

FINITE ELEMENT SIMULATION OF REINFORCED CONCRETE DEEP BEAM  
WITH OPENING STRENGTHENED WITH FRP LAMINATES

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*This project is dedicated to my loving father and mother  
who have taken great pains to see me prosper in life.*

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## ABSTRACT

Deep beams are typically used in tall buildings, transfer floors and offshore structures and have a lot of advantages which make their use the most feasible option in many situations. The building codes do not give any explicit guidance to designing this type of structure with opening. Furthermore the theoretical model for concrete deep beam strength in shear composite FRP is very limited in the literature. A series of experimental tests from previous literature were carried out to investigate the behavior and performance of reinforced concrete deep beams with opening and concrete beams strengthened in shear with composite material such as steel fiber reinforced (SFR), aramid fiber reinforced polymer (AFRP), glass fiber reinforced polymer (GFRP) and carbon fiber reinforced polymer (CFRP) and others. This current study aims at developing a 3D finite element model of deep beam with opening and deep beam with opening strengthened in shear externally with CFRP sheets and analyzing them using explicit method in ANSYS. It will be used for modeling shear strength and comparison between them for load - deflection curve. The failure load and deformation that acquired from the 3D finite element model will be verified against published experimental data. That comparison between the numerical and the experimental results asserted that good correlation for the load and deflection curves was recorded, ANSYS can predict the crack patterns which it have good agreement with those observed in experimental work. Moreover numerical simulation results were obtained stress and strain in all specimen and that cannot be easily obtained in experimental tests.

## ABSTRAK

Rasuk dalam biasanya digunakan dalam bangunan tinggi, lantai dan struktur luar pesisir dan mempunyai banyak kelebihan yang membuatkan penggunaannya sebagai pilihan yang paling sesuai dalam kebanyakan keadaan. Kod bangunan tidak memberi apa-apa petunjuk jelas tentang bentuk anggota struktur ini dengan bukaan. Satu siri ujikaji yang diperolehi daripada literatur telah digunakan untuk mengkaji tingkah laku dan prestasi rasuk konkrit dalam bertetulang dengan bukaan dan rasuk konkrit diperkukuh dalam ricihan dengan bahan rencam seperti gentian keluli bertetulang (SFR), polimer bertetulang gentian aramid (AFRP), polimer bertetulang gentian kaca (GFRP) dan polimer bertetulang gentian karbon (CFRP) dan lain-lain. Kajian ini bertujuan membangunkan model 3D unsur terhingga rasuk dalam dengan bukaan dan rasuk dalam dengan bukaan diperkukuhkan dalam ricihan menggunakan CFRP, seterusnya menganalisis rasuk tersebut menggunakan kaedah explicit dalam ANSYS. Ia akan digunakan dalam permodelan kekuatan ricih dan perbandingan akan dibuat untuk lengkung beban-pesongan. Beban gagal dan ubahbentuk yang diperolehi daripada model unsur terhingga 3D akan disahkan dengan data ujikaji. Perbandingan antara keputusan berangka dan keputusan data ujikaji menunjukkan hubungan adalah baik untuk lengkung beban-pesongan. ANSYS dapat meramal corak keretakan dan persetujuan yang baik dicapai melalui pemerhatian ujikaji. Keputusan simulasi berangka juga didapati dapat meramal tegasan dan terikan untuk kesemua specimen yang tidak dapat diukur melalui ujikaji.

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## LIST OF SYMBOLS

FEM	-	Finite Element Method.
FRP	-	Fiber reinforced polymer.
CFRP	-	carbon fiber reinforced polymer.
AFRP	-	aramid Fiber reinforced polymer.
GFRP	-	Glass Fiber reinforced polymer.
STM	-	Strut and Tie Method.
RC	-	Reinforcement Concrete.
ACI	-	American Concrete Institute.
$V_u$	-	Shear force because of the factored loads.
$V_n$	-	The nominal shear strength.
$V_c$	-	The nominal shear strength provided by concrete.
$V_s$	-	The nominal shear strength provided by steel shear reinforcement.
$f_c$	-	Specified compressive strength of concrete.
$b_w$	-	Web width.
$d$	-	Distance from extreme compression fiber to centroid of longitudinal tension reinforcement.
$V_u$	-	Factored shear force at critical section.
$\rho$	-	Ratio of tension reinforcement.
$M_u$	-	Factored moment occurring simultaneously with $V_u$ at the critical section.
$l_n$	-	clear span
$A_v$	-	Area of shear reinforcement perpendicular to flexural tension reinforcement within a distance $s$ .

