter, γ. Results shown that when internal heat generation, γ is increased, the velocity and temperature profiles are increased. Limiting cases of present results are compared with the published results and found to concur very well.

ABSTRAK

Kajian berkenaan aliran bedalir likat-kenyal kini menjadi semakin menarik dan bertambah penting kerana kegunaannya yang meluas dalam kejuruteraan dan juga dalam pelbagai industri pemprosesan. Dalam tesis ini, olakan campuran lapisan sempadan bagi bendalir likat-kenyal melintasi silinder membulat dengan fluks haba malar dengan penjanaan haba dalaman telah dibincangkan. Persamaan lapisan sempadan menakluk diubah ke dalam sistem persamaan tak bermatra dengan menggunakan pemboleh ubah tak bermatra. Persamaan menakluk tak bermatra seterusnya diselesaikan secara berangka menggunakan kaedah kotak-Keller dengan menambah syarat sempadan tambahan di infiniti. Penyelesaian berangka dipaparkan secara grafik untuk profil halaju dan suhu dengan kesan daripada parameter olakan campuran, λ, parameter likat-kenyal, K, nombor Prandtl, Pr, dan parameter penjanaan haba dalaman, γ. Keputusan menunjukkan apabila penjanaan haba dalaman meningkat, γ halaju dan juga suhu meningkat. Perbandingan di antara kes terhad dari keputusan terkini dengan keputusan yg telah diterbitkan menunjukkan persetujuan yg baik.

TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENT	iv
	ABSTRACT	v
	ABSTRAK	vi
	TABLE OF CONTENTS	vii
	LIST OF TABLES	ix
	LIST OF FIGURES	x
	LIST OF SYMBOLS	xii
	LIST OF APPENDIX	xiv
1	**INTRODUCTION**	
	1.1 Introduction	1
	1.2 Background of the Study	1
	1.3 Problem Statements	3
	1.4 Objectives of Study	3
	1.5 Scopes of Study	3
	1.6 Significance of the Study	4
	1.7 Thesis Outline	5
2	**LITERATURE REVIEW**	
	2.1 Introduction	6
	2.2 Effects of Internal Heat Generation	6
	2.3 Boundary Layer in Mixed Convection Flow	9
	2.4 Viscoelastic Fluid	11

2.5 Keller-box Method 13

3 MATHEMATICAL FORMULATION

3.1 Introduction 15

3.2 Basic Equations 15

3.2.1 Continuity Equation 18

3.2.2 Momentum Equation 19

3.2.3 Energy Equation 23

3.3 Solution Procedure 25

3.3.1 Continuity Equation 26

3.3.2 Momentum Equation 27

3.3.3 Energy Equation 29

4 RESULTS AND DISCUSSION

4.1 Introduction 32

4.2 The Validation of the Results 32

4.3 Results and Discussion 36

5 CONCLUSION

5.1 Introduction 46

5.2 Summary of Research 46

5.3 Suggestion for Future Research 48

REFERENCES 49

Appendix A 52

LIST OF TABLES

TABLE NO.	TITLE	PAGE
4.1	Comparison the values of $f''(0)$ and $\theta(0)$ for various values of λ when *Pr* = 1 and *K* = 0 (Newtonian fluid).	33
4.2	Comparison the values of $f''(0)$ and $\theta(0)$ for various values of λ when *Pr* = 7 and *K* = 0 (Newtonian fluid)	33

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
3.1	Physical model and coordinate system	16
4.1	Comparison of the local skin friction C_f with x for $K = 0.2$, Pr = 1 and various values of λ.	34
4.2	Comparison of the local heat transfer $\theta_w(x)$ with x for $K = 0.2$, Pr = 1 and various values of λ.	35
4.3	Comparison of the local skin friction C_f with x for $K = 1$, Pr = 1 and various values of λ.	35
4.4	Comparison of the local heat transfer $\theta_w(x)$ with x for $K = 1$, Pr = 1 and various values of λ.	36
4.5	Velocity profile f' for various values of λ at $K = 0.2$, Pr = 1 and $\gamma = 0.3$.	38
4.6	Temperature profile θ for various values of λ at $K = 0.2$, Pr = 1 and $\gamma = 0.3$.	39
4.7	Velocity profile f' for various values of λ at $K = 1$, Pr = 1 and $\gamma = 0.3$.	39
4.8	Temperature profile θ for various values of λ at $K = 1$, Pr = 1 and $\gamma = 0.3$.	40
4.9	Velocity profile f' for various values of K at $\lambda = 1$, Pr = 1 and $\gamma = 0.3$.	40
4.10	Temperature profile θ for various values of K at $\lambda = 1$, Pr = 1 and $\gamma = 0.3$.	41
4.11	Velocity profile f' for various values of Pr at $\lambda = 1$, K = 0.2 and $\gamma = 0.3$.	41

