ALTERNATING GROUP EXPLICIT METHOD FOR EDGE DETECTION ON BRAIN AND BREAST TUMOUR IMAGES

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requirements for the award of the degree of
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To my beloved family,

Md. Zubaidin Muhamad @ Mamat

Rohaya Ismail

Zawani Md. Zubaidin

Md. Zulkarami Md. Zubaidin

Zatil Syarafana Md. Zubaidin.

With love and much thanks.

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ABSTRACT

In this research, we used Geodesic Active Contour (GAC) model to detect the edges of brain and breast tumor on MRI images. An additive operator splitting (AOS) method is employed in the two dimensional GAC model to maintain the numerical consistency and makes the GAC model computationally efficient. The numerical discretization scheme for GAC model is semi-implicit and unconditional stable lead to sparse system matrix which is a block tridiagonal square matrix. The proposed AOS scheme capable to decompose the sparse system matrix into a strictly diagonally dominant tridiagonal matrix that can be solved very efficiently likes a one dimensional problem. Gauss Seidel and AGE method is used to solve the linear system equations. The AGE employs the fractional splitting strategy which is applied alternately at each half (intermediate) time step on tridiagonal system of difference scheme and it is proved to be stable. This advanced iterative method is extremely powerful, flexible and affords users many advantages. MATLAB has been choosing as the development platform for the implementations and the experiments since it is well suited for the kind of computations required. In the implementation of GAC-AOS model for edges detection of tumor, the experimental results demonstrate that the AGE method gives the best performance compared to Gauss Seidel method in term of time execution, number of iterations,.RMSE, accuracy and computational cost.

ABSTRAK

Model kontur aktif Geodesik digunakan dalam kajian ini untuk menjejak sisi-sisi tumor bagi barah otak dan payudara pada imej MRI. Kaedah agihan separa tersirat (AOS) diguna dalam model GAC dua dimensi untuk mengekalkan kekonsistenan berangka dan membolehkan pengiraan dibuat secara berkesan bagi model GAC. Skema pendiskritan berangka bagi GAC model ialah dalam skema separuh tersirat dan stabil secara tidak mutlak. Hal ini menghasilkan sistem matrik yang jarang dan besar. Sistem matrik adalah dalam bentuk segi empat yang mempunyai bilangan baris dan kolum yang sama dan merupakan matrik blok yang mempunyai tiga unsur pada pepenjuru. Skema AOS mampu menguraikan sistem matrik yang jarang kepada sistem matrik yang hanya mempunyai tiga unsur pada pepenjuru. Sistem matrik ini boleh diselesaikan secara berkesan sepertimana menyelesaikan masalah satu dimensi. Kaedah Gauss Seidel dan AGE digunakan untuk menyelesaikan persamaan sistem linear. Kaedah AGE adalah berorientasikan strategi belahan paras masa terkini secara berselang-seli bagi sistem persamaan linear tiga pepenjuru dan terbukti stabil. Kaedah lelaran yang maju ini adalah sangat berkuasa, fleksibel, dan memberi banyak kelebihan kepada para pengguna. Perisian MATLAB dipilih sebagai platform pembangunan kerana ia sesuai untuk semua pengiraan yang diperlukan. Dalam pelaksanakan model GAC-AOS untuk mengesan sisi-sisi tumor, hasil kajian menunjukkan bahawa kaedah AGE memberi persembahan yang bagus berbanding kaedah Gauss Seidel dalam aspek pelaksanaan masa, bilangan lelaran, RMSE, ketepatan dan kompleksiti pengiraan.

