ADOMIAN DECOMPOSITION METHOD FOR TWO-DIMENSIONAL NONLINEAR FREDHOLM INTEGRAL EQUATION OF THE SECOND KIND

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To my beloved parents

Wan Mohammad Ayub bin Wan Ismail

and

Rohani binti Mohamad.

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ABSTRACT

Nonlinear phenomena's that appear in many applications in science fields such as fluid dynamic, plasma physics, mathematical biology and chemical kinetics can be modeled by integral equation. Nonlinear integral equation usually produces a considerable amount of difficulties. This problem can be handling with some method such as Adomian decomposition method (ADM) and modified Adomian decomposition method (MADM). In this research, ADM and MADM are applied to solve two-dimensional nonlinear Fredholm integral equation of the second kind (FIE). We used ADM to find the exact solution and MADM to find the numerical approximation. From the observation with some example are presented in this research, the first five terms convergent numerical approximations give the good approximation.

ABSTRAK

Fenomena tak linear muncul dalam banyak peggunaan bidang sains seperti bendalir dinamik, fizik plasma, biologi matematik dan kimia kinetic boleh dijadikan model oleh persamaan kamiran. Persamaan kamiran tak linear biasanya menghasilkan nilai yang agak besar. Masalah ini dapat diatasi dengan sesetengah model seperti kaedah penghuraian Adomian (ADM) dan modifikasi kaedah penghuraian Adomian (MADM). Dalam kajian ini, ADM dan MADM akan digunakan untuk menyelesaikan persamaan kamiran Fredholm jenis kedua (FIE) dua-dimensi tak linear. ADM digunakan untuk mencari penyelesaian yang tepat dan MADM pula untuk mencari nilai penghampiran. Daripada pemerhatian dengan beberapa contoh yang telah dibuat didalam kajian ini, nilai penghampiran untuk lima penggal pertama memberikan nilai penghampiran yang terbaik.

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LIST OF SYMBOLS

λ	-	Parameter
α, β	-	Spatial variable
H	-	Hilbert space
$K(x,t)$	-	Kernel
$\|x\|$	-	Norm of x
α	-	Constant
Ω	-	Finite interval $[a,b] \subseteq \Re$
$\Re$	-	Real number
$\mu(t)$	-	Lebesgue measure
$F[u(x,t)]$	-	Nonlinear function
$\hat{u}_t(\omega,t)$	-	Fourier transform

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CHAPTER 1

INTRODUCTION

1.1 Background of the study

An integral equation is an equation in which the unknown function $u(x)$ appears under an integral sign. According to Bocher [1914], the name *integral equation* was suggested in 1888 by du Bois-Raymond. A general example of an integral equation in $u(x)$ is

$$u(x) = f(x) + \int K(x,t)F[u(t)]dt \tag{1.1}$$

where $K(x,y)$ is a function of two variables called the *kernel* of the integral equation. The integral equation can be classified into two classes. First, it is called Volterra integral equation (VIE) where the Volterra's important work in this area was done in 1884-1896 and the second, called Fredholm integral equation (FIE) where the Fredholm's important contribution was made in 1900-1903. Fredholm developed the theory of this integral equation as a limit to the linear system of equations.

There are many problems which can be modeled using integral equation. Examples are as following:

i. Population competition

$$u(x) = f(x) + \int_a^b K\big(x - t, u(s,t)\big)u(t)dt$$

ii. Quantum scatting: close-couple calculations

$$u(x) = f(x) - \lambda \int_0^\infty K_1(x,z) \int_0^\infty K_2(z,t)u(t)dtdz$$

iii. Currents in a superconducting strip

$$f(x) = \frac{1}{\pi} \int_0^1 \frac{t - x}{(t - x)^2 + a^2} u(t)dt$$

iv. Flow round a hydrofoil

$$u(x) = f(x) + \lambda \int_c K(x,t)u(t)dt$$

The most general linear integral equation in $u(x)$ can be written as the following

$$h(x)u(x) = f(x) + \lambda \int_a^{b(x)} K(x,t)F[u(t)]\, dt \tag{1.2}$$

where λ is a scalar parameter. The integral equation is said to be singular if either the domain $a \le x \le b$, $a \le t \le b$ in equation (1.2) of definition is infinite, or if the *kernel*, $K(x,t)$ has a singularity within its region of definition.