4.12	Temperature profile θ for various values of *Pr* at $\lambda = 1,$ *K* = 0.2 and $\gamma = 0.3.$	42
4.13	Velocity profile f' for various values of *Pr* at $\lambda = 1,$ *K* = 1 and $\gamma = 0.3.$	42
4.14	Temperature profile θ for various values of *Pr* at $\lambda = 1,$ *K* = 1 and $\gamma = 0.3.$	43
4.15	Velocity profile f' for various values of γ at $\lambda = 1,$ *Pr* = 1 and *K* = 0.2.	43
4.16	Temperature profile θ for various values of γ at $\lambda = 1,$ *Pr* = 1 and *K* = 0.2.	44
4.17	Velocity profile f' for various values of γ at $\lambda = 1,$ *Pr* = 1 and *K* = 1.	44
4.18	Temperature profile θ for various values of γ at $\lambda = 1,$ *Pr* = 1 and *K* = 1.	45

LIST OF SYMBOLS

a	-	Radius of cylinder
C_f	-	Local skin friction coefficient
$f(\eta), F(x,y)$	-	Dimensionless stream functions
K	-	Viscoelastic parameter
g	-	Acceleration due to gravity
G_r	-	Grasshof number
Pr	-	Prandtl number
R_e	-	Reynolds number
Q_0	-	Heat generation constant
Q_w	-	Heat transfer coefficient
T	-	Fluid temperature
T_w	-	Wall temperature
T_∞	-	Ambient temperature
$\bar{u}, \bar{v}$	-	Velocity components along $\bar{x}$, $\bar{y}$ directions
u, v	-	Velocity components along x, y-axes
$\bar{u}_e(\bar{x})$	-	Velocity outside boundary layer
$u_e(x)$	-	Non-dimensional velocity outside boundary layer
U_0	-	Dimensional constant
U_∞	-	Free stream velocity
x	-	Coordinate in direction of surface motion
y	-	Coordinate in direction normal to surface motion

Greek Symbols

α	-	Thermal diffusity of the fluid
β	-	Volumetric thermal expansion coefficient of the fluid
λ	-	Mixed convection parameter
η	-	Similarity variable
μ	-	Dynamic viscosity of the fluid
υ	-	Kinematic viscosity of the fuid
ρ	-	Fluid density
$\theta(x,y),\theta(\eta)$	-	Dimensionless temperatures
$\theta_w(x)$	-	Local wall temperature distribution
ψ	-	Stream function

Subscripts

w	-	Condition at the surface
∞	-	Condition at the ambient medium

Superscripts

‘	-	Differentiation with respect to η

LIST OF APPENDIX

APPENDIX	TITLE	PAGE
Appendix A	Fortran 77 Programming for Finding the Solutions of Mixed Convection Boundary Layer Flow in a Viscoelastic Fluid with Internal Heat Generation	52

CHAPTER 1

INTRODUCTION

1.1 Introduction

Discussion in this chapter begins with Section 1.2 which contains background of study. Next, Section 1.3 mentions about problem statements of the study while objectives of the study are then states in Section 1.4. Then, scopes of the study are then discussed in Section 1.5. Significance of the study are discussed in Section 1.6 while Section 1.7 consist of thesis outlines.

1.2 Background of Study

Mixed convection is the combination between forced and natural convection in boundary layer flow. Natural convection or also known as a free convection is caused naturally such as buoyancy effects due to density differences. This type of convection happens because of the temperature variations in the fluid. An example of natural convection is the cooling process in heat exchanger components. Meanwhile, forced convection occurs when a fluid flow is induced by an external force such as pump, fan or mixer.

Boundary layer theory is the platform of modern fluid dynamics as founded by Ludwig Prandtl in 1904. It gives a physical explanation of the flow of air and other fluids of small viscosity under circumstances of interest in many engineering applications. The boundary layer theory finds its application mostly in engineering such as in the calculation of the drag of a flat plate at zero incidences, the form drag of a ship and of an airplane wing, and a turbine blade. Not only that, it also used in the prediction of the maximum lift of an aerofoil and the heat transfer along a heated plate in uniform flow.