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LIST OF ABBREVIATIONS

ACM	-	Active contour model
ADI	-	Alternating Direction explicit
AGE	-	Alternating Group Explicit
AOS	-	Additive Operator Splitting
C-N	-	Crank-Nicholson
GAC	-	Geodesic Active Contour
GE	-	Group explicit
GGAC	-	Generalized Geodesic Active Contour
GS	-	Gauss Seidel
IADE	-	Iterative Alternating Decomposition Explicit
JPEG	-	Join Photographic Experts Group
LSE	-	Linear system equations
MRI	-	Medical Resonance Image
PDE	-	Partial differential equation
RMSE	-	Root mean square error
SOR	-	Successive Over Relaxation

LIST OF SYMBOLS

u	-	Edge detection of tumor
x, y	-	The space at coordinate system
ε	-	Tolerance value
E_{snake}	-	Energy of snake
E_{int}	-	Internal energy
E_{ext}	-	External energy
E_{image}	-	Image forces
E_{con}	-	External constraint forces
E_{line}	-	Line functional
E_{edge}	-	Edge functional
E_{term}	-	Termination functional
α, β, w	-	Weighted for energy of snake
∇	-	Gradient operator
Ω	-	Image domain
$\mathcal{I}$	-	Pixel of interest
$\mathcal{J}$	-	Neighbourhood pixel of pixel of interest
$\mathcal{N}(\mathcal{I})$	-	The set of four neighbourhood pixel
ρ	-	Acceleration parameter
g	-	Stopping function
v	-	Positive constant
τ	-	Time
k	-	Number of iterations

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CHAPTER 1

INTRODUCTION

1.1 Introduction

In the past several years, active contour models have been widely applied in computer vision especially in image processing since they were first introduced by Kass *et al.* (1988). It is an effective tool for image segmentation, object tracking, shape recognition, edge detection and stereo matching.

According to representation and implementation, active contours can be classified into two types which are parametric active contours Kass *et al.*(1988), Cohen(1991), Eviatar and Samorjai(1996), Xu *et al.*(2000), Wang *et al.*(2009) and geometric active contours Caselles *et al.*(1993), Caselles *et al.*(1997), Xu *et al.*(2000), Goldenberg *et al.*(2001).

Parametric active contours are represented explicitly as parameterized curves or splines. Geometric active contours are represented implicitly as level sets of two-dimensional distance functions which its evolution does not depend on particular parameterization. These models are based on the curve evolution theory and geometric flows, Caselles *et al.*(1993)

In (x, y) plane, the contour is defined as a parametric curve (Rosdiana, 2012),

$$v(s) = (x(s), y(s)) \tag{1.1.1}$$

where $x(s)$ and $y(s)$ are the coordinates throughout the contour as shown in Figure 1.1. Parameter s is independent and with domain $s \in [0,1]$.

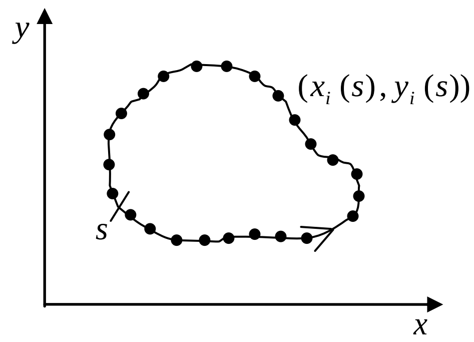

Figure 1.1: Parametric curve in (x, y) plane

In 1988, Kass *et al.* make a contribution in image processing field with the introduced Snake active contour model. This model is a parametric active contour model. The contour of Snake model is a controlled continuity spline associated to its energy functional which is the sum of two terms of internal and external forces.

Snake is called as an active model because it always minimizing its energy functional to develop the contour line. The implementation of Snake model is based on the image processing to the targeting region. The energy functional of Snake model is defined as (Kass *et al.*,1988),

$$\begin{aligned} E^*_{snake} &= \int_0^1 E_{snake}(v(s))\,ds \\ &= \int_0^1 E_{\text{int}}(v(s)) + E_{ext}(v(s))\,ds \\ &= \int_0^1 E_{\text{int}}(v(s)) + E_{image}(v(s)) + E_{con}(v(s))\,ds \end{aligned} \tag{1.1.2}$$

where

E_{snake}	: Energy functional of Snake
E_{int}	: Internal energy of Snake to smooth the edge curve
E_{ext}	: External Snake forces lead the curve to the edges of object in the image.
E_{image}	: Image forces pushing the Snake toward the desired object.
E_{con}	: External constraint forces

