# THE FORD FULKERSON ALGORITHM ON A CASE STUDY OF THE DISSERTATION SCHEDULING PROBLEM

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To my beloved

*mak, Kamariah Sarwan*

*akak, Raja Nazira Raja Mohd Nazir*

*Muhamad Azmi Saru*

and

*all my friends*

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# ABSTRACT

Scheduling problem are often hard and time consuming to solve. It is considered as one of the most difficult problem. Therefore, the development of a network model for timetabling problem is necessary to solve the problem by determining the maximum flow in the network. The purpose of this study is to develop a schedule of students versus examiner for proposal presentation exercise at Mathematical Sciences Department, Faculty of Science, UTM. In this research one of the network flow algorithm called Ford Fulkerson algorithm is applied in order to find the maximum flow of the network. Given a set of constraint consisting of a set of programs, a set of students, a set of lecturers, a set of days and periods and a set of classrooms, the problem is to assign programs to students, students to lecturer, lecturer to days and period, and lecturer to classroom subject to a set of hard and soft constraints. The computer software, MATLAB version 7.11.0 will be used to generate the Ford Fulkerson Algorithm. This is because it manages to successfully satisfy most of the constraints and optimizes the utilization of classrooms effectively and efficiently.

# ABSTRAK

Masalah penjadualan biasanya sukar dan memerlukan masa untuk diselesaikan. Ia dianggap sebagai salah satu masalah sukar yang dihadapi oleh pihak universiti pada hari ini. Oleh itu, pembangunan model rangkaian bagi masalah penjadualan adalah perlu untuk menyelesaikan masalah ini dengan menentukan aliran maksimum dalam rangkaian tersebut. Kajian ini dilakukan bertujuan menbangunkan model jaringan bagi masalah penjadualan yang melibatkan pelajar dan pensyarah untuk pembentangan proposal di Jabatan Matematik Fakulti Sains, UTM. Dalam kajian ini salah satu algoritma aliran rangkaian dipanggil Ford Fulkerson algoritma digunakan untuk mencari aliran maksimum rangkaian. Diberi satu set program, set pelajar, pensyarah, set hari dan tempoh dan menetapkan bilik darjah masalahnya ialah untuk memberikan program untuk pelajar, pelajar kepada pensyarah, pensyarah kepada hari dan tempoh, dan pensyarah kepada kelas tertakluk kepada satu set batasan keras dan lembut. Perisian komputer iaitu MATLAB versi 7.11.0 akan digunakan dalam membina algorithma Ford Fulkerson. Ini adalah kerana jadual tersebut berjaya memenuhi kebanyakan batasan serta nebgoptimumkan penggunaan kemudahan sedia ada secara efektif dan efisien.

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# INTRODUCTION

## 1.1 Introduction

Operational research involves the study of mathematical models for complex organizational systems. In other words, operational research describes the application of information technology for informed decision making. During World War II, the allies invented a new scientific field which is Operational Research (OR) to help complex military organizations cope with rapid technological change(Rau, 2005). The allies also used operational research to determine how best to use radar devices. This field area is very important in testing and validating tactical doctrines in several branches of warfare.

Operational research become more advance after the World War II. Todays global markets and instant communications mean that customers expect high-quality products and services when they need them, where they need them. The techniques began to be applied more widely to problems in business, industry and society.

Operations research is a discipline based on the formulation of models and the development and application of algorithms for solving problems in decision making. Verifying and validating alternatives are an important component of decision making process. When a decision is reached, it may not be the absolute optimum, the point where all goal have been achieved.

This study involves the construction of mathematical model. The collection of logic and mathematical relationship are presented in the model for easy understanding. The formulation phase is the most crucial, because if the problem is not set up correctly, the solution found may be the correct answer to the wrong problem. Element that are not very important to the problem are to be ignored, hopefully the solution obtained from the model has value with regard to the original problem. The technological advances of computer in operational research modeling has make this field become much easier. Line control and mathematical programming model can be stimulated in large scale by increasing computing power.

Multi-criteria decision making (MCDM) is the most well known branch of decision making. According to many authors MCDM is divided into Multi-Objective Decision Making (or MODM) and Multi-Attribute Decision Making (or MADM). MODM studies decision problems in which the decision space is continuous. A typical example is mathematical programming problems with multiple objective functions. The first reference to this problem, also known as the ”vector-maximum” problem, is attributed to (Boland and Goh, 1992). On the other hand, MADM concentrates on problems with discrete decision spaces. In these problems the set of decision alternatives has been predetermined. Several commonly used method such as the TOPSIS (Technique for order performance by similarity to ideal solution) and RR (Relative Ratio) were proposed to solve the MADM problems (Triantaphyllou and Ray, 1998).

Some of the MCDA methods are analytic hierarchy process, analytic network process, data envelopment analysis and inner product of vectors. A primary advantage to MCDA is the provision of a highly structured decision-making technique. Within a decision problem, objectives (criteria) are used to evaluate the performance of an alternative. We have then the classic form of an optimization problem: the objective function is the single criterion; the constraints are the requirements on the alternatives. Depending on the form and functional description of the optimization problem, different optimization techniques can be used for the solution.