$A_{vh}$	- Area of shear reinforcement parallel to flexural tension reinforcement within a distance $s_2$ .
$A_{fv}$	- Area of FRP shear reinforcement with spacing $s$ , ( $\text{mm}^2$ )
$f_{fe}$	- Effective stress in the FRP; stress level attained at section failure, (MPa)
$\alpha$	- Effective stress in the FRP; stress level attained at failure, (MPa)
$d_f$	- Depth of FRP shear reinforcement
$s_f$	- Spacing FRP shear reinforcing
$n$	- Number of plies of FRP reinforcement
$t_f$	- Nominal thickness of one ply of the FRP reinforcement. (mm)
$w_f$	- Width of the FRP reinforcing plies.
$\varepsilon_{fe}$	- Effective strain level in FRP reinforcement; strain level attained at section
$E_f$	- Failure.(mm / mm)
$k_v$	- Tensile modulus of elasticity of FRP.(MPa)
$k_1, k_2$	- Bond-reduction coefficient
$W_{int.}$	- Internal work (strain energy)
$W_{ext.}$	- External work (work done by the applied force)
$\{\varepsilon\}$	- Elements of virtual strain vector
$\{\sigma\}$	- Elements of real stress vector
$dV$	- Infinitesimal volume of the element
$[D]$	- Constitutive matrix
$[N]$	- Shape function matrix
$\{a\}$	- Unknown nodal displacements vector (local displacements)
$\{U\}$	- Body displacements vector (global displacements).
$[B]$	- strain-nodal displacement relation matrix, based on the element shape functions
$\{F\}$	- nodal forces applied to the element
$[K^e]$	- element stiffness matrix
$[K]$	- overall structural stiffness matrix
$n$	- total number of elements
$f_c$	- stress at any strain $\varepsilon$ , MPa
$\varepsilon$	- strain at stress $f$

- $\varepsilon_o$  - strain at ultimate compressive stress and  $f'_c$
- $E_c$  - concrete elastic modulus, MPa
- $\sigma_{xp}$  - principal stresses in principal directions
- $\sigma_{yp}$  - principal stresses in principal directions
- $\sigma_{zp}$  - principal stresses in principal directions
- $F$  - function of principal stress state ( $\sigma_{xp}, \sigma_{yp}$  &  $\sigma_{zp}$ )
- $f_t$  - ultimate uniaxial tensile strength.
- $f_{cb}$  - ultimate biaxial compressive strength
- $f_1$  - ultimate compressive strength for a state of biaxial compression superimposed on hydrostatic stress state ( $\sigma_h^a$ ).
- $f_2$  - ultimate compressive strength for a state of biaxial compression superimposed on hydrostatic stress state ( $\sigma_h^a$ )
- $\sigma_h^a$  - ambient hydrostatic stress state.
- $\sigma_h$  - hydrostatic stress state

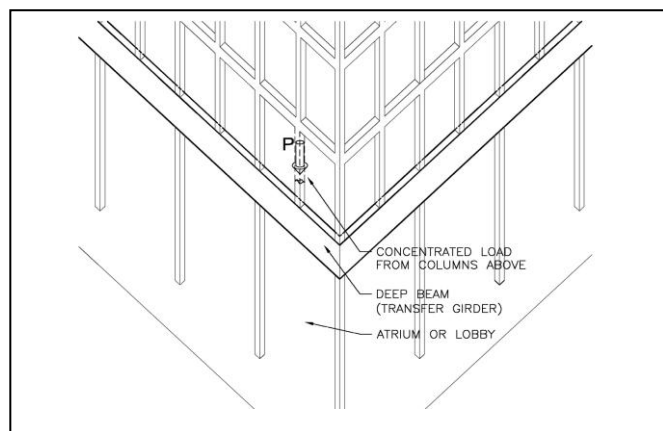
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## CHAPTER 1

### INTRODUCTION

Reinforced concrete deep beams are commonly found in a range of structural elements in building construction. They are usually used in tall building functioning as transfer girders, to transfer and distribute the load from one or more columns or piles due to its high stiffness (Figure 1.1), Concrete wall is also often categorized as a deep beam for instance the shear wall in a bunker or in a ground floor, water tanks and silos since they all have same performance for the deep beam.