Definition 1 (Chama Abdoulkadri, [10])

Let Ω be a measurable set in a measurable space D, and let μ be a positive measure defined on D. The Fredholm integral equations divided into two groups, referred to as Fredholm integral equations of the first and second kind. It has the following general expression:

$$\int_{\Omega} K(x,t,u(t))d\mu(t) = F(x,u(t)) \tag{1.3}$$

where K and F are known functions. K is called the kernel of the integral equation, Ω is a finite interval $[a,b] \subseteq \Re$ and $\mu(t)$ become a Lebesgue measure defined by $d\mu = dt$.

Consider Fredholm integral equation (FIE) of the following form

$$a(x)u(x) - \int_{\Omega} K(x,t,u(t))d\mu(t) = f(x) \tag{1.4}$$

It satisfies the following two conditions:

a) a, K, and F are known functions.

b) u is an known function to be determined.

If $a(x) \equiv 0$ for all x, the equation is FIE of the second kind.

If $a(x) \neq 0$ the equation can be written as

$$u(x) = f(x) + \int_{\Omega} K(x,t,u(t))d\mu(t) \tag{1.5}$$

In the previous researcher, Adomian decomposition method has been used for solving the Volterra integral equation of the second kind. In this research we seek to extend the application of Adomian decomposition method to solving the Fredholm integral equation of the second kind.

1.2 Problem Statement

Recently, the Adomian decomposition method (ADM) and modified Adomian decomposition method (MADM) has been applied for solving systems of linear and nonlinear Volterra integral equation of the second kinds [6, 7].

In [8] this method gives a better result when applied to the Volterra integral equation and it is shown in [11]. In this dissertation, we try to solve the method to solving two-dimensional Fredholm integral equation (FIE) of the second kind in $u(x,y)$. Xie.W and Lin.F [22], stated that

$$u(x,y) = f(x,y) - \int_a^b \int_c^d K(x,y,s,t) F[u(s,t)] ds dt \qquad (x,y) \in D \tag{1.6}$$

is a two-dimensional FIE of the second kind.

where $K(x,y,s,t)$ is a kernel and $u(x,y)$ is in $D = [a,b] \times [c,d]$.

Convergence of the methods will also be studied.

1.3 Objectives

The objectives of this study are as follows:

1) To study the concept of Adomian decomposition method and modified Adomian decomposition method for solving two-dimensional nonlinear Fredholm integral equation of the second kind.
2) To design an algorithm to find the exact solution of two-dimensional nonlinear Fredholm integral equation of the second kind using Adomian decomposition method.
3) To design an algorithm to generate the numerical solution of the two-dimensional Fredholm integral equation of the second kind using modified Adomian decomposition method.
4) To find the convergence of the Adomian decomposition method when applied to a systems of Fredholm integral equation of the second kind.

1.4 Scope of the study

This research will focus on the two-dimensional nonlinear Fredholm integral equation for the second kind. This study will be limited to finding the exact solution and convergence using Adomian decomposition method or the modified Adomian decomposition method. We will use MATHEMATICA 7.0 software to implement the algorithm in this research.

1.5 Significance of study

Recently, Adomian decomposition method and modified Adomian decomposition method were popular among the researchers who studied integral equations. From the findings this dissertation, it is hoped that the present work can be used as a reference for the future study.

1.6 Project Outline

This study consists of five chapters, which are Chapter 1, Chapter 2, Chapter 3, Chapter 4 and Chapter 5. In Chapter 1, discuss about the background of the study, the problem statement, the objectives, the scope of the study and the significance of this study to the other researcher.

Chapter 2 explained some literature and previous work that had been done by other researchers. Chapter 3, the method which will be used in this study and analyzed using some examples of Fredholm integral equations. It consists of the exact and numerical solutions of two-dimensional Fredholm integral equation of the second kind and also the error of the method.

In Chapter 4, the results obtained from this study are summarized. Finally, in Chapter 5, the conclusions and recommendations for further research given.

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