## 1.2 Research Background

Effective decision making depends on reliable up to date data available on databases and computer network. An amazing useful technique, network flow programming method describes a type of model that is a special case of the more general linear program. Network flow problems are central problems in operations research, computer science, and engineering and they arise in many real world applications. Many classical network models can be solved very quickly, they have naturally integer solutions, and they provide a modeling language for real world problems that is easier to understand than the language of linear programming.

Network model are also applicable to an enormous variety of decision problems that can be modeled as networks optimization problems and solved efficiently and effectively. Some of these decision problems are really physical problems such as transportation or flow of commodities. However, many network problems are more of an abstract representation of processes or activities such as the critical path activity network in project management.

Problem can be included as a network flow programs such as the shortest path problem, transportation problem, the maximum flow problem and the pure minimum cost flow problem. Network flow models and algorithms are useful to solve scheduling problems. The problem of scheduling are usually very difficult ones with a large computational complexity.

Presentation timetable problem are mainly about allocating resources such as students, lecturers, program, classroom and time slots to present. The resources are limited to the associated capacity and the distribution of the resources should be well-organized to ensure that all the requirement are fulfilled. The requirements in this context refer to the availability of teachers, classrooms and duration of time slots. The presentation scheduling problem can be classified as a maximum flow problem in

network. The network is based on a multipartite graph, whose set of vertices is formed by the source, the sink and subsets with elements to be placed in scheduling.

The maximum flow technique is to find the maximum flow of any quantity or substance through a network. This problem arises in a variety of situations in which the arc capacities are not fixed but are functions of a single parameter, and the goal is to find the value of the parameter such that the corresponding maximum flow (or minimum cut) meets some side condition. The usual approach to solving such problems is to use a maximum flow algorithm as a subroutine and to use either binary search, monotonic search, or some other technique, to find the desired value of the parameter.

Many alternatives have been proposed to solve the maximum flow problem. Some alternatives are Ford Fulkerson algorithm, Dicnic algorithm and Dinamic algorithm. The maximum flow problem was first formulated in 1954 by T. E. Harris as a simplified model of Soviet railway traffic flow. In 1955, Lester R. Ford and Delbert R. Fulkerson created the first known algorithm, the Ford Fulkerson algorithm (Andrew V. Goldberg and Tarjan, 1990). The Ford-Fulkerson algorithm , being the first algorithm created for the maximum flow problem, is not as complex as some of the later algorithms. It was used as a stepping-stone towards more efficient and elegant algorithms with more variables taken into consideration.

Generally, the decision of feasible value are delimited by a set of constraints that are described by mathematical functions of the decision variables. The objective function is an equation to be optimized given certain constraints and with variables that need to be minimized or maximized using nonlinear programming techniques. In some problem there may be more than one competing objectives and need to trade-off the objectives against each other. Since Ford Fulkerson algorithm is widely used in solving the maximum flow problem compared to other methods which is more complicated, this makes the Ford Fulkerson algorithm as the first choice to solve the timetabling problem. In addition, Ford Fulkerson algorithm is the pioneer to all alternative stated in solving maximum flow problem and this research is interested in exploring the most basic method to solve the timetabling problem.

## 1.3 Problem Statement

The aims of this research is to develop a schedule of students versus examiners for the proposal presentation exercise at the Mathematical Sciences Department Faculty of Science, UTM. The data will be taken from the problem of scheduling students versus examiners for the proposal presentation exercise at the Mathematical Sciences Department, Faculty of Science, UTM.

## 1.4 Objectives of the Study

The objectives of this research are:

(i) To model the dissertation scheduling problem as a network flow problem.

(ii) To apply Ford Fulkerson algorithm to dissertation scheduling problem.

(iii) To analyse the effectiveness of Ford Fulkerson algorithm approach.

## 1.5 Scope of the Study

The scope of this study is to focus on the construction of a schedule of student versus examiners for the proposal presentation exercise at the Mathematical Sciences Department, Faculty of Science, UTM. The constraints of this research involve course work (Dissertation 1),research students and PHD (assessment 1). In this research of constraints are the set of programs, the set of students, the set of lecturers, the set of days and periods and the set of classrooms. The problem is to assign programs to students, students to lecturers, lecturers to days and periods, and lecturers to

classrooms subject to a set of hard and soft constraints. In addition, this study will make all lecturers involved in the presentation.

The Ford Fulkerson algorithm will be applied in a scheduling problem. Since the algorithm is widely used and simple compared to Dicnic algorithm and Dinamic programming which is more complicated, this makes the Ford Fulkerson algorithm as the first choice to solve the scheduling problem.

MATLAB will be used to develop the coding in solving the scheduling problems. MATLAB is chosen because of is high-level language and interactive environment that performs computationally intensive tasks faster than traditional programming languages.

## 1.6 Significance of Findings

This study will en-change one more collection of research on solving timetable problem, especially the construction of presentation timetable using approach graph theory and more specific, the application of Ford Fulkerson algorithm. Thus, application of Ford Fulkerson algorithm using MATLAB is the main purpose of this study to see whether this approach is good enough for helping the problem of proposal presentation exercise in Mathematical Sciences Department, Faculty of Science, UTM.

## 1.7 Thesis Organization

The first chapter serves as an introduction to the whole thesis. The contents of this thesis have been arranged by chapter.

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