**Figure 1.1:** Deep beams found in multi-story buildings.



Many pipes and ducts are required in the modern building construction to provide essential services like electricity, telecommunications cables and air-conditioning, ,sewage, water supply and others, these ducts and pipes are usually placed under the soffit of the beam and thus covered by false ceiling, “Dead area” is a term used to refer to the area between the slab and false ceiling, when deep beam in the construction is used with services under the soffit of the deep beam it increases the height for the “dead area “ and reduces the clear building story height, thru creating an opening in the deep beam for services to pass which helps keep the height of clear building story.

A prevalent obstacle in deep beam with web opening is obviously visible cracks with load about 30% to 50% of the total load in conditions of serviceability; this has effect on the aesthetics and durability of the structure.

Numerous methods have been established and applied in practical ways to strengthened or repaired reinforced concrete (RC), for the previous twenty five years the common method was bonding the critical or damaged areas at sides or soffits in the beam by epoxy bonded steel plate to enhance the flexural and/or shear strength, the main weaknesses of this method is the steel disadvantages, the most obvious of which it is the amount and weight of the steel added to the structure, and the case of steel corrosion when being exposed to a harsh environment, Also, the weight of the plate needs special tools and equipment to properly install it, in the end of the last century, the fiber reinforced polymer (FRP) was an alternative to steel, for (FRP) offers higher strength than steel, high modulus for some kinds of (FRP) as carbon fiber reinforced polymer (CFRP) and it's not difficult to handle and install it due to its light weight.

## **1.1 Fiber Reinforced Polymers (FRP):**

Fiber reinforced polymer (FRP) materials are new materials that attracted structural engineers in the field of concrete construction, mainly for the usage as strengthening materials for reinforced concrete (RC) beams.

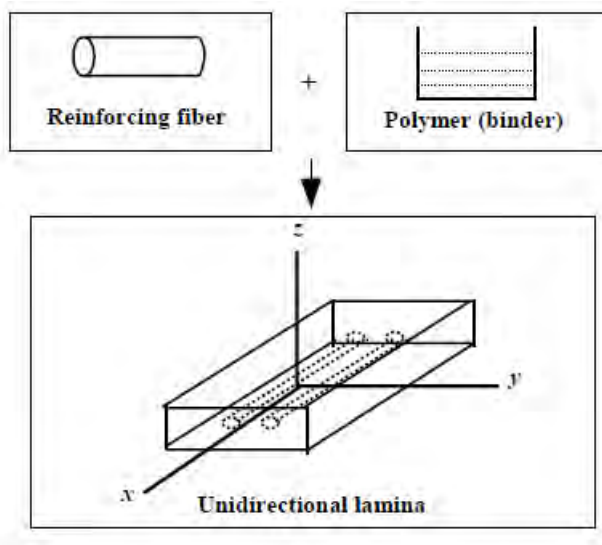
FRP is a compound material made of high polymer matrix and strength fibers. The FRPs is mainly used for civil engineering applications which involve carbon fiber reinforced polymer (CFRP), aramid fiber reinforced polymer (AFRP) and glass fiber reinforced polymer (GFRP). The matrix is used to join the fibers together, transfer the forces between the fibers and to protect the fibers from environmental and external mechanical damage.

It is significant that the matrix are capable of taking higher tension than the fibers, otherwise, there will be cracks in the matrix so the fibers will fail and the fiber will be unprotected (Nordin, H., 2003). Figure 1.2 shows a schematic of FRP composites.

### **1.1.1 Advantages of FRP**

Compared to steel, FRP materials have higher strength and lower density. When these properties are put together they lead to fiber composites having a strength/weight ratio, in some cases, higher than steel plate. The installation and handling of FRP is considerably easier than steel due to its lower weight. These properties are certainly

significant when installation in cramped locations is done. Further works like works on building floor slabs and soffits of bridges are conducted through man-access platforms instead of full scaffolding.