Boundary layers may be either laminar (layered), or turbulent (disordered) depending on the value of the Reynolds number. For lower Reynolds numbers, the boundary layer is laminar and the stream wise velocity changes uniformly as one move away from the wall. For higher Reynolds numbers, the boundary layer is said to be turbulent and the stream wise velocity is characterized by unsteady swirling flows inside the boundary layer.

Viscoelastic fluid exhibit both viscous and elastic characteristics. Viscous materials like honey resist shear and strain linearly with time when a stress is applied. The viscous property is due to the transport phenomenon of the fluid molecules. Meanwhile, elastic materials strain instantaneously once stretched and it will quickly return back to the original state when the stressed is removed. The elastic property is due to the chemical structure and configuration of the polymer molecule.

Internal heat generation is one of the affects that give a great importance on heat transfer. Internal heat generation plays a significant role in many practical situations such as Joule heating due to the flow of an electric current through a conducting fluid, radioactive heating as well as cooling of molten glass and heating of water in a solar collector.

In this study, we will be interested to investigate the effects of internal heat generation in mixed convection boundary layer flow in a viscoelastic fluid past a horizontal circular cylinder.

1.3 Problem Statement

The study will explore the following questions. How the internal heat generation affects the fluid flow characteristics such as velocity and temperature profiles? Is the constant surface heat flux influence the mixed convection viscoelastic boundary layer flow?

1.4 Objectives of Study

The following objectives will be achieved from this project:

- To determine the mathematical formulation of mixed boundary layer flow in a viscoelastic fluid past a horizontal circular cylinder.
- To transform the dimensional governing equations into non dimensional governing equations.
- To observe the effects of internal heat generation in viscoelastic fluid with constant heat flux through graphs and tables.

1.5 Scope of Study

This study will take into consideration of two dimensional incompressible viscoelastic fluid problems. For this study, the problem will be narrow down to mixed boundary layer flow of a viscoelastic fluid past a horizontal circular cylinder with heat flux with the effects of internal heat generation. The numerical method chosen is two dimensional Keller-box schemes in order to obtain numerical results. Solutions are then compared with published results for accuracy purposes.

1.6 Significance of Study

Applications involving viscoelastic fluid are quite broad and include such areas as micro dispensing of bioactive fluids through high throughput injection devices, creation of cell attachment sites, scaffolds for tissue engineering, coatings and drug delivery systems for controlled drug release, and viscoelastic blood flow past valves.

The boundary layer problem of viscoelastic fluid theory has generated many interests and become more important in their recent years because their wide applications in industrial manufacturing processes such as in petroleum drilling, manufacturing of foods and papers. In engineering problem, viscoelastic fluid is possible to reduce frictional drags on the hulls of ships and submarines.

In environment and engineering field, mixed convection over a cylinder is an essential problem. The study of heat generation in all types of fluid is very important in viewing several physical problems include those dealing with chemical reaction and those concerned with dissociating fluid. The possible heat generation can affects the temperature distribution and particle deposition rate. This may occurs in such applications related to nuclear reactor cores, fire and combustion modelling, electronic chips and semi conductor wafers.

Thus, the study of the effects on internal heat generation on mixed convection boundary layer flow in a viscoelastic fluid past a horizontal circular cylinder with constant heat flux is important due to the strong applications in real life. It can be very useful to create an effective and efficient heat exchanger component and design. Besides, the study of heat generation to the fluid flow will be very helpful in order to solve the problem related to Computational Fluid Dynamics problems.

1.7 Thesis Outline

In general, this report consists of five chapters. Chapter 1 discusses the background of study, problem statements, objectives of the study, scopes and significance of study in numerous applications especially in engineering. Then, in Chapter 2, the literature review has been extensively studied. Further, Chapter 3 contains the mathematical formulation of the mixed convection boundary layer flow of viscoelastic fluid. The transformations of the governing equations are also showed from the dimensional to non dimensional equations in this chapter. Later, results and discussions are discussed in Chapter 4. In this chapter, results are presented in the form of tables and graphs. Finally, Chapter 5 contains a summary of the study followed by some recommendations for the future research.

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