The internal energy can be expressed as (Kass et al.,1988),

$$E_{\text{int}} = \frac{(\alpha(s)\left| v_s(s) \right|^2 + \beta(s)\left| v_{ss}(s) \right|^2)}{2} \tag{1.1.3}$$

The first-order term v_s is controlled by $\alpha(s)$ and the second-order term v_{ss} is controlled by $\beta(s)$. The function of first-order term is to make the Snake act like a membrane while the second-order term is to make Snake act like a thin-plate.

The relative importance of the membrane and thin-plate terms can be control by adjusting the weighted $\alpha(s)$ and $\beta(s)$. By reviewing some previous researches, a constant applied as a coefficient for the first-order term in (1.1.3) i.e., $\alpha(s) = \alpha$, Wang *et al.*(2009). While the weight of $\beta(s)$ need to set as zero. This is to make sure that Snake can be second-order discontinuous and extract a corner.

The total image energy is a weighted combination of the three energy functionals. This energy can be represented as follows

$$E_{image} = w_{line}E_{line} + w_{edge}E_{edge} + w_{term}E_{term} \tag{1.1.4}$$

The three different energy functional ($E_{line}, E_{edge}, E_{time}$) can attract a the contour of Snake to lines, edges, and terminations. A wide range of Snake can be created by adjusting the weights (w_{line} , w_{edge}, w_{term}).

A line functional E_{line} is the image intensity itself. It is defined as,

$$E_{line} = I(x, y) \tag{1.1.5}$$

where $I(x, y)$ is the image function and it is viewed as a function of continuous position variables (x, y), Cheng *et al.* (2007). It is depending on the sign of w_{line} so that the contour or Snake will be attracted to either low level prediction contour or high level of prediction contour.

The edges of the image can be found by the energy functional (1.1.6), Kass *et al.*(1988). This allows Snake model to be attract to contours with large image gradient.

$$E_{edge} = -\left|\nabla I(x, y)\right|^2 \tag{1.1.6}$$

A different edge functional (1.1.7) issued by Kass *et al.*(1988) in order to show the relationship of scale-space continuation to the theory of edge-detection by Marr and Hildreth(1980).

$$E_{edge} = -\left(G_\sigma(x, y) * \nabla^2 I(x, y)\right)^2 \tag{1.1.7}$$

where $G_\sigma(x, y)$represent as a two dimensional Gaussian of standard deviation σ and ∇ is the gradient operator. The functional lies on zero-crossing of $G_\sigma * \nabla^2 I$. The location is minima which defined edges in the Marr-Hildreth theory. The Snake will attract to zero-crossing if we add the edge functional term (1.1.7) to the existing equation (1.1.4). Despite of adding this term to Snake model, it is still constrained by its own smoothness.

Curvature of level lines in equation (1.1.8) is used to find the terminations of line segments and corners in a slightly smoothed image.

$$
\begin{aligned}
E_{term} &= \frac{\partial \theta}{\partial n_{\perp}} \\
&= \frac{\partial^2 C / \partial n_{\perp}^2}{\partial C / \partial n} \\
&= \frac{C_{yy}\, C_x^2 - 2 C_{xy}\, C_x\, C_y + C_{xx}\, C_y^2}{\left(C_x^2 + C_y^2\right)^{3/2}}
\end{aligned}
\tag{1.1.8}
$$

where $C(x, y) = G_{\sigma}(x, y) * I(x, y)$ is the smoothed image, $\theta = \tan^{-1}\left(C_y / C_x\right)$ is the gradient angle, and $n = (\cos\theta, \sin\theta)$ and $n_{\perp} = (-\sin\theta, \cos\theta)$ be unit vectors and perpendicular to the gradient direction. From the iterations in numerical implementation, the combination of E_{edge} and E_{term} full fill the convergent criterion.

1.2 Background of the Problem

The image processing problem in this research is to detect the edges of object on medical resonance image (MRI). The traditional active contour or Snake model has some drawbacks. Basically, it depends on its parameterization. The characteristic of active contour parameterization is limited ability to draw the geometrical regularity of contour. Other problem is the model cannot deal with changes in topology directly and impossible to detect all the objects in an image.

To overcome the problem of Snake, Caselles *et al.* (1993) proposed geometric models of active contours based on the curve evolution theory and the level set method. The proposed model is as follows,

$$
\frac{\partial u}{\partial t} = g(x) \left| \nabla u \right| \left(div \left(\frac{\nabla u}{\left| \nabla u \right|} \right) + v \right) \qquad (r, x) \in \left[0, \infty\right[\times \Re^2 \tag{1.2.1}
$$

with the initial data as,

$$u(0,x) = u_0(x) \qquad x \in \Re^2 \tag{1.2.2}$$

$$g(x) = \frac{1}{1 + (\nabla G_\sigma * g_0)^2} \tag{1.2.3}$$

where

$|\nabla u|$: To controls the interior and exterior of contour

$div\left(\frac{\nabla u}{|\nabla u|}\right)$: To ensures that the grey level at a point increase proportionally to the algebraic curvature. Also responsible in regularizing effect of the model and done its rule in internal energy (1.1.3)

v : A positive real constant and a correction term so that $div\left(\frac{\nabla u}{|\nabla u|}\right) + v$ remains always positive.

$G_\sigma * g_0$: The convolution of the image g_0 and $G_\sigma(x) = C\sigma^{-1/2} \exp\left(-|x|^2 / 4\sigma\right)$

$g(x)$: Stopping function. The aim is to stop the evolving curve when it arrives to the object edges

The improvement of geometric active contour model is not dependent on the curve's parameterization. The implementation level-set based on numerical algorithm (Osher and Sethian, 1988) is allowed changes in the topology automatically .So, the good implementation of geometric active contour is several objects can be detected simultaneously.

Other alternative model proposed by Caselles *et al.*(1997) was geodesic active contour model. It is a geometric model and also energy functional minimization.

Caselles *et al.* (1997) suggested the model of geodesic active contour as follows

$$\frac{\partial u}{\partial t} = |\nabla u| \, div\left(g(x) \frac{\nabla u}{|\nabla u|} \right) + v |\nabla u| g(x) \qquad (t,x) \in [0,\infty[\times \Re^2 \qquad (1.2.4)$$

The real fact is, geodesic active contour model yields the same result as that of a simplified Snake model. It is up to arbitrary constant that depends on the initial parameterization (Goldenberg *et al.* 2001).

However, geodesic active contour also has its drawbacks that we need to consider in this research. The main disadvantage is its nonlinearity that will cause bad implementation.

To linearize the geodesic active contour model, we apply the additive operator splitting (AOS) scheme based on the Weickert *et al.* (1998). It can be defined as follows,

$$u^{k+1} = \frac{1}{m} \sum_{l=1}^{m} \left(I - m\tau A_l \left(u^k \right) \right)^{-1} u^k \qquad (1.2.5)$$

where

k	: Number of iteration
m	: Dimension of the problem
l	: Index running over the dimension
I	: Unit matrix
τ	: Time step

This numerical scheme is an unconditionally stable for nonlinear diffusion for image processing problem. It is consistent, first order and semi-implicit scheme. In this research, we are going to consider the two dimensional model of active contour. So the AOS scheme for two dimensional cases is given by,

$$u^{k+1} = \frac{1}{2}\sum_{l=1}^{2}\left(I - 2\tau A_l\left(u^k\right)\right)^{-1} u^k \qquad (1.2.6)$$

where $A_l = \left(a_{ijl}\right)_{ij}$ (Rosdiana,2012) corresponds to derivative along the l -th coordinate axis. Even though the problem to be overcome is in two dimensional cases, AOS scheme will easily turn that problem into one dimensional case, Weickert (1998). All coordinate axes can be treated in exactly the same manner since the AOS is an additive splitting scheme.

Therefore in this research, we will consider to use the geodesic active contour model based on additive operator splitting scheme to detect the edges of brain and breast tumor on medical images.

1.3 Statement of the Problem

In this study, we use GAC model based on AOS scheme to detect the edges of tumor on MRI images. To implement this model, it needs to be descretized first. Hence we tend to use the finite different method in order to discretize the model. From the discretized version of GAC-AOS model, we could derive the linear system equations. We should get the tridiagonal and diagonally dominant matrix system so that we can solve easily by using AGE and GS method. The solution of the matrix system by AGE and GS method would give the different numerical results in term of time execution, number of iterations, root mean square error, accuracy, rate of convergence, and computational cost. Based on the numerical result performances, the best iterative method between AGE and GS method can be determined.

1.4 Objectives of the Study

The objectives of the study are:

i) To detect the edges of brain and breast tumor on MRI images.

ii) To apply some iterative methods (AGE and Gauss Seidel) to solve the linear system equations.

iii) To compare the numerical analysis of the iterative methods (AGE, and Gauss Seidel) in term of time execution, number of iterations, computational complexity, root mean square error (RMSE), convergence rate and accuracy.

1.5 Scope of the Study

This study will focus on detecting the edges of tumor by using Geodesic active contour (GAC) model based on additive operator splitting (AOS) scheme. The solution for linear system of equation (LSE) can be done by using some iterative methods. The iterative methods under consideration are Gauss-Seidel, alternating group explicit (AGE). This experiment will be applied to brain and breast tumor on MRI images. The MRI images are the real image of two patients from Hospital Kubang Kerian, Kelantan and Hospital Kuala Batas, Pulau Pinang. The algorithm will be run using MatlabR2011a.

1.6 Significance of the Study

From this study, it is hope that we can detect the edges of brain and breast tumor on medical resonance image (MRI). Other than that, the numerical analysis results can be the measurement in proving that AGE method is the best iterative method with accuracy (2, 4) than Gauss Seidel method with accuracy (2, 2).

1.7 The Organization of the Dissertation

This dissertation consists of six chapters. Chapter 1 describes the introduction of active contour models. In this chapter, we included the problem formulation, active contour model under consideration, objectives, scope, and significance of the research.

Chapter 2 focuses on the literature review. This chapter describes the use and application of GAC model by different researchers year by year. There also have descriptions about AOS scheme, finite difference method, Gauss Seidel method, and AGE method. We also explain briefly about the numerical analysis of sequential algorithm and the computational platform used in this research. At the end of the chapter, we show the chart of our research scope.

Chapter 3 describes the discretization process for GAC-AOS model by using finite differences method. From the discretized version of GAC-AOS, we derived the linear system equations. Because of the AOS scheme, the linear system can be solves for two directions separately. At the end of the chapter, we explain the flowchart of the sequential algorithm for edge detection problem on MRI image.

In Chapter 4, we describe the solution of tridiagonal and diagonal matrix system using AGE and GS methods. We show how the formulation of AGE and GS method could solve the matrix system for two directions which are x-direction and y-direction. We also included the computational molecule for each AGE and GS method.

Chapter 5 presents the results of edge detection on MRI images. We analyse the results based on number of iterations, time execution, root mean square error, rate of convergence, accuracy and computational complexity. All the numerical results are shown in the form of table while the visualization results of the captured edge of tumor by contour line are shown by images.

The last chapter is the Chapter 6. In this chapter, we state the conclusions of this research based on the results that we showed in Chapter 5 and relate them with our objectives in Chapter 1. Then, there are some suggestions and recommendations for the future researchers.

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