**Figure 1.2:** Schematic of FRP composites (Kaw, A. K., 1997)

It is known that steel plates need heavy lifting gear and are to be clenched in place while the bonding agent gets its strength while bolts are fixed through the steel plate into the parent concrete to uphold the plate while the paste dries. On the contrary, the use of FRP plate or sheet material resembles placing wallpaper; when it has been carefully rolled on, to remove entrapped air and extra adhesive, it may remain unsupported. At this point, no bolts are necessary; actually, unless extra cover plates are attached; the use of bolts would earnestly weaken the material. Additionally, there is no jeopardy of damaging the current reinforcement since there is no need to drill into the structure to install bolts or other mechanical anchors. Fibre compound materials are available in very long lengths whereas steel plate is usually limited to 6 m. The availability of long length materials and the flexibility of the material also make installation simpler:

- They do not require laps and joints.
- The material can go in for irregularities in the concrete surface shape.
- The material can follow a bent shape while steel plate has to be pre-bent to the needed radius.
- The material can be easily installed behind existing services.
- Since the material is thin, Overlapping, which is required when strengthening in two directions, is not a problem in this case.

If correctly specified, the materials fibers and resins are durable, and do not need much maintenance. It is relatively simple to repair them, if they are damaged in service, which is achieved by supplementing an additional layer. Fiber composites, when used, do not considerably increase the structures' weight or the members' dimensions. This may be mainly significant for bridges and structures with limited headroom as well as for tunnels.

As for sustainability and environmental impact, research shows that the energy needed to make FRP materials is not as much of that for conventional materials. The transport of FRP materials has lower environmental impact, due to their light weight. These numerous factors combined result in an obvious quicker and simpler strengthening process in comparison of when using steel plate. This is mainly important for bridges due to the high costs of lane closures and possession times on railway lines and major highways. In Switzerland, as a result of these factors, it has been assessed that almost 90% of the market for plate strengthening has been taken by carbon plate systems.

### **1.1.2 Disadvantages of FRP**

The risk of fire, vandalism or accidental damage, unless the strengthening is protected, is considered the core weaknesses of externally strengthening structures with fibre composite materials. The risk of soffit reinforcement being hit by over-height vehicles is a precise concern when building bridges over roads. The relatively high cost of the materials is considered a remarkable disadvantage of using FRP for strengthening. Nevertheless, there should be comparison conducted based on the complete strengthening exercise; in particular cases, the expenses can be a lesser amount than that of steel plate bonding. The lack of experience of the techniques and suitably qualified staff to accomplish the work will be a disadvantage in the perception of many clients. As a final point, an important disadvantage is the shortage of accepted design standards.

## **1.2 Problem statement**

Shear failure of reinforced concrete (RC) is extremely dangerous and takes place suddenly with no warning. Thus, it is significant to comprehend the behavior of shear of reinforced concrete deep beams with opening. This takes account of the behavior of the beam before and after the cracking. Usually, steel plate as the external strengthened mechanism is used for repair of reinforced concrete. However, steel plate shows disadvantages in terms of steel plate corrosion caused by the environment. For that reason, the present study tends to use CFRP as it has enhanced performance concerning the reaction with the environment in comparison to steel plate. The CFRP composite strips were utilized to reinforce concrete externally at a known failure plane to endure shear stresses in shear friction.

Due to the desire of development of a reliable analytical software-model, preceding researchers established some models based on combination of Software simulation using element types, analysis types, and material properties. The outcomes could not totally match experimental data, even though some were very close by via interface elements and explicit analysis. The present study is attempts to produce a model which closely resembles the experimental data available in previous literature. Currently there is no modeling data available by using FEM software in the literature on the applicability of employing externally bonded FRP sheets to intensify the shear capacity of RC deep beams containing openings.

### **1.3 Research Aim and Objectives**

The aim of the current study is to use interface finite elements for modeling shear strengthening in RC deep beam with opening using CFRP wrapping. In order to reach this aim, the following objectives must be fulfilled:

1. To investigate the effectiveness of using externally bonded CFRP strips in repairing and strengthening of RC deep beam with opening.
2. To study the behavior of RC deep beam with opening strengthened in shear with CFRP sheet by using finite element analysis.
3. To provide numerical evidence that would lead to better understanding of the interrelationship between the opening location, size, shear strength, and failure mode of RC deep beams strengthened with FRP sheets.
4. To make a comparison between the experimental results from the literature and theoretical results obtained from three-dimensional nonlinear finite element analysis adopted from ANSYS (Release 14.0) computer